SCATTERING FUNCTION OF A POINT SOURCE IN THE ATMOSPHERE

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The problem of transfer of radiation from an isotropic glowing element of a surface in a spherical atmosphere by the Monte-Carlo method in order to determine the scattering function of a point source which is equivalent to the optical transfer function (OFT) of the atmosphere. Winter and summer models of the atmosphere[5] as well as the variant of the Elterman's model with visibility range. S_m = 15 km were employed in the calculation.

It is shown based on the data obtained that when an object of the order of 600 m in size is observed at the nadir taking the optical transfer function into account can give corrections exceeding 25%. When small (20–100 m in size) objects are observed the atmosphere has the effect of changing the contrast of the object without destroying its spatial structure.

In evaluating the effect of the external medium on observation conditions the atmosphere can be regarded as one element of a chain along which information about the spatial structure of the observed object is transmitted. This element can be analyzed in terms of the optical transfer function (OTF), i.e., some complex function acting on the brightness and structure of the image (atmospheric turbulence is taken into account precisely in this manner[1]). Scattering (molecular and aerosol) should also change the spatial structure of the image. This factor is usually ignored in practice though it has been studied in detail[2,3] and the results have been reduced to comparatively simple final formulas. The applicability of these formulas, however, is restricted by the substantial simplification of the conditions employed in their derivation (single scattering in a homogeneous layer containing large particles[2]; diffusion and small-angle approximation in the solution of the transfer equation in a homogeneous medium[3]). The effect of the scattered radiation in exoatmospheric observation problems is analyzed in Ref. 4.

Reference 9 .where a quite simple formula is derived for the. OTF based on unpublished Monte-Carlo calculations as well as experimental data from analysis of cosmic images is of special interest. However, there are many objections to this formula. The scattering function of a point source (SFP) derived by the Fourier transformation of the OTF[9] does not have an asymptotic form $F(\rho)\sim\frac{1}{\rho}$ in the limit $\rho \to 0$,as follows from the transfer theory[2] (according to formula (9) from Ref. 9 $F(\rho) \le F_{\max}$). In addition the representation of the OTF and, therefore the SFP as a sum of components determined by the aerosol and molecular scattering is completely unsubstantiated They are probably best represented as a product of the OTF (convolution of the SFP[9]).

The purpose of this report differing from and supplementing the results of Ref. 4, is to formulate practical recommendations as regards the size of the object for which the scattered radiation can be ignored. Our calculations were performed for conditions that approximate as closely as possible the optical properties of the real atmosphere without the mathematical simplification characteristics for Refs. 2–4.

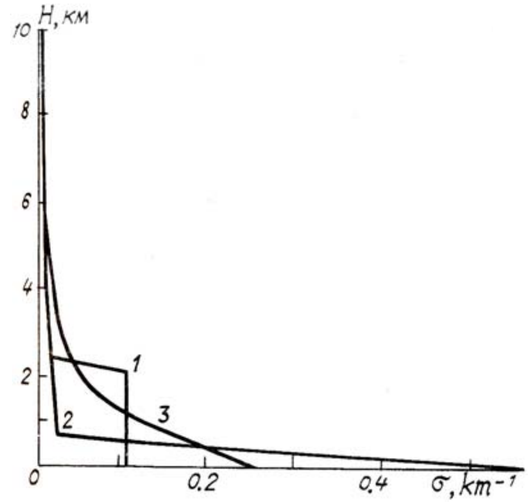

FIG. 1. Vertical profiles of the atmosphere attenuation coefficient: 1 – summer model, 2 – winter model; 3 – Elterman's model.

As an example we shall study the cases of observations at the nadir from, altitudes 20 and 250 km with different states of the atmosphere. The calculations incorporate the optical characteristics of the atmosphere corresponding to the optical atmospheric model of Ref. 5 (two-weather models were studied: summer-daytime and winter) as well as a variant of Elterman's model[8], which is widely employed in the solution of problems of radiation transfer in the atmosphere. The form of the profiles of the aerosol attenuation coefficient up to the altitude

$H = 10$ km, employed in our calculations, is shown in Fig. 1. The calculations were made for the wavelength $\lambda = 0.55$ μm, and the SPF, employed in the calculations, corresponded closely to Dermendjan's haze H model[7] with the real part of the refractive index $n = 1.5$. The photon survival probability Λ was taken to be 0.96.

