LASER BEAM PROPAGATION ALONG EXTENDED VERTICAL AND SLANT PATHS IN THE TURBULENT ATMOSPHERE

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In this paper we present analysis of the mean intensity, relative variance, and scales of temporal and spatial correlations of the intensity fluctuations of focused and collimated beams in the far zone of diffraction, when propagating through the turbulent atmosphere along slant and vertical paths whose lengths essentially exceed the thickness of a distorting layer.

It is shown that for such paths the time scale of correlation of the intensity fluctuations is identically characterized by the time required for the atmospheric inhomogeneities to travel across the initial laser beam cross section, irregardless of the level of turbulent distortions in the layer. The spatial scale of correlations coincides with diffraction size of the beam in the observation plane, if the amplitude fluctuations of the optical radiation within the layer are small compared with the phase ones, otherwise it is proportional to the radius of correlation of the field at the exit from the distorting layer.

It is also shown in this paper that maximum value of the mean intensity, if determined in the coordinate system with the origin at the energy center of gravity of a beam, can essentially exceed the mean intensity due to random wanderings.

This paper is devoted to the study of laser radiation propagation through the turbulent atmosphere along vertical and slant paths whose lengths essentially exceed the thickness of a distorting layer. By mean intensity of a laser beam we understand, in this paper, the intensity averaged over its instantaneous cross section in the coordinate system whose origin coincides with the instantaneous energy center of gravity of the beam. This allows only the intensity variations due to diffraction and scattering on small scale turbulent inhomogeneities of the refractive index of air to be separated out for study. Variance, spatial and temporal correlation of the optical radiation intensity are also analyzed for the case of propagation along inhomogeneous paths.

Rigorous expression for complex amplitude $U(z, \rho, t)$ wave field propagating in a medium with large–scale fluctuations of its refractive index $\tilde{n}(z, \rho, t)$ can be represented in the form of the path integral[1–4]

$$U(z, \rho, t) = \frac{\kappa}{2\pi i z} \int\limits_{-\infty}^{+\infty} d^2\rho' U_0(\rho') \exp\left\{\frac{i\kappa}{2z}(\rho - \rho')^2\right\} \times$$

$$\times \lim_{N\to\infty} \left(\frac{\kappa}{2\pi i z}\right)^N \int\limits_{-\infty}^{\infty}\!\!\int d^2b_1\, d^2b_2 \ldots d^2b_N \exp\left\{\frac{i\kappa}{2z}\sum_{j=1}^{N} b_j^2\right\} \times$$

$$\times \exp\left\{ i\kappa \int\limits_0^z dz'\, \tilde{n}\left(z', \left(1 - \frac{z'}{z}\right)\rho' + \frac{z'}{z}\rho + \frac{\sqrt{2}}{\pi}\sum_{j=1}^{N} \frac{\sin\left(j\pi \frac{z'}{z}\right)}{j}\mathbf{b}_j, t \right)\right\}, \quad (1)$$

where z is the length of the path; ρ is the two–dimensional radius–vector in the plane perpendicular to the axis of propagation, t is time, $\kappa = 2\pi/\lambda$, λ is the radiation wavelength, and $U_0(\rho)$ is the complex amplitude of the wave field in the plane $z' = 0$. However, direct use of Eq. (1) for making calculations in problems of laser radiation propagation through the turbulent atmosphere is accompanied by a lot of difficulties, while, at the same time, it can be essentially simplified, if it is used for solving the problem under study.

Let a source of radiation be on the Earth's surface while a receiver of radiation be at a sufficiently large distance from the effective distorting layer of the atmosphere. In this case it would be convenient to divide the propagation path into two portions, that is, the portions $[0, z_0]$ and $[z_0, z]$. Let us also assume that the wave field undergoes the main distortions, due to the turbulent distortions of the refractive index $\tilde{n}$, only within the atmospheric layer from the Earth's surface to the altitude z_0, and propagates practically in vacuum ($\tilde{n} \approx 0$) in the second portion of the path. According to models[5] of vertical profiles of the structure characteristic of the refractive index C_n^2 the effective length of the distorting layer of the atmosphere is defined as follows:

