# RELATIONS OF THE EXPANSION COEFFICIENTS OF THE SCATTERING PHASE MATRIX ELEMENTS OVER THE GENERALIZED SPHERICAL FUNCTIONS AND MICROSTRUCTURE PARAMETERS OF THE NEAR-GROUND AEROSOL

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*Based on most common models of the low atmospheric aerosol we have studied the information content of the expansion coefficients of the scattering phase matrix elements over the generalized spherical functions. By modeling the stochastic relations among the informative characteristics of the expansion coefficients and aerosol microstructure parameters, under conditions of aerosol transformations, we have estimated the corresponding correlation coefficients. Regression equations are constructed for estimates of the effective radii of particles in aerosol fractions and their mixtures, as well as for relative content of the components at different stages of aerosol transformation.*

It is known quite well that the elements of scattering phase matrices of aerosol have high information content regarding the aerosol microstructure and its optical properties.[1] However, if the solution of the equation of radiation transfer uses expansions over the orthogonal basis of the generalized spherical functions (GSF), the elements of the scattering phase matrix (SPhM) manifest themselves directly in the expansion coefficients.[2,3] In this connection, it is of practical interest to study the information content of the expansion coefficients as well as their relations to the aerosol microstructure. This would certainly be helpful when developing optical methods for identifying the aerosol states during its transformations.

The aim of this work is to analyze the distributions of the expansion coefficients and reveal their properties that are closely related, either stochastically or analytically, to the effective size of aerosol particles (and its fractions). We also aimed in this study at revealing how these coefficients may be connected with the input parameters determining the aerosol transformations. It was among our primary goals either to obtain regression functions under conditions of accumulation of the measurement errors and errors due to incorrect *a priori* information.

## MODELS OF THE NEAR-GROUND AEROSOL

The objects of our study are most common models of the low atmospheric aerosol recommended by the International Association for Meteorology and Atmospheric Physics (IAMAP).[4] According to Ref. 4, different stages of aerosol transformation are characterized by changes in the ratios of main aerosol components: D (dust-like particles), W (water-soluble particles), S (soot, carbon aerosol of anthropogenic origin). During the process of aerosol transformation from the continental model C (continental) to the urban model U (urban/industrial), relative volume content $C_v$ of the main aerosol components, D, W, and S, vary within the limits of 0.70–0.17, 0.29–0.61, and 0.01–0.22, respectively. Using data from Refs. 4–7, we have interpolated the relations between D, W, and S when the content of the dust component $C_v^{\rm D}$ varies from 0.05 to 0.9. In addition to aerosol models of C and U (see Ref. 4) types, the models with the following values of $C_v^{\rm D}$, $C_v^{\rm W}$, and $C_v^{\rm S}$ are considered: a) 0.05; 0.65; 0.30; b) 0.30; 0.55; 0.15; c) 0.50; 0.44; 0.06; d) 0.90; 0.10; 0.00. Here and below, like in Ref. 4, the term "aerosol modelB is used to denote each possible set of aerosol components D, W, S obtained as a result of interpolation.

Aerosol components have the following values of the refraction and absorption indices at the wavelength $\lambda = 0.55$ μm: $n = 1.53$ (D and W) and 1.75 (S); $\varkappa = 0.008$ (D), 0.06 (W), 0.44 (S). Distributions of particles over radii $r$ of the $k$th aerosol component ($k$ = D, W, S) are approximated by the single-mode lognormal law

$$f_k(r) = \frac{1}{\sigma_k \sqrt{2\pi}} \frac{1}{r} \exp\left[ -\frac{\ln^2 (r/r_0^k)}{2\,\sigma_k^2} \right], \tag{1}$$

where the parameter $r_0^k$ takes the values 0.5, 0.005, and 0.0118 μm for the components D, W, and S, respectively; $\sigma_k = 1.09527$ (D and W) and 0.69315 (S). The dust component describes the coarse-disperse fraction with the effective radius, i.e., the ratio of the third moment of the distribution to the second one,

