On sodar measurements of dynamic turbulence parameters

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Received July 28, 2003

Presented in this paper are the equations derived for estimating standard errors and 90% confidence intervals in sodar measurements of the turbulence intensity and anisotropy of the wind field. The equations apply to any of the three-channel sensing scheme. Two (direct and indirect) methods of constructing the processing algorithms are also presented. The use of these equations allows one to estimate the degree of uncertainty in the data obtained, that is, to correctly interpret the results of acoustic sensing of the atmosphere. Vertical profiles of dynamic turbulence parameters are determined from sodar data using these equations.

Introduction

Determination of the mean values $M(\cdot)$, standard deviations $\sigma(\cdot)$, asymmetry $\gamma(\cdot)$ and excess $\varepsilon(\cdot)$ coefficients of the longitudinal u and transverse v components of the horizontal wind velocity in the processing system of a Volna-3 sodar were described in Ref. 1. Two methods for estimation of these parameters were considered. In the first one, the so-called direct method, the characteristics sought are estimated from the predetermined ensembles of instantaneous uv-components:

$$u(i) = \sum_{r=1}^{3} u_r V_r(i); \quad v(i) = \sum_{r=1}^{3} v_r V_r(i), \qquad (1)$$

where u_r, and v_r are the coefficients of transition from the radial components of the wind velocity $V_r(i)$ that are directly measured by the sodar to the current uv-coordinates. They are determined by the direction of the mean vector of the horizontal wind velocity and the sensing geometry used.[1] In this processing method, the current values of the uv-components obtained by Eqs. (1) are taken as results of direct measurements. The second, purely indirect, method is based on obtaining the functional relations between the statistical moments of the uv-components and the moments of the radial components V_r from Eqs. (1). It was shown that for practical purposes the random character of the coefficients u_r, and v_r could be neglected.

For more thorough characterization of the atmosphere as a random medium, other, not mentioned above, parameters are also widely used.[2] Thus, different variation coefficients, for example, $I_w = \sigma(w)/M(u)$ and $I_u = \sigma(u)/M(u)$ characterize the turbulence intensity of the corresponding component (w is the vertical component of the wind velocity vector). The degree of anisotropy of the turbulent wind field in the vertical and horizontal direction (with respect to the u-component) can be estimated by the following parameters[2]:

$$A_{wu} = \sigma(w)/\sigma(u),\ A_{wv} = \sigma(w)/\sigma(v),$$
$$A_{vu} = \sigma(v)/\sigma(u).$$

The vertical profiles of I_w measured with a sodar for different classes of thermodynamic stability of the atmosphere are presented in Ref. 3. However, it is difficult to judge on the actual significance of these height dependences of I_w, because neither interval nor point estimates of measurement errors in these profiles are given.

This paper is an extension of Ref. 1. Its aim is to obtain standard errors and 90% confidence intervals for the variation and anisotropy coefficients directly from experimental data for any three-channel sensing schemes and for the two mentioned methods of construction of measurement algorithms. (Below the estimates corresponding to the direct method are marked by the subscript "dir.")

To solve the problem formulated, assume that $V_r(i)$ values measured by a sodar at any fixed height form a set of independent sampled values corresponding to some continuous distribution $W_r(V_r)$. Assume also that the channels for measuring V_r are statistically independent by pair, that is, the distribution of random values of radial components obtained in one channel is not determined by the V_r values in the other channel. As was noted in Ref. 1, for the sensing geometries with separation of measurement channels in space and time used most often this probabilistic dependence must be so weak that it can be neglected for practical purposes.

1. Analysis of turbulence intensity estimates

First, consider the estimate of the turbulence intensity for the longitudinal component:

$$\hat{I}_u = \hat{\sigma}(u)/\hat{M}(u) = \sqrt{\hat{D}(u)}\,/\,\hat{M}(u)\,. \qquad (2)$$

For this purpose, the direct method uses unbiased estimates of the sample mean $\hat{M}_{\text{dir}}$ and the variance