$$z_0 = \int\limits_0^{\infty} d z'\, C_n^2(z') / C_n^2(0)\,, \quad (2)$$

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and for the case of propagation along vertical paths it is about 500–1000 m. In the case of not diverging and not very narrow beams the condition $ka^2/z_0 \gg 1$ is valid at the distance z_0, that is, the wave propagating along this path is practically plane. Note that here a is the initial radius of the beam. The propagation through the layer $[0, z_0]$ is accompanied by the distortions of the wave amplitude $A(z_0, \rho) = |U(z_0, \rho)|$ and of the phase $s(z_0, \rho) = i \ln(U(z_0, \rho)/|U(z_0, \rho)|)$. In the case of vertical paths and when $z' = z_0$ we usually have conditions under which the following relationship for relative variance of the intensity is valid, $\sigma_i^2 = \langle I^2\rangle/\langle I\rangle^2 - 1 \ll 1$. For this reason it would be convenient for further analysis to introduce some parameters characterizing the level of amplitude and phase fluctuations of a plane wave ($\kappa a^2/z_0 \gg 1$) occurring inside the distorting layer which could be calculated using the first approximation of the smooth perturbation method[6] and assuming that $C_n^2(z') = C_n^2(0) = \text{const}$ and $z' \in [0, z_0]$, like the relative variance of the intensity

$$\beta_0^2 = \langle A^4(z_0, 0)\rangle/\langle A^2(z_0, 0)\rangle^2 - 1 = 1.24\, C_n^2(0)\, \kappa^{7/6} z_0^{11/6} \quad (3)$$

and the structure function of phase,

$$D = \langle [S(z_0, \rho_1) - S(z_0, \rho_2)]^2\rangle|_{|\rho_1-\rho_2|=a} = 2.92 C_n^2(0)\, \kappa^2 a^{5/3} z_0 \,. \quad (4)$$

Taking into account the condition that

$$z \gg z_0 \,, \quad (5)$$

let us formally direct the path length in the last exponent of Eq. (1) to infinity ($z \to \infty$). This allows the integration over all variables $\mathbf{b}_j$ to be done what, in turn, results in the following formula:

$$U(z, \rho, t) = \frac{\kappa}{2\pi\, i\, z} \int_{-\infty}^{+\infty} d^2\rho'\, U_0(\rho') \times$$

$$\times \exp\left\{\frac{i\kappa}{2\, z} (\rho - \rho')^2\right\} \exp\left\{i\kappa \int_0^z dz'\, \tilde{n}(z', \rho', t)\right\}. \quad (6)$$

This formula for the wave field is known as the phase screen approximation.[6–11] Note that condition (5) is sufficient for a justified use of formula (6) when analyzing the mean intensity $\langle I\rangle$.

Mean intensity $\langle I\rangle = \langle UU^*\rangle$. Let us now derive an expression describing the mean intensity $\langle I\rangle$ on the beam axis ($\rho = 0$) and at the center of gravity of the intensity distribution

$$\rho_c(z) = \left(\int_{-\infty}^{+\infty} d^2\rho\; I(z,\rho)\rho\right) \Big/ \left(\int_{-\infty}^{+\infty} d^2\rho\; I(z,\rho)\right) . \quad (7)$$

In doing so let us assume the beam to be Gaussian in the emission plane, that is,

$$U_0(\rho) = A_0 \exp\left\{-\frac{1}{2}\left(\frac{1}{a^2} + i\frac{\kappa}{F}\right)\rho^2\right\} , \quad (8)$$

where F is the focal length.

From Eqs. (6)–(8) we have

$$\rho_c(z) = \frac{2\, z}{\pi\, a^4} \int_0^{+\infty} d\, z' \int_{-\infty}^{+\infty} d^2 R\; \tilde{n}(z', \mathbf{R})\, \mathbf{R}\, e^{-\mathbf{R}^2/a^2} \,. \quad (9)$$