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$r_{eff}^{D}$ = 10.033 μm. The components W and S describe the fine aerosol fraction ($r_{eff}^{W}$ = 0.1003 μm, $r_{eff}^{S}$ = = 0.0395 μm). Each of the aerosol models is a polydisperse mixture of the D, W, and S components with a three-mode size-distribution function

$$f(r) = \sum_{k=1}^{3} C_k f_k(r) , \qquad (2)$$

effective radius of particles

$$r_{eff} = \frac{\sum_{k=1}^{3} C_k m_3^k}{\sum_{k=1}^{3} C_k m_2^k} = \frac{1}{\sum_{k=1}^{3} C_v^k r_{eff}^k} \qquad (3)$$

and modes $r_m^k = r_0^k \exp(-\sigma_k^2)$. In Eqs. (2) and (3), $C_k = 1 \Big/ \left( \frac{m_3^k}{C_v^k} \sum_{k=1}^{3} \frac{C_v^k}{m_3^k} \right)$ is the weighting factor; $m_2^k$ = $= (r_0^k)^2 \exp(2\sigma_k^2)$ and $m_3^k = (r_0^k)^3 \exp(4.5\sigma_k^2)$ are the second and the third moments of the $k$th distribution.

## ANALYSIS OF THE EXPANSION COEFFICIENTS

In the approximation of spherical symmetric particles, the scattering phase matrix of aerosol is a partitioned diagonal matrix

$$\| x_{ij}(\theta) \| = \left\| \begin{matrix} x_{11} & x_{12} & 0 & 0 \\ x_{21} & x_{22} & 0 & 0 \\ 0 & 0 & x_{33} & x_{34} \\ 0 & 0 & x_{43} & x_{44} \end{matrix} \right\| . \qquad (4)$$

Taking into account the symmetry of the elements $x_{ij}$, the relations $x_{11} = x_{22}$, $x_{33} = x_{44}$, $x_{12} = x_{21}$, $x_{43} = -x_{34}$ are valid. In this case the scattering phase matrix is completely defined by four angular functions: the diagonal, $x_{11}(\theta)$ and $x_{44}(\theta)$, and off-diagonal, $x_{12}(\theta)$ and $x_{34}(\theta)$, ones. The element $x_{11}(\theta)$ is normalized according to the expression $\frac{1}{2} \int_{-1}^{1} x_{11}(\mu)\, d\mu = 1$ where $\mu = \cos\theta$, $\theta$ is the scattering angle.

For each of the aerosol models the scattering phase matrix is presented as a weighted average characteristic of a mixture of three components according to the expression

$$x_{ij}(\theta) = \sum_{k=1}^{3} C_v^k \frac{\langle Q_k \rangle\, r_{eff}}{\langle Q \rangle\, r_{eff}^k} x_{ij}^k(\theta) , \qquad (5)$$

where $\langle Q_k \rangle$ is the section-averaged scattering efficiency factor for the $k$th component:

$$\langle Q_k \rangle = \frac{1}{m_2^k} \int_0^\infty Q_k r^2 f_k(r)\, dr , \qquad (6)$$

$\langle Q_k \rangle$ is the corresponding value for the mixture.

Figure 1 illustrates the elements $x_{11}(\theta)$ of the scattering phase matrix calculated by the Mie formulae[8] for the initial components D, W, S of the C and U models of the near-ground aerosol. It is natural that the dust component D has a strongly forward peaked scattering phase function. For the dust component the asymmetry coefficient, that is the ratio fluxes scattered into the forward and backward hemispheres, equals 22.1. The scattering phase functions of the W and S components are less forward peaked, have similar angular behavior at lower values of the asymmetry coefficient (8.1 and 2.7, respectively).

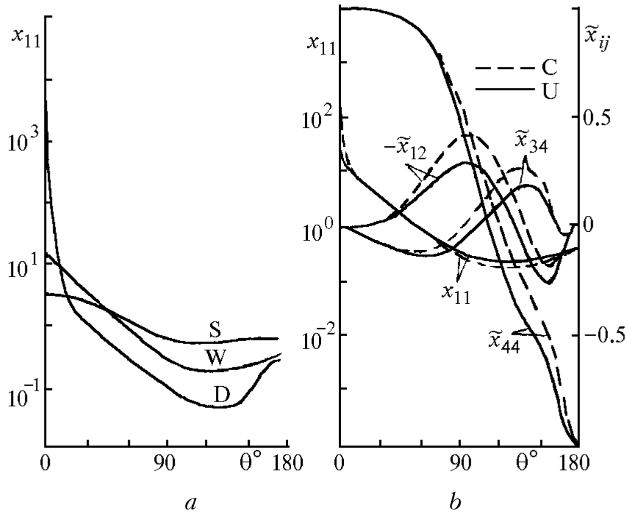


*FIG. 1. The elements of the scattering phase matrix for the components* D, W, and S *(a) and aerosol models* C and U *(b) for the lower atmosphere of the Earth (* $\tilde{x}_{ij} = x_{ij}/x_{11}$*).*