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$$\hat{D}_{\rm dir}(u) = [N/(N-1)]\, m_2(u),$$

where $m_2(\cdot)$ is the central second-order sample moment[4,5]; N is the number of readings of $u(i)$. The similar estimates for the indirect method have the form[1]:

$$\hat{M}(u) = \sum_{r=1}^{3} u_r \hat{M}(V_r) \text{ and } \hat{D}(u) = \sum_{r=1}^{3} u_r^2 \hat{D}(V_r),$$

where $\hat{M}(V_r)$ is the sample mean of the rth radial component of the wind velocity vector calculated by its N_r readouts;

$$\hat{D}(V_r) = [N_r/(N_r-1)]\, m_2(V_r).$$

To study the bias of the estimate $\hat{I}_u$, expand the nonlinear function (2) into a Taylor series in the vicinity of the mean values of its arguments up to the second power terms inclusive.[5] For the direct method after the needed averaging with regard for the above assumptions and unbiasedness of $\hat{M}_{\rm dir}(u)$ and $\hat{D}_{\rm dir}(u)$, we obtain

$$M(\hat{I}_u) = I_u\left[1 - \frac{\varepsilon(u)-1}{8N} + \frac{I_u^2}{N} - \frac{I_u\gamma(u)}{2N}\right] = I_u + O(N^{-1}).$$

Similar equations are also valid for the method of indirect estimation. Note that in the most practical situations $I_u < 1$, and therefore, N usually achieves several tens and higher. Therefore, the bias of this estimate can be neglected.

To find the standard error $\hat{I}_u$, let us use the linearization method.[4,5] Consideration of the corresponding nonlinear terms in this case is unpractical because of the further need in using higher-order sample moments, which are estimated with large errors in case of limited number of observations N. Then, after averaging, we obtain the sought equation valid for both the direct and indirect method of estimating I_u:

$$\sigma(\hat{I}_u) = \frac{1}{M(u)} \times$$
$$\times\left\{\frac{D[\hat{D}(u)]}{4D(u)} + I_u^2 D[\hat{M}(u)] - \frac{\mathrm{cov}[\hat{D}(u),\hat{M}(u)]}{M(u)}\right\}^{1/2}. \quad (3)$$

The equations for variance and covariance of the estimates $\hat{M}_{\rm dir}(u)$ and $\hat{D}_{\rm dir}(u)$ follow from Refs. 1, 4–6:

$$\mathrm{cov}[\hat{D}_{\rm dir}(u), \hat{M}_{\rm dir}(u)] = \mu_3(u)/N,$$

where $\mu_3(u)$ is the third central moment of the u-component;

$$D[\hat{M}_{\rm dir}(u)] = D(u)/N;$$

$$D[\hat{D}_{\rm dir}(u)] = D^2(u)\,[\varepsilon(u) - (N-3)/(N-1)]/N.$$

Restricting the last equation to the terms of the order of $O(N^{-1})$, for the direct method we obtain a simpler version of Eq. (3):

$$\sigma(\hat{I}_{u\,\rm dir}) = I_u\sqrt{\varepsilon(u) - 1 + 4I_u^2 - 4I_u\gamma(u)}\,/\,2\sqrt{N}.$$

For the indirect method, the variances of $\hat{M}(u)$ and $\hat{D}(u)$ can be found according to Ref. 1:

$$D[\hat{M}(u)] = \sum_{r=1}^{3} u_r^2 D[\hat{M}(V_r)] = \sum_{r=1}^{3} u_r^2 D(V_r)/N_r,$$

$$D[\hat{D}(u)] = \sum_{r=1}^{3} u_r^4 D[\hat{D}(V_r)],$$

where according to Refs. 4–6:

$$D[\hat{D}(V_r)] = D^2(V_r)[\varepsilon(V_r) - (N_r-3)/(N_r-1)]/N_r.$$

The equation for covariance of these estimates follows directly from their definition and the results of Ref. 4 for $\mathrm{cov}[\hat{D}(V_r), \hat{M}(V_r)]$ is as follows:

$$\mathrm{cov}[\hat{D}(u),\hat{M}(u)] = \sum_{r=1}^{3} u_r^3 \mu_3(V_r)/N_r,$$

where $\mu_3(V_r)$ is the third central moment of the corresponding radial component.