Assuming also that components of the vector ρ_c obey normal distribution law from Eqs. (6), (8), and (9) after averaging, we obtain

$$\langle I_\alpha(z)\rangle = \langle I(z, \alpha\, \rho_c)\rangle = A_0^2 \left(\frac{\kappa}{2\pi z}\right)^2 \int_{-\infty}^{+\infty} d^2\rho_1\, d^2\rho_2 \times$$

$$\times \exp\left\{-\frac{1}{2\, a^2}(\rho_1^2 + \rho_2^2) - i\frac{\kappa}{2\, z}\left(1 - \frac{z}{F}\right)(\rho_1^2 - \rho_2^2) - \right.$$

$$- \frac{\pi}{2}\kappa^2 \times \int_0^{\infty} d\, z' \int_{-\infty}^{+\infty} d^2\kappa\; \Phi_n(z', \kappa, 0) \times$$

$$\times\left[1 - \exp\left(ik\,(\rho_1 - \rho_2)\right) - \alpha\, i\, \kappa\, (\rho_1 - \rho_2) \exp\left(-\frac{a^2}{4}\kappa^2\right) \times\right.$$

$$\times \left(\exp(-i\kappa\rho_1) - \exp(-i\kappa\rho_2)\right) + \frac{1}{2}\alpha\left[\kappa(\rho_1 - \rho_2)\right]^2 \times$$

$$\left.\left.\times \exp\left(-\frac{a^2}{2}\kappa^2\right)\right]\right\} . \quad (10)$$

At $\alpha = 0$ this formula describes the mean intensity at a point on the beam propagation axis ($\rho = 0$), while at $\alpha = 1$ it represents the average of instantaneous intensity values at the beam center of gravity; the function $\Phi_n(z', \kappa, 0)$ is the three–dimensional spectrum of the refractive index fluctuations. In the case of Kolmogorov spectrum (see Refs. 6 and 12–14) $\Phi_n(z', \kappa, 0) = 0.132 C_n^2(z')|\kappa|^{-11/3}$ and from Eq. (10), we obtain

$$\langle I_\alpha(z)\rangle = A_0^2\left(\frac{\kappa a^2}{2\pi z}\right)^2 \int_{-\infty}^{+\infty} d^2R'\, d^2\rho' \exp\left\{-\mathbf{R}'^2 - \frac{1}{4}\rho'^2 - i\frac{\kappa a^2}{z} \times\right.$$

$$\times \left(1 - \frac{z}{F}\right)\mathbf{R}'\rho' - \frac{1}{2} D\left[|\rho'|^{5/3} - \alpha\, 1.566\, \rho'\left(\left(\mathbf{R}' + \frac{1}{2}\rho'\right) \times\right.\right.$$

$$\times {}_1F_1\left(\frac{1}{6}\,;\, 2;\, -\left(\mathbf{R}' + \frac{1}{2}\rho'\right)^2\right) - \left(\mathbf{R}' - \frac{1}{2}\rho'\right) {}_1F_1 \times$$

$$\left.\left.\left.\times \left(\frac{1}{6}\,;\, 2;\, -\left(\mathbf{R}' - \frac{1}{2}\rho'\right)^2\right)\right) + \alpha\, 0.697\, \rho'^2\right]\right\} , \quad (11)$$

where ${}_1F_1(a, b, c)$ is the confluent hypergeometric function.

In order to describe the intensity decrease due to turbulence let us introduce the factor

$$\eta_a = \langle I_a(z)\rangle \,/\, I_D(z) \,, \quad (12)$$

where

$$I_D(z) = A_0^2\,(\kappa a^2/z)^2 \big/ \left[1 + (1 - z/F)^2\,(\kappa a^2/z)^2\right]$$

is the intensity of radiation on the beam axis in the absence of turbulence on the propagation path ($D = 0$). Using polar system of coordinates for the integration variables in Eq. (11) one can reduce the estimation of η_0 value to calculation of a single integral and of the value η_1 to calculation of a triple integral.

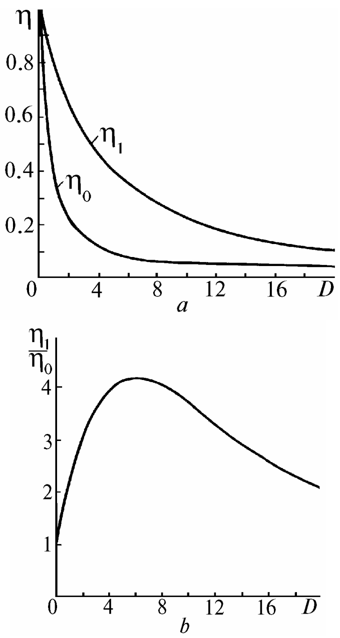

FIG. 1. Dependences of the factors η_0 and η_1 (a), as well as of their ratio η_1/η_0 (b) describing a decrease of the beam intensity due to atmospheric turbulence on the parameter D.