The elements of the scattering phase matrix calculated by Eq. (5) for the models of urban (U) and continental (C) aerosol are presented in Fig. 1*b*. As seen from the figure, angular functions corresponding to different states of aerosol have qualitatively similar form and only weakly resemble the variations of the input parameters.

Presenting optical information in the form of the distributions of the expansion coefficients of scattering phase matrix elements, over the GSFs, over the discrete argument $l$ is more convenient for analyzing the aerosol transformations. When calculating the expansion coefficients, taking into account the symmetry relations $x_{11} = x_{22}$ and $x_{33} = x_{44}$ which are valid in the approximation of spherical particles, one can restrict oneself to two generalized spherical functions, $P_{00}^l(\mu)$ and $P_{20}^l(\mu)$. The expansion coefficients are calculated by the following formula[3]:

$$x_l^{ij} = \frac{2l+1}{2} \int_{-1}^{1} x_{ij}(\mu) P_{00}^l(\mu) d\mu, \; ij = \{11, 44\}; \quad (7)$$

$$x_l^{ij} = \frac{2l+1}{2} \int_{-1}^{1} x_{ij}(\mu) P_{20}^l(\mu) d\mu, \; ij = \{12, 34\}. \quad (8)$$

Calculation of the integrals (7) and (8) faces some difficulties due to strongly oscillating behavior of the functions $P_{mn}^l(\mu)$ at large $l$. We used the technique proposed in Ref. 9. This techniques guarantees a high reliability of the calculations.

Figure 2 presents the distributions over $l$ of the expansion coefficients of the diagonal elements $x_{11}(\theta)$ and $x_{44}(\theta)$ of the scattering phase matrix. The distributions are obtained by Eqs. (7) for the initial components D, W, S. Besides, the figure depicts the distribution for six models of the near-ground aerosol that reflect different stages of the aerosol transformations. For $l \geq 10$, distributions of the coefficients $x_{11}(\theta)$ and $x_{44}(\theta)$ coincide within the limits of the graphical representation errors. Some differences may be observed only in the domain of small values of $l$ (curve *4'* corresponds to $x_l^{44}$). So the further analysis is performed for the distributions of $x_l^{11}$; the index "11B at $x_l^{11}$ is omitted.

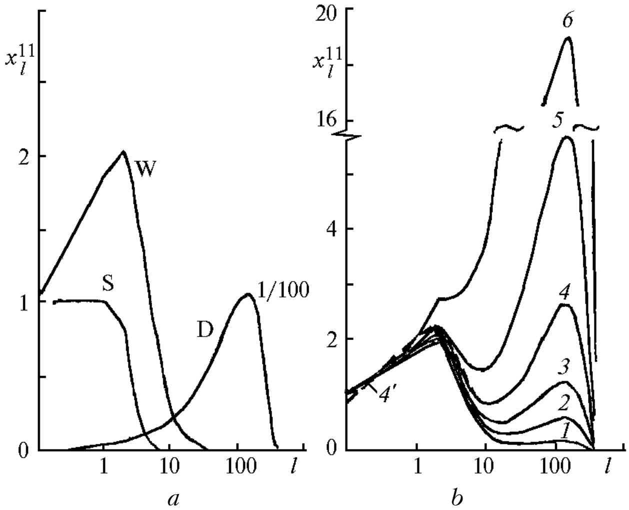


*FIG. 2. Distributions over l of the coefficients of the expansion of the diagonal elements of the scattering phase matrix of the components* D, W, S over the GSF *(a) and of the aerosol models (b) at different stages of the aerosol transformation:* $C_v^D = 0.05$ *(1);* 0.17 (U) *(2);* 0.3 *(3);* 0.5 *(4, 4');* 0.7 (C) *(5);* 0.9 *(6).*

As seen from Fig. 2, the aerosol components as well as different aerosol models have considerably different distributions of the coefficients $x_l$. The components D, W, and S (Fig. 2*a*) correspond to the single-mode distributions with separate positions of $l_m$ maxima. For instance, the integer $l_m$ values for the components S and W with the effective diffraction parameters $\rho_{eff} = 2\pi r_{eff}/\lambda = 0.45$ and 1.14 equal to 1 and 2, respectively; for the D component ($\rho_{eff} = 114.5$), the values of $l_m$ are ~ 135–140.