For a comparison of the standard errors of the two methods for estimation of I_u considered, assume first that the number N_r of significant readouts of $V_r(i)$ in each sodar measurement channel is the same and equals to N. In Ref. 1 it was shown that the estimates of the mean in this case are identical, in particular,

$$D[\hat{M}_{\rm dir}(u)] = D[\hat{M}(u)].$$

It is also valid that

$$\mathrm{cov}[\hat{D}(u),\hat{M}(u) = \mathrm{cov}[\hat{D}_{\rm dir}(u),\hat{M}_{\rm dir}(u)].$$

But, according to Ref. 1, $D[\hat{D}_{\rm dir}(u)] > D[\hat{D}(u)]$. Finally, the condition $\sigma(\hat{I}_{u\,\rm dir}) > \sigma(\hat{I}_u)$ is always true. However, in practice the increase of $\sigma(\hat{I}_{u\,\rm dir})$ with respect to $\sigma(\hat{I}_u)$ is affected much more strongly by the following factor: at a weak echo signal in at least one of the sodar radial channels in some sensing cycles, some instantaneous values of the uv-components (1) remain uncalculated at realization of the direct method.[1] As a result, the number N of significant readouts of $u(i)$ can be much smaller than even the minimum number of the readouts N_r obtained, that is, some information may be lost. At the same time, the indirect method always uses the entire ensemble of data on $V_r(i)$ obtained. Finally, this leads to the additional increase of the random error of the direct method with respect to the indirect one. The influence of this factor is most significant for the parameters estimated using short averaging times.

The statistical characteristics of the turbulence intensity estimation for the vertical components $\hat{I}_w$ are obtained similarly to the case of $\hat{I}_u$. Note only that in using the most typical sensing geometry with one vertical channel and at $N_r = N$ the estimates $\hat{I}_{w\,\mathrm{dir}}$ and $\hat{I}_w$, in contrast to $\hat{I}_{u\,\mathrm{dir}}$ and $\hat{I}_u$, are fully identical. Otherwise, because of the factor mentioned above, $\sigma(\hat{I}_{w\,\mathrm{dir}}) > \sigma(\hat{I}_w)$.

2. Analysis of the turbulence anisotropy estimates

Consider the estimate of turbulence anisotropy in the transverse direction[2]:

$$\hat{A}_{vu} = \hat{\sigma}(v)/\hat{\sigma}(u) = \sqrt{\hat{D}(v)}\,/\,\sqrt{\hat{D}(u)}, \qquad (4)$$

where $\hat{D}(v)$ is similar to the above estimates $\hat{D}(u)$, that is,

$$\hat{D}_{\mathrm{dir}}(v) = [N/(N-1)]\, m_2(v)$$

and

$$\hat{D}(v) = \sum_{r=1}^{3} v_r^2 \hat{D}(V_r).$$

In this case, the bias $\hat{A}_{vu}$ for both of the processing methods can be neglected, because the following equation is valid:

$$M(\hat{A}_{vu}) = A_{vu} + O(N^{-1}).$$

This statement can be proved similarly to that for the estimate I_u in Section 1. The equation for the standard error in determination of the parameter A_{vu} can be obtained by the linearization method:

$$\sigma(\hat{A}_{vu}) = \frac{1}{2\sigma(u)\sigma(v)} \times$$

$$\times \left\{ D[\hat{D}(v)] + A_{vu}^4 D[\hat{D}(u)] - 2A_{vu}^2 \operatorname{cov}[\hat{D}(u), \hat{D}(v)] \right\}^{1/2}. \qquad (5)$$

For the indirect method the equation for $D[\hat{D}(u)]$ is presented above, and

$$D[\hat{D}(v)] = \sum_{r=1}^{3} v_r^4 D[(V_r)].$$

The equation for covariance of these estimates follows directly from their definitions and the above assumptions:

$$\operatorname{cov}[\hat{D}(u), \hat{D}(v)] = \sum_{r=1}^{3} u_r^2 v_r^2 D[\hat{D}(V_r)].$$

The equation for $D[\hat{D}_{\mathrm{dir}}(u)]$ if used in the direct measurement method is presented in Section 1. For $D[\hat{D}_{\mathrm{dir}}(v)]$ it is quite similar. Deriving the equation for covariance of these estimates in this case is a more complicated problem. Therefore, let us consider this in a more detail. First, with regard for their unbiasedness, one can write

$$\operatorname{cov}[\hat{D}_{\mathrm{dir}}(u), \hat{D}_{\mathrm{dir}}(v)] =$$

$$= M[\hat{D}_{\mathrm{dir}}(u)\hat{D}_{\mathrm{dir}}(v)] - D(u)D(v). \qquad (6)$$