Figure 1*a* shows the dependences of η_0 and η_1 on the parameter D calculated assuming that $(\kappa a^2/z)(1 - z/F) = 0$. Figure 1*b* clearly demonstrates the difference between long– and short–exposure mean intensities, $\langle I_0\rangle(\eta_0)$ and $\langle I_1\rangle(\eta_1)$, at maxima of their distributions under different turbulent conditions of propagation characterized by the parameter D. Thus at $D \simeq 6$ the intensity $\langle I_1\rangle$ exceeds the value of $\langle I_0\rangle$ by more than 4 times. This means that the effective beamwidth determined by averaging instantaneous intensity distributions over sufficiently long time is, on the average, nearly twice as large as the instantaneous one. The ratio η_1/η_0 (or $\langle I_1\rangle/\langle I_0\rangle$) then slowly falls down to unity with increasing D.

Intensity fluctuations. In contrast to the mean intensity calculations the calculations of intensity fluctuations need some extra condition, apart from condition (5), in order to make use of the phase screen approximation (formula (6)). Asymptotic analysis of the expression for σ_i^2 obtained from Eq. (1), carried out for the cases of weak and strong intensity fluctuations, has shown that for a focused beam or in the far zone of diffraction ($(\kappa a^2/z)(1 - z/F \simeq 0$) the variance σ_i^2 is a function of two parameters β_0 and D. Under condition that

$$(\beta_0^2)^{12/5} \ll D^{6/5} \qquad (13)$$

this expression coincides with the formula for σ_i^2 obtained using relation (6).

Thus, we can state that the phase screen approximation is applicable to analysis of the intensity fluctuations if the phase distortions of a beam occurring inside the layer $[0, z_0]$ essentially exceed the amplitude ones. The fluctuations of the beam intensity in the plane $z' = z_0$ can be rather strong ($\beta_0^2 > 1$) in this case. One can surely assume that for vertical and slightly slant paths condition (13) is satisfied since practically always we have conditions when $\beta_0^2 < 1$ and $\beta_0^2 > 1$ and $\kappa a^2/z_0 > 1$.

In the case of the Kolmogorov spectrum of the refractive index fluctuations one can easily obtain, using the Taylor hypothesis of frozen turbulence[12–14] from Eqs. (6) and (8) the following expression for the second moment of intensity:

$$\left\langle I\left(z,\, \mathbf{R} + \frac{1}{2}\rho,\, t\right) I\left(z,\, \mathbf{R} - \frac{1}{2}\rho,\, t + \tau\right)\right\rangle =$$

$$= (2\pi)^{-3}\left(\frac{\kappa a^2}{z}\right)^4 \int\limits_{-\infty}^{+\infty} \mathrm{d}^2R_2\,\mathrm{d}^2R_3\,\mathrm{d}^2R_4 \times$$

$$\times \exp\left\{-\frac{1}{2}\left[\mathbf{R}_2^2 + \mathbf{R}_3^2 + \left(1 + \left(\frac{\kappa a^2}{z}\right)^2\left(1 - \frac{z}{F}\right)^2\right)\mathbf{R}_4^2\right] + \right.$$

$$+ i\,\frac{\kappa a^2}{z}\left(1 - \frac{z}{F}\right)\mathbf{R}_2\,\mathbf{R}_3 - i\,\frac{\kappa a}{z}\left(\mathbf{R}_3\,\rho + 2\,\mathbf{R}_4\,\mathbf{R}\right) - \frac{1}{2}D \times$$

$$\left.\times \left[\,|\mathbf{R}_3 + \mathbf{R}_4|^{5/3} + |\mathbf{R}_3 - \mathbf{R}_4|^{5/3}\right]\right\} \exp\left\{1.46\,\kappa^2\,a^{5/3} \times\right.$$

$$\times \int\limits_0^{\infty} \mathrm{d}z' C_n^2(z')\left(\left|\mathbf{R}_2 + \mathbf{R}_3 + \langle \mathbf{V}_\perp(z')\rangle \frac{\tau}{a}\right|^{5/3} + \right.$$

$$+ \left|\mathbf{R}_2 - \mathbf{R}_3 + \langle \mathbf{V}_\perp(z')\rangle \frac{\tau}{a}\right|^{5/3} - \left|\mathbf{R}_2 + \mathbf{R}_4 + \right.$$

$$\left.\left. + \langle \mathbf{V}_\perp(z')\rangle \frac{\tau}{a}\right|^{5/3} - \left|\mathbf{R}_2 - \mathbf{R}_4 + \langle \mathbf{V}(z')\rangle \frac{\tau}{a}\right|^{5/3}\right)\right\}, \qquad (14)$$

where $\langle V_\perp\rangle$ is transverse, with respect to the beam axis, component of the wind velocity.