The purpose of our calculations was to determine the scattering function of a point source $F(x, y)$, where x and y are the coordinates on the underlying surface. Giving the function $F(x, y)$ is equivalent to determining the OTF. The effect of the atmosphere on the transfer of an image of a flat object can be expressed in terms of the integral convolution transformation

$$B'(x,y) = \iint_{-\infty}^{\infty} B(x',y')F(x-x',y-y')dx'dy' + B(x,y)T_{dir}, \qquad (1)$$

where $B(x, y)$ and $B'(x, y)$ are, respectively, the true and observed brightnesses of the object, and T_{dir} is the direct transmission owing to unscattered radiation. For a point source of unit power

$$B_{ps}(x,y) = \delta(x'-x, y-y'),$$

then

$$B_{ps}(x,y) = F(x,y) + \delta(x'-x, y'-y)T_d .$$

Therefore the problem reduces to finding the brightness distribution of the scattered radiation owing to the presence of a luminous, orthotropic, infinitesimal element of the surface, and T_{dir}.

The calculations were performed by the Monte-Carlo method, i.e., the emission of "particles", which after passage through a spherical atmosphere impinged against a "receiver" situated at an altitude H, from a point source situated on a surface with albedo A was modeled. Simple modeling in problems with small detectors cannot be done in practice, so that we employed a modification of the Monte-Carlo method "local calculation"[8]. All radiation reaching the detector was recorded with appropriate "counters"; the indications of these counters were used to construct histograms of the angular distribution of radiation ΔT_1 relative to the optical axis passing through the detector and the source. For the conditions of observations at the nadir, studied below,

$$\Delta T_1 = 2\pi \int_{\alpha_1}^{\alpha_1+1} B(\alpha)\sin\alpha \, d\alpha . \qquad (2)$$

The function $F(x, y)$ is related to the diffuse transmission owing to scattered radiation $T_{ыс}$ by the relation

$$\iint_{-\infty}^{\infty} F(x,y)dxdy = T_{sc} \approx \sum_{1}^{1_{max}} \Delta T_1 . \qquad (3)$$

Since the scattering radiation reaching the "detector" is concentrated in angles $\alpha \ll \frac{\pi}{2}$ it may be assumed that $\alpha \approx \frac{\rho}{H}$ (ρ is a distance from the source to a point on the underlying surface; H is the observation altitude). Therefore

$$\Delta T_1 \approx 2\pi \int_{\rho_1}^{\rho_1+1} B(\rho)\rho d\rho . \qquad (4)$$

This approximation in the region 4710 m < ρ < 13156 m holds to within 10–15%, in all other cases the error is significantly smaller.

Nearly 5×10^4 tests were performed and the error was, in the main, less than 105%.

Table 1 gives the results of calculations of the quantities ΔT_{sci} for different models of the atmosphere.

TABLE 1.

The results of calculations of the parameters $\Delta T_{\rho i} = 2\pi \int_{\rho_i}^{\rho_{i+1}} F(\rho)\rho d\rho$ *for the scattering fund Ion of a point source*

$\rho_{1+1}-\rho_1$, m	ΔTp_1: MODEL "WINTER"	MODEL "SUMMER - DAY"		ELTERMANs MODEL	
	A=0.7 H=20km	A=0.15 H=20km	A=0.3 H=20km	A=0.15 H=250km	A=0.3 H=250km
29 - 0	0.017	0.0065	0.066	0.0106	0.0106
58 - 29	0.015	0.0057	0.0058	0.0094	0.0094
116 - 58	0.021	0.0106	0.0107	0.0185	0.0187
291 - 116	0.038	0.0262	0.0262	0.039	0.040
581 - 291	0.032	0.033	0.034	0.042	0.043
1165 - 581	0.03	0.043	0.044	0.045	0.047
2327 - 1165	0.035	0.047	0.050	0.040	0.042
3520 - 2327	0.025	0.023	0.025	0.019	0.022
4710 - 3520	0.015	0.014	0.016	0.013	0.0145
13156 - 4710	0.050	0.040	0.045	0.037	0.042
∞ - 13156	0.03	0.027	0.029	0.035	0.041