Passing on from the u-components to their centered values $\dot{u}$ and replacing the estimates $m_2(u)$, $m_2(v)$ in the equations for $\hat{D}_{\mathrm{dir}}(u)$, $\hat{D}_{\mathrm{dir}}(v)$ by their statistical equivalents,[4,5] one obtains

$$M[\hat{D}_{\mathrm{dir}}(u)\,\hat{D}_{\mathrm{dir}}(v)] = (N/N-1)^2\, M\,[a_2(\dot{u})\, a_2(v) -$$

$$- a_2(\dot{u})\,\hat{M}^2(v) - a_2(v)\,\hat{M}^2(\dot{u}) + \hat{M}^2(\dot{u})\,\hat{M}^2(v)], \qquad (7)$$

where $a_2(\cdot)$ is the corresponding initial second-order sample moment. In this case, it is fulfilled that

$$M\,[a_2(\dot{u})] = D(u) \text{ and } M[a_2(v)] = D(v).$$

Then consider individual terms of Eq. (7):

$$M[a_2(\dot{u})a_2(v)] = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} M[\dot{u}^2(i)v^2(j)] =$$

$$= \frac{1}{N} M(\dot{u}^2 v^2) + \frac{N-1}{N} D(u)D(v).$$

The equation for $M(\dot{u}^2 v^2)$ can be obtained using Eq. (1) and the results of Ref. 1:

$$M(\dot{u}^2 v^2) = D(u)D(v) + \sum_{r=1}^{3} u_r^2 v_r^2 \mu_4(V_r) -$$

$$-\sum_{r=1}^{3} u_r^2 v_r^2 D^2(V_r) + 4\sum_{r<k} u_r v_r u_k v_k D(V_r)D(V_k).$$

It is also valid that

$$M\,[a_2(\dot{u})\,\hat{M}^2(v)] = \frac{1}{N^3}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N} M[\dot{u}^2(i)v(j)v(l)] =$$

$$= \frac{1}{N^2} M(\dot{u}^2 v^2) + \frac{N-1}{N^2} D(u)D(v) = \frac{M[a_2(\dot{u})a_2(v)]}{N},$$

$$M\,[a_2(v)\,\hat{M}^2(\dot{u})] = \frac{1}{N^3}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N} M[v^2(i)\dot{u}(j)\dot{u}(l)] =$$

$$= \frac{1}{N^2} M(\dot{u}^2 v^2) + \frac{N-1}{N^2} D(u)D(v) = \frac{M[a_2(\dot{u})a_2(v)]}{N};$$

$$M\,[\hat{M}^2(\dot{u})\hat{M}^2(v)] =$$

$$= \frac{1}{N^4}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N}\sum_{m=1}^{N} M[\dot{u}(i)\dot{u}(j)v(l)v(m)] =$$

$$= \frac{1}{N^3} M(\dot{u}^2 v^2) + \frac{N-1}{N^3} D(u)D(v) + \frac{2(N-1)}{N^3} M^2(\dot{u}v) =$$

$$= \frac{1}{N^2} M[a_2(\dot{u})a_2(v)] + \frac{2(N-1)}{N^3} M^2(\dot{u}v),$$

where, as follows from Ref. 1,

$$M^2(\dot{u}v) = \operatorname{cov}^2(u,v) =$$

$$= \sum_{r=1}^{3} u_r^2 v_r^2 D^2(V_r) + 2\sum_{r<k} u_r v_r u_k v_k D(V_r)D(V_k).$$

Substituting the obtained equations into Eqs. (6) and (7) and after some transformations, one has

$$\mathrm{cov}[\hat{D}_{\mathrm{dir}}(u),\hat{D}_{\mathrm{dir}}(v)] =$$

$$= \sum_{r=1}^{3} u_r^2 v_r^2 D[\hat{D}(V_r)] + \frac{4}{N-1}\sum_{r<k} u_r v_r u_k v_k D(V_r) D(V_k) =$$

$$= \mathrm{cov}[\hat{D}(u),\hat{D}(v)] + \frac{4}{N-1}\sum_{r<k} u_r v_r u_k v_k D(V_r) D(V_k).$$

So all the terms in Eq. (5) needed for calculation of the standard error of measurement of the turbulence anisotropy in the transverse direction by a sodar are determined. For other directions these equations can be obtained in a similar way. Note that the errors in the direct method are again higher than that characteristic of the indirect one. This is largely determined by the above-mentioned factor of the probable decrease in the number of instantaneous readouts N of the uv-components with respect to the potentially achievable number.[1]