The process of aerosol transformation visually manifests itself in deformation of the distributions $x_l$. constructed using different models from Fig. 2*b*. The content $C_v^D$ of the dust component ($0.05 < C_v^D < 0.90$) is the input parameter governing different stages, *1–6*, of the aerosol transformation. One can see that, in contrast to the components D, W, and S, the distributions of $x_l$ for the aerosol mixtures have a characteristic two-mode behavior. It is evident that the first maxima of the distribution is a sum of two coinciding maxima and reflects the presence of S and W components of the aerosol simultaneously; the second maximum means the presence of the dust component D. Here the components W and S are considered jointly as the fine fraction; the D component is considered as the coarse-dispersion fraction. It should be expected that the presence of an additional fraction with the effective diffraction parameter of the intermediate value (~10) leads to a three-mode distribution. Preliminary estimates show that the possibility of distinguishing among different components of a mixture by the distributions of expansion coefficients may be practical only if the ratio of effective radii of the fractions exceeds certain limiting value $K_{lim} = 4–5$. For the components W and S, the ratio is 2.53 what is insufficient for their separation.

According to calculations, the positions of maxima $l_{mk}$ do not depend on the relative content of the components as well as on the variation of the efficient values of optical constants $n$ and $k$ within the frames of the accepted model of aerosol transformation. Weak variability of the maxima positions is indicative of a functional connection between $l_{mk}$ and the efficient radii of particles $r_{eff\,k}$ of the aerosol fractions under study. The connection can be represented as an estimate $r_{eff\,k} \sim c\lambda l_{mk}$ where $c = 0.11–0.14$. Here $k = \{1, 2\}$ is the number of a fraction.

The ratio of amplitudes of $x_l$ at the corresponding maxima gradually varies with variation of $C_v^D$. At limiting values of $C_v^D$ which are equal to 0.05 and 0.9, the distributions of the coefficients become similar to the single-mode ones what is characteristic of the aerosol components.

The process of aerosol transformation, as seen from Fig. 2*b*, was monitored at the characteristic points corresponding to zero values of the first and second derivatives. The characteristics monitored are: the value of the expansion coefficient in the second maximum $x_l(l_{m2})$, the value of the expansion coefficient in the minimum $x_l(l_{min})$, the position of the minimum $l_{min}$, the slope, tan α, at the inflection point between the minimum and the second maximum. These information bearing features are denoted below as A, B, C, and D, respectively.

## ANALYSIS OF THE STOCHASTIC RELATIONS

In the general case, each of the above information bearing features may be affected by a number of random factors such as the calculation errors $\delta_{calc}$, measurement errors in the matrix's elements $\delta_{meas}$, inaccuracies of assigning the input parameters of the model $C_v^D$, $C_v^W$, or $r_{eff}$; so their ratios should be characterized with the use of mathematical statistics concepts.[11] Analysis of calculated data indicates that the information bearing features, $Y = \{x_l(l_{m2}), x_l(l_{min}), l_{min}, \tan\alpha\}$ are nonlinear functions of each of the input parameters $X = \{\delta_{calc}, \delta_{meas}, C_v^D, C_v^W, r_{eff}\}$. However, in the neighborhoods of the fixed values of mathematical expectation of the argument, the nonlinear function $y = f(x)$ can be approximated by a linear function whose derivative $f'_x$ defines the value of the dimensionless coefficient of the sensitivity of a characteristic to variations in the input parameter[10]:

$$\xi = \frac{x}{y} f'_x = \frac{\Delta y / y}{\Delta x / x} . \qquad (9)$$

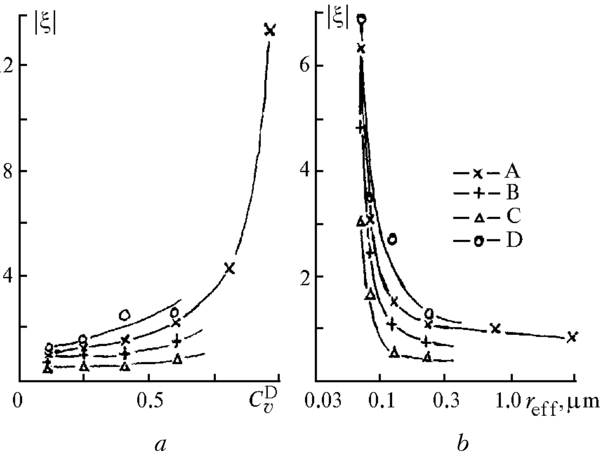


*FIG. 3. The sensitivity coefficients of the informative characteristics* (A, B, C, D) *to input parameters* $C_v^D$ *(a) and* $r_{eff}$ *(b).*

Coefficients of sensitivity which are presented in Fig. 3 enable one to estimate relative limits $\Delta y/y$ of variation for every information bearing feature $n$, given the parameter of the model $x$ and the range of its variation $\Delta x$. For instance, if the aerosol is transformed from the continental model to the model defined by the concentration $C_v^D = 0.5$, one can easily estimate variations using Fig. 3*a*: 70% for $x_l(l_{m2})$, 50% for $x_l(l_{min})$, 30% for $l_{min}$. The most sensitive characteristic is tanα; however, as it is demonstrated below, it can be used only in some cases. Figure 3*b* that presents the data on the sensitivity of the informative characteristics to variation of $r_{eff}$ does not require additional explanations.

Let us estimate the effect of calculation errors $\delta_{calc}$ upon the values of $C_v^D$ and $r_{eff}$. Relative error in calculations of the expansion coefficients by the technique from Ref. 9 does not exceed 1% for $l < 40$ and 10% for $l > 140$. Assuming that the relative error of determining $x_l(l_{m2})$ is 10% and that for $x_l(l_{min})$ it is 1%, calculation errors $\delta_{calc}$ were estimated for $l_{min}$ and tanα. For instance, $\delta_{calc}$ varied from 3% (model U) to 12% (model C); for tanα , it was ~3%. The values $\delta_{calc}$ obtained for each of the characteristic at different stages of the aerosol transformation, with allowance for the sensitivity coefficients $|\xi|$, were then used to determine the corresponding relative errors of input parameters. The errors of $C_v^D$ and $r_{eff}$ that are caused by calculation errors are given in Tables I and II.

*TABLE I. Calculation errors* $C_v^D$, *%, in different ranges for the characteristics* A, B, C, *and* D.

| $C_v$ | 5–17 | 17–30 | 30–50 | 50–70 | 70–90 | 90–100 |
|---|---|---|---|---|---|---|
| A | 9.83 | 7.82 | 6.95 | 4.61 | 2.33 | 0.75 |
| B | 1.28 | 1.02 | 1.00 | 0.65 | – | – |
| C | 8.40 | 9.20 | 16.7 | 10.5 | – | – |
| D | 2.73 | 2.11 | 1.15 | 1.19 | – | – |

*TABLE II. Calculation errors* $r_{eff}$, *%, in different ranges for the characteristics* A, B, C, *and* D.

| $r_{eff}$, μm | 0.063–0.075 | 0.075–0.094 | 0.094–0.151 | 0.151–0.299 | 0.299–0.920 | 0.920–10.03 |
|---|---|---|---|---|---|---|
| A | 1.57 | 3.25 | 6.47 | 9.10 | 9.49 | 11.82 |
| B | 0.20 | 0.42 | 0.92 | 1.28 | – | – |
| C | 1.34 | 3.80 | 15.4 | 20.8 | – | – |
| D | 0.44 | 0.87 | 1.07 | 2.34 | – | – |

As seen from the Tables, for the diagonal elements of the matrix, errors in the input parameters $C_v^D$ and $r_{eff}$ connected with calculation errors of $x_l$ vary approximately from one percent for B to several percents for A and D characteristics; in a particular case, they reach 20.8% (for C in the range $0.151 < r_{eff} < 0.299$ μm). Thus, in the cases A, B, and D, the calculation errors $\delta_{calc}$ have a comparatively weak effect upon the value of the input parameters.