Consider now relative variance of the intensity σ_i^2 on the beam axis ($\rho = 0$). Under condition that $(\kappa a^2/z)(1 - - z/F) = 0$ the value σ_i^2 is governed solely by the parameter D. Moreover, if $D \ll 1$ or $D \gg 1$ it is possible, using the known techniques,[12–14] to derive asymptotic formulas for σ_i^2 from Eq. (14). Thus, in the case of weak intensity fluctuations ($D \ll 1$) (Ref. 14)

$$\sigma_I^2 \approx 2^{10/3}\left[(1/2)\,\Gamma^2(8/3) + (1/2)\,\Gamma^2(11/6) - (3/4)^{8/3}\Gamma^2(11/6) \times\right.$$

$$\left.\times\, {}_2F_1\,(11/6,\, 11/6\,;\, 1;\, 1/4)\right] D^2 \approx 1.58\, D^2, \qquad (15)$$

and for strong ones ($D \gg 1$) (Ref. 16)

$$\sigma_I^2 \approx 1 + (13/12)\,2^{16/15}(7/6)^2\left[\Gamma^2(7/5)/\Gamma^2(6/5)\right]\Gamma^2(5/3)\,D^{-2/5} \approx$$

$$\approx 1 + 2.6\,D^{-2/5}, \qquad (16)$$

where $\Gamma(a)$ is gamma function; ${}_2F_1(a, b; c; d)$ is Gauss hypergeometric function.

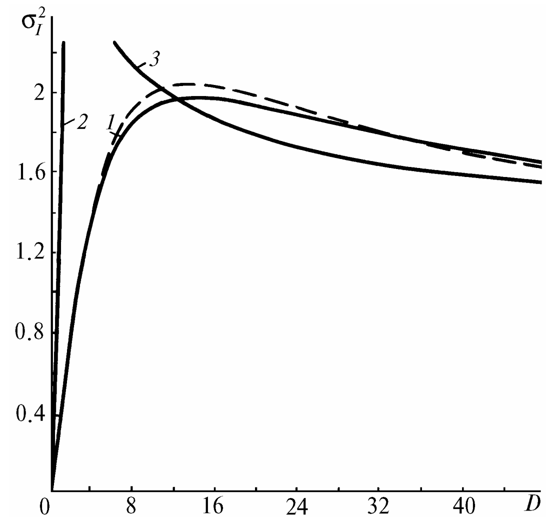

FIG. 2. Relative variance of the intensity fluctuations calculated by the Monte Carlo method (curve 1), by formulas (15) and (16) (curves 2 and 3, respectively), and using relation (17) (dashed line).

To calculate the sixfold integral in Eq. (14) at arbitrary values of the parameter D we used the Monte Carlo method.[14,15] Figure 2 shows data of numerical calculations of relative variance σ_i^2 for the case of $(\kappa a^2/z)(1 - z/F) = 0$ (curve *1*). Curves *2* and *3* stand for calculations by formulas (15) and (16), respectively. It can be seen from this figure that the used asymptotics provide the accuracy of 15% in the regions $D < 0.5$ (formula (15)) and $D > 10$ (formula (16)). Based on the results obtained by the Monte Carlo method (curve *1*) we have constructed an empirical formula for σ_i^2

$$\sigma_I^2 = \frac{1 + D^{-2/5}\left[2.6 + 23.5/D\right]}{1 + D^{-2/5}\left[\dfrac{70.1}{(1 + D/2)\,D} + \dfrac{14.9}{D^3}\right]} \tag{17}$$

which yields a maximum error of 3% and the results coinciding with those obtained by asymptotics (15) at $D \ll 1$ and (16) at $D \gg 1$. The curve $\sigma_i^2(D)$ calculated by formula (17) is shown in Fig. 2 by dashed line.