3. Interval estimates of the dynamic turbulence parameters

To have a clear idea of the accuracy and reliability of the considered point estimates $\hat{g}$ of the dynamic turbulence parameters (2) and (4), let us pass on to the corresponding interval characteristics. Similarly to Refs. 1 and 6, let us realize the known approach that employs the properties of 90% confidence intervals. Using the methods and criteria from Ref. 6, determine the minimum sample size $N_{\min}$, starting from which the measurement result on the parameter g can be presented in the following form with the 90% confidence probability

$$g_{0.9} = \hat{g} \pm 1.6\sigma(\hat{g}). \qquad (8)$$

Similarly to Refs. 1 and 6, as the initial distribution of the radial components $W_r(V_r)$ we used the uniform distribution and the Gauss, Laplace, and Rayleigh distributions (with the parameters characteristic of sodar measurements). The simulation showed that the resulting distributions of all errors $W(\hat{g})$ have positively asymmetric behavior. This is also valid for other distributions in view of strictly positive values of these parameters. This determines the asymmetric arrangement of 5-% ($p_{0.05}$) and 95-% ($p_{0.95}$) quantiles about the center of $\hat{g}$ grouping. Consequently, the accuracy of calculation of the lower L_d and upper L_u confidence boundaries by use of approximate equation (8) will be different, especially, at small values of N_r or N. This situation is similar to that in estimation of the excess of the radial components considered in Ref. 6. In such a case, the values of L_d are always smaller than the corresponding values of $p_{0.05}$, that is, the length of the left part of the 90% confidence interval obtained by Eq. (8) can be considered as an upper estimate with respect to the true value. On the other hand, the value of L_u is determined with a much smaller error. Therefore, in this case, as in Ref. 6 for estimation of the excess of the radial components, the selection of $N_{\min}$ was based on the comparison of $p_{0.95}$ and L_u.

The simulation suggests that in the most practical situations Eq. (8) for L_u is valid accurate to 15%, if $N_r \geq N_{\min} = 10$ is fulfilled for all sodar channels. For the direct measurement method, the condition is more strict, $N \geq 10$. Note that at a certain combination of the types and parameters of $W_r(V_r)$ the estimated length of the left part of the confidence interval may exceed the true value by no more than 15%. At a smaller N_r or N, one can use the corresponding standard error as a measure of uncertainty in the measured values of g.

4. Experimental results

Let us illustrate the above-said using, as an example, the vertical profiles of some dynamic turbulence parameters measured with a Volna-3 sodar by the two methods considered above (Figs. 1–3).

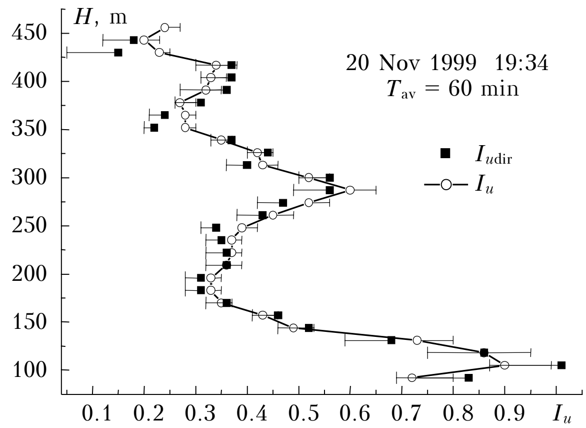

Fig. 1. Turbulence intensity I_u.

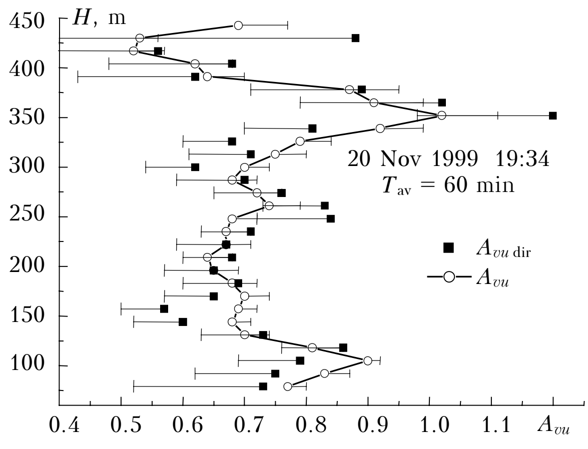

Fig. 2. Turbulence anisotropy in the transverse direction A_{vu}.