In real experiments, together with the above-mentioned errors, one should take into account random errors $\delta_{meas}$ of measurements of the angular functions of the matrix elements. For this purpose, statistical errors of measurements were simulated by a random-number generator, and coefficients $K$ of error amplification were calculated (they express the sensitivity of the information bearing features to such variations). The results obtained for the model C are presented in Table III.

Relative error of measurements $\delta_{meas}$ was presented by a discrete sequence of values 0.02, 0.04, 0.06, 0.08, 0.10 given in the upper row of the Table III. The simulated random numbers had uniform distributions what is characteristic of a "bad measurementB As seen from Table III, high values of the coefficients $K$ reaching several units are characteristic of the features C and D (the jump in the coefficient $K_C$ can be explained by the discrete nature of the characteristic). The coefficients $K_C$ and $K_D$ have comparable magnitudes; in some cases, they exceed the corresponding sensitivity factors $|\xi|$ presented in Fig. 3. Here, the variations $K\delta_{meas}$ of a characteristic may fall, due to the measurement errors, out of the limits of the value $|\xi|\delta x$ determined by the input parameter $X$. The values of $C_v^D$ and $r_{eff}$ corresponding to these cases can be easily estimated by the information from Fig. 3 and Table III. Analysis of the sensitivity coefficients, with the allowance for measurement errors, demonstrates that the expansion coefficient in the second maximum $x_l(l_{m2})$ has the largest information content regarding the variations of the microstructure parameters.

*TABLE III. Coefficients of error amplification K for the characteristics* A, B, C, D, *and* $\delta_{meas}$.

| $\delta_{meas}$ | 0.02 | 0.04 | 0.06 | 0.08 | 0.10 |
|---|---|---|---|---|---|
| $K_A$ | 0.83 | 1.00 | 1.20 | 1.25 | 1.20 |
| $K_B$ | 0.50 | 0.63 | 0.83 | 0.91 | 0.77 |
| $K_C$ | 0 | 3.23 | 2.17 | 1.61 | 1.30 |
| $K_D$ | 1.89 | 2.78 | 3.33 | 3.45 | 3.23 |

In practice, data on the input parameters are always inaccurate. To take this fact into account, absolute variations of $C_v^D$ and $C_v^W$ within the limits $\Delta C_v^{DW} = \pm 0.05$ were simulated with a random-number generator for each aerosol models considered. The carbon component was presented as the difference $C_v^S = 1 - C_v^D - C_v^W$. In this case, as follows from Eq. (3), efficient radii of the mixture particles $r_{eff}$ are also random values.

Regression of $Y$ on $X$ and $X$ on $Y$ ($X = \{C_v^D, r_{eff}\}$, $Y = x_l(l_{m2})$) was analyzed with the allowance for random nature of $\delta_{meas}$, $C_v^D$, and $C_v^W$. In the general case, stochastic connection between the values may be multiple, and one should take into account paired correlations.[10] However, according to calculations of $x_l(l_{m2})$ under variation of the parameters $X_2 = C_v^W$ and $X_3 = \delta_{meas}$, random values $x_l(l_{m2})$ and $C_v^W$, $x_l(l_{m2})$ and $\delta_{meas}$ are in fact not pair-wise correlated and the regression functions $\hat{\eta}_{YX}(x)$ and $\hat{g}_{XY}(y)$ can be restricted to two summands[10]

$$\hat{\eta}_{YX}(x) = \alpha_{YX} + \beta_{YX}x;$$

$$\hat{g}_{XY}(y) = \alpha_{XY} + \beta_{XY}y \ . \qquad (10)$$

for each aerosol model. Here $x$ and $y$ are random values $X = X_1 = \{C_v^D, r_{eff}\}$ and $Y = x_l(l_{m2})$; $\alpha_{YX}$ and $\alpha_{XY}$ are constants; $\beta_{YX} = r_{YX}\sigma_Y/\tilde{\sigma}_X$ is the coefficient of regression of $Y$ upon $X$; $\beta$ is the coefficient of regression of $X$ on $Y$; $\sigma_X$ and $\sigma_Y$ are rms deviations. The correlation coefficient

$$r_{XY} = r_{YX} = \frac{1}{\sigma_X\,\sigma_Y\,(N-1)} \times$$

$$\times \sum_{i=1}^{N} (x_i - m_x)(y_i - m_y) \qquad (11)$$

($m_x$ and $m_y$ are mathematical expectations of $X$ and $Y$) expressing the degree (measure) of the relationship between $X$ and $Y$ was determined by a sample $N > 100$ what guaranteed satisfactory (< 5%) error of calculations.