Analysis of the data in the table shows that the function F(ρ) for $\rho \le \rho_{max}$ can be approximated by a simple formula that is convenient for practical applications:

$$F(\rho) = a_i \rho^{-k_i}, \tag{5}$$

where

$$i = \begin{cases} 1 \text{ when } \rho < 116 \text{ m}, \\ 2 \text{ when } \rho > 116 \text{ m}. \end{cases}$$

The parameters a_i and k_i for our conditions are presented in Table 2. We note that $k_i = 1$ for small values of ρ, and the parameters a_1 and a_2 are related with one another by the condition of "joining" at the point ρ = 116 m, i.e., formula (3) contains only two Independent parameters. The average error of approximation (5) in the interval ρ ≤ 0 ≤ 3520 m, where up to 75% of a scattered energy is concentrated, is 15–20%. In the interval 3520 m ≤ ρ ≤ 4710 m the error is higher.

TABLE 2.

Parameters of the scattering function F(ρ) (ρ_{max} = 4710 m) and values of the direct T_d and diffuse T_s transmission.

MODELS	a_1	a_2	k_1	k_2	T_d	T_s
"WINTER" A=0.7	$7.14 \cdot 10^{-5}$	$1.106 \cdot 10^{-2}$	1	2.06	0.58	0.31
"SUMMER - DAY" A=0.15	$3.107 \cdot 10^{-5}$	$5.58 \cdot 10^{-4}$	1	1.607	0.60	0.28
"SUMMER - DAY" A=0.3	$3.158 \cdot 10^{-5}$	$5.75 \cdot 10^{-4}$	1	1.610	0.60	0.29
ELTERMAN's A=0.15	$5.25 \cdot 10^{-5}$	$2.78 \cdot 10^{-3}$	1	1.834	0.58	0.30
ELTERMAN's A=0.50	$5.47 \cdot 10^{-5}$	$2.46 \cdot 10^{-3}$	1	1.800	0.58	0.363

In conclusion we shall present the results of calculations of the characteristic sizes of the object for which the scattered radiation must be taken into account in the formation of the image, i.e., the second term on the right side of (1) is important. As a simplification we shall study an object in the form of a circle with radius *r* situated in a homogeneous background

$$B(\rho) = \begin{cases} B_{ob} \text{ when } \rho < r, \\ B_b \text{ when } \rho > r. \end{cases} \tag{6}$$

For observation at the nadir it can be shown, based on simple physical considerations, that the contribution of the first term in (1) will be greatest at the center of the image, and we shall make our calculations for this point. For our conditions the following relation holds:

$$B(0) = B_{ob} T_{dir} + B_b T_{sc} + (B_{ob} - B_b) 2\pi \int_0^r F(\rho) \rho d\rho \tag{7}$$

The last term in (7) is determined by radiation scattered by the object itself. We introduced the parameter η(*r*) characterizing this perturbation

$$\eta(r) = \frac{(B_{ob} - B_b) 2\pi \int_0^r F(\rho) \rho d\rho}{B_{ob} T_{dir} + B_b T_{sc}} \tag{8}$$

Equation (8) is especially easy to analyze in two cases: $B_{ob} = 0$ and $B_F = 0$. In the first case (a dark object on a light background)

$$\eta(r) = \frac{2\pi \int_0^r F(\rho) \rho d\rho}{T_{sc}}, \tag{9}$$

in the second case (a light object on a dark background)

$$\eta(r) = \frac{2\pi \int_0^r F(\rho) \rho d\rho}{T_{dir}}. \tag{10}$$

The results of the calculations performed using the formulas presented above are given in Table 3. It follows from the results that under our conditions the scattered radiation can appreciably change the contrast characteristics of the object, whose size is of the order of 500 m and larger. Special calculated showed that as the observation altitude is varied from 20 m to 250 m, the form of the function *F*(ρ) remains virtually unchanged, i.e., the data in the table can be used for all observation altitudes above 20 km.

TABLE 3.

The dependence of the correction $\eta(r)$ (per cent) on the size of an object for observations at the nadir

r, m	WINTER MODEL H=20 km A=0.7 (formula (8))	SUMMER MODEL H=20 km A=0.15 (formula (10))	ELTERMAN's MODEL H=250 km A=0.15 (formula (10))
29	5.4	0.65	1.7
58	10.2	1.22	3.2
116	16.8	2.3	6.05
291	28.2	4.9	12.2
581	38.3	8.2	18.8
1165	47.5	12.5	26.0

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