The measured turbulence intensity $\hat{I}_u$ is denoted in Fig. 1 as I_u, the anisotropy in the transverse direction

$\hat{A}_{vu}$ is denoted as A_{vu} (Figs. 2 and 3), and that in the vertical direction is denoted as A_{wu} (Fig. 3).

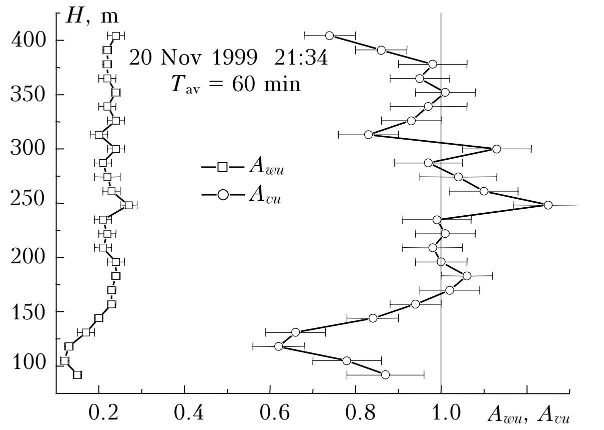

Fig. 3. Turbulence anisotropy in the vertical A_{wu} and horizontal A_{vu} directions.

Not pretending to a detailed physical interpretation of the data obtained, compare the used estimation methods and demonstrate their actual accuracy characteristics. For this purpose, let us draw the corresponding 90% confidence intervals on the plots using Eq. (8). In Figs. 1 and 2, only the left part of the 90% confidence interval is shown for the direct method, and only the right one is shown for the indirect method. Note that the profiles shown in the figures and Figs. 2–5 of Ref. 1 were obtained at the same time and the same place (Tomsk suburbs) and for the same averaging interval $T_{av} = 60$ min. Therefore, the vertical profile of I_u is fully determined by the height dependences of the mean $M(u)$ and the standard deviation $\sigma(u)$ of the u-component that are shown in Figs. 2 and 3 of Ref. 1. In this case, a rather complex profile of $M(u)$ shows the principal effect on I_u.

The large values of I_u in the lower part of the height range are likely explained by the effect of the surface at the measurement site. It is worthy to note quite close agreement and correlation between the data obtained by the direct and indirect methods, which is a consequence of the good agreement of measured $M(u)$ with $M(u)_{\rm dir}$ and $\sigma(u)$ with $\sigma(u)_{\rm dir}$ as was noted in Ref. 1. Only at the height $H \approx 352$ m the confidence intervals for I_u and $I_{u\,\rm dir}$ do not overlap. However, the difference between the point values of I_u and $I_{u\,\rm dir}$ is insignificant – only 0.06. (Similar fact for this height was also noticed in Ref. 1 in analysis of $\sigma(u)$ measurements). In general, it should be noted that the measurements of the turbulence intensity by the direct method are characterized by a somewhat higher uncertainty in the data obtained as compared to the indirect approach. This is mostly determined by the factor noted above in analysis of the estimates $\hat{I}_u$ in Section 1.

In principle, similar conclusions are also valid for estimation of the vertical profiles of A_{vu} (see Fig. 2). Again, the deviations of A_{vu} from $A_{vu\,\rm dir}$ are either insignificant or their confidence intervals overlap. But in this case the direct measurements of these parameters are characterized by a far higher uncertainty of the data obtained as compared to the indirect approach. This is especially pronounced in the upper part of the height range as the signal-to-noise ratio decreases.

It is also worth noting the significant difference of the coefficients A_{vu} from unity at all heights (except for $H \approx 352$ m), which is indicative of the lack of isotropy along the considered direction at a given time. However, 2 hr later the situation in the atmosphere changed. It is illustrated in Fig. 3, which shows the profiles of A_{wu} and A_{vu} obtained by the indirect method. Thus, in the range of heights roughly from 157 to 378 m the state of the wind field in the horizontal direction can already be considered as isotropic, but this is not true for the vertical direction. Note that the profiles of A_{wu} almost did not change with time, that is, stable vertical anisotropy was observed. So, according to the sodar data, the wind turbulence during the period of measurement had a different character in the vertical and horizontal directions.

Thus, the equations presented in this paper can be used to estimate the uncertainty in the dynamic turbulence parameters measured with a sodar, which allows correct interpretation of the data of acoustic sensing of the atmosphere.

Acknowledgments

The author is thankful to S.L. Odintsov for useful discussions.

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