Coefficients for the regression equations (10) calculated for $\delta_{meas} = 0.1$ and $\Delta C_v^{DW} = \pm 0.05$ are presented in Table IV.

*TABLE IV. Coefficients for the regression eCuations (10).*

| $T$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $m_X$; $X = C_v^D$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| $m_{X_2}$; $X_2 = C_v^W$ | 0.63 | 0.59 | 0.55 | 0.49 | 0.44 | 0.37 | 0.29 | 0.20 | 0.10 |
| $m_X$; $X = r_{eff}$ | 0.08 | 0.09 | 0.11 | 0.13 | 0.17 | 0.22 | 0.31 | 0.48 | 0.92 |
| $m_Y$ | 0.36 | 0.77 | 1.25 | 1.88 | 2.66 | 3.83 | 5.70 | 9.27 | 18.4 |
| $r_{XY}$; $X = C_v^D$ | 0.77 | 0.82 | 0.83 | 0.75 | 0.79 | 0.85 | 0.85 | 0.86 | 0.86 |
| $r_{XY}$; $X = r_{eff}$ | 0.34 | 0.40 | 0.47 | 0.47 | 0.43 | 0.57 | 0.62 | 0.58 | 0.79 |
| $\alpha_{XY}$; $X = C_v^D$ | 0.04 | 0.08 | 0.12 | 0.22 | 0.32 | 0.38 | 0.51 | 0.66 | 0.83 |
| $\alpha_{YX}$; $X = C_v^D$ | 0.01 | −0.13 | −0.17 | −0.49 | −3.23 | −3.76 | −9.55 | −31.4 | −147 |
| $\alpha_{XY}$; $X = r_{eff}$ | 0.01 | 0.07 | 0.06 | 0.05 | 0.04 | 0.02 | 0.02 | 0.08 | −0.08 |
| $\alpha_{YX}$; $X = r_{eff}$ | −0.12 | 0.25 | 0.55 | 1.19 | 2.01 | 2.47 | 3.35 | 5.59 | 7.81 |
| $\beta_{XY}$; $X = C_v^D$ | 0.17 | 0.15 | 0.14 | 0.10 | 0.07 | 0.06 | 0.03 | 0.02 | 0.01 |
| $\beta_{YX}$; $X = C_v^D$ | 3.53 | 4.49 | 4.76 | 5.93 | 11.8 | 12.6 | 21.8 | 50.8 | 184 |
| $\beta_{XY}$; $X = r_{eff}$ | 0.02 | 0.03 | 0.03 | 0.04 | 0.05 | 0.05 | 0.05 | 0.04 | 0.05 |
| $\beta_{YX}$; $X = r_{eff}$ | 6.38 | 5.87 | 6.58 | 5.30 | 3.94 | 6.13 | 7.54 | 7.64 | 11.6 |

The number $T$ in the Table IV corresponds to different stages of aerosol transformation ($C_v^D = 0.1\,T$). Two upper rows contain input parameters of the models. One can see that for $X = C_v^D$ the values of the correlation coefficients $r_{XY}$ fall within the range 0.7–0.9 what corresponds to a strong correlation according to accepted classification.[11] At $X = r_{eff}$, the correlation coefficients mainly correspond to a medium correlation. With increasing $T$, the correlation grows to values corresponding to a strong correlation. It is evident that the correlation coefficient grows with a decrease in the errors $\delta_{meas}$ and $\Delta C_v^D$, and the connection is close to a functional one.

The calculated data for the regression coefficients α and β in the lower part of the Table IV for *a priori* models of the near-ground aerosol enable one to obtain (by use of Eq. (10)) a prognostic estimate of the conditional mathematical expectation $C_v^D$ and $r_{eff}$ (or $x_l(l_{m2})$) by non-random values $x_l(l_{m2})$ (or $C_v^D$ and $r_{eff}$) obtained when processing separate realizations of the diagonal elements of the scattering phase matrix.