From Eq. (14) it follows that in contrast to σ_i^2 the radius of spatial correlation r_1 of intensity determined from the fall off of the correlation coefficient down to e^{-1} level both for focused and collimated beams in the far zone of diffraction ($\kappa a^2 \ll z$) depends, in the region behind the phase screen, on the path's length z and is determined by the diffraction radius of the beam[16]

$$r_I \approx \sqrt{2}\, z/(\kappa\, a)\,. \tag{18}$$

Analysis of temporal correlation of the intensity can be significantly simplified if the approximation

$$1.46\,\kappa^2 a^{5/3} \int\limits_0^\infty \mathrm{d}z' C_n^2(z') \left| \mathbf{R} + \langle \mathbf{V}_\perp(z') \rangle \tau/a \right|^{5/3} \approx$$
$$\approx (D/2) \left| \mathbf{R} + \upsilon_{\mathrm{eff}}\, \tau/a \right|^{5/3} \tag{19}$$

is used, where υ_{eff} is the vector of effective velocity of transfer of the turbulent inhomogeneities of the refractive index across the beam cross section. Absolute value of this vector is determined by the formula

$$|\upsilon_{\mathrm{eff}}| = \left[\frac{\int\limits_0^\infty \mathrm{d}z' C_n^2(z') \left| \langle \mathbf{V}_\perp(z') \rangle \right|^{5/3}}{\int\limits_0^\infty dz' C_n^2(z')} \right]^{3/5}, \tag{20}$$

while its direction is determined by the direction of near–ground wind and by the direction of beam propagation.

By using Eqs. (14) and (19) we have calculated by the Monte Carlo method the coefficient of temporal correlation of the intensity

$$K(\tau) = \left(\langle I(t)\, I(t + \tau)\rangle - \langle I\rangle^2\right) \big/ \left(\langle I^2\rangle - \langle I\rangle^2\right) \tag{21}$$

for different values of the parameter D assuming that $(\kappa a^2/z) \times (1 - z/F) = 0$. It can be seen from this figure that the time of the intensity correlation τ_c defined as the width of the function $K(\tau)$ at the level e^{-1} is of the order of time required for the refractive index inhomogeneities to travel across the beam $\tau_0 = 2a/|\upsilon_{\mathrm{eff}}|$. Thus at $D \ll 1$ the value $\tau_c \approx 3\,\tau_0$ and at $D \gg 1$ $\tau_c \approx \tau_0$.

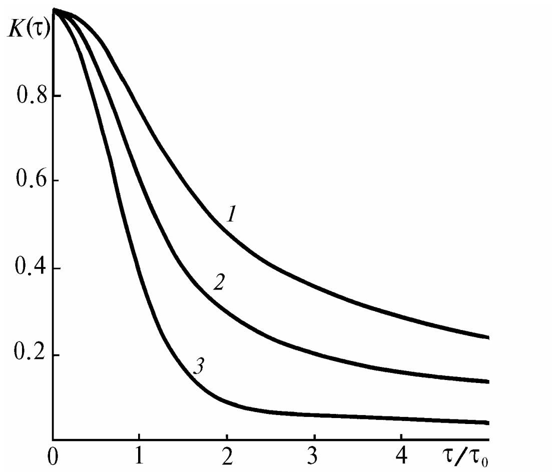

FIG. 3. Coefficient of time correlation of the intensity calculated for different D values: D = 0.2 (curve 1), D = 2.0 (curve 2), and D = 20 (curve 3).

An increase of the zenith angle θ of a beam propagation direction results in an increase of the thickness z_0 of the disturbing layer and, hence, in an increase of the intensity fluctuations β_0^2. As a consequence it happens so that starting with certain value of θ condition (13) may be violated. In this case the calculations of strong intensity fluctuations may be done using formula (1) according to the technique described in Ref. 4. In the limiting case of $(\beta_0^2)^{12/5} \gg D^{6/5}$ and for $(\kappa a^2/z)(1 - z/F) \approx 0$ the variance σ^2 has the form

$$\sigma_I^2 \approx 1 + 0.33\, \beta_0^{-4/5}. \tag{22}$$

It can be seen from this formula that intensity fluctuations in the plane $z' = z$ are entirely determined by the distortions of the wave amplitude occurring within the interval $[0, z_0]$ and do not depend on the initial size a of the beam.

From Eq. (1) and under condition that $(\kappa a^2/z)(1 - z/F) \approx 0$ for the radius of spatial correlation of the