## EXPANSION COEFFICIENTS FOR OFF-DIAGONAL ELEMENTS

The expansion coefficients for the off-diagonal elements $x_{12}(\mu)$ and $x_{34}(\mu)$ of the scattering phase matrix are presented in Fig. 4. They exhibit a weak sensitivity of distributions $x_l^{ij}$ to variations of the aerosol structure parameters $C_v^D$ and $r_{eff}$. One can see that aerosol transformation from the continental model C to the urban model U that is being accompanied by variation of $C_v^D$ from 0.70 to 0.17 leads, in general, to insignificant variations of the distributions. Some differences are observed only in the ranges $l = 2–6$ for $x_l^{12}$ and $l = 6–20$ for $x_l^{34}$. The maximum value of the dimensionless sensitivity factor does not exceed 0.24 for $C_v^D$ and 0.4 for $r_{eff}$ what is significantly less than the values obtained for the diagonal elements of the matrix in the case of the characteristic A.

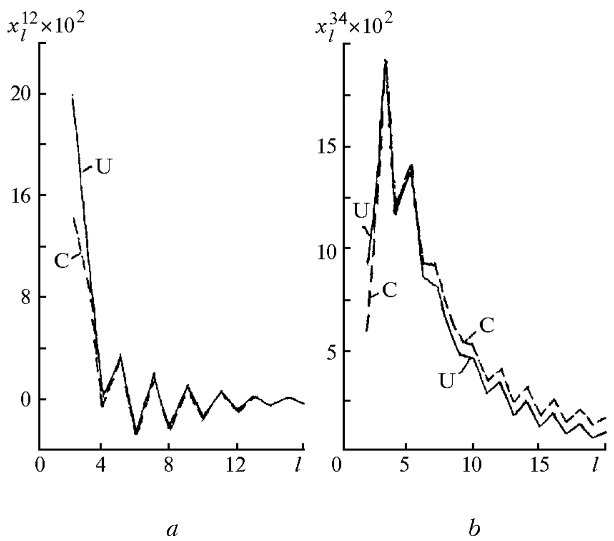


*FIG. 4. Coefficients of expansion over the GSF for the off-diagonal elements $x_{12}$ (a) and $x_{34}$ (b) of the scattering phase matrix and the models of continental (C) and urban (U) aerosol.*

## RESULTS OF THE STUDY

The information content of the expansion coefficients of scattering phase matrix elements over the GSFs have been studied in relation to variations of the aerosol microstructure parameters using most common aerosol models for the lower Earth's atmosphere.[4–7]

Analysis of the distributions of the expansion coefficients of the diagonal elements of the scattering phase matrix made it possible to reveal some information bearing features that are closely related (stochastically or functionally) with the microstructure and input parameters determining the aerosol transformation. For instance, the number of maxima in the distributions coincides with that of the aerosol fractions.

The numbers of the expansion coefficients in the first and second maxima are linearly connected with the effective radii of particles from the fine and coarse fractions, respectively. The value of the expansion coefficient in the second maximum is in a close stochastic connection with the concentration $C_v^D$ of the coarse fraction determining different stages of the transformation; the medium connection is characteristic of the effective radius $r_{eff}$ of the mixture particles.

We have analyzed the connections between an information characteristic of a distribution and the input parameters of the models. We have obtained quantitative data on the sensitivity coefficients of these characteristics to input parameters, coefficients of error amplification due to random measurement errors, and the errors of calculation of the expansion coefficients. Based on these data, we have revealed the characteristic $x_l(l_{m2})$ which is most informative with respect to variations of the parameters $C_v^D$ and $r_{eff}$.

Under conditions accumulation of the measurement errors and inaccuracy of *a priori* information about the models, the values of the coefficients of correlation between the input parameters $C_v^D$, $r_{eff}$, and the characteristic $x_l(l_{m2})$ were obtained, and prognostic functions of regression constructed for different stages of the aerosol transformation.

Expansion coefficients for the off-diagonal elements $x_{12}(\theta)$ and $x_{34}(\theta)$ of the scattering phase matrix were calculated. They are shown to be weakly sensitive to variation of the aerosol parameters $C_v^D$ and $r_{eff}$ during the process of aerosol transformation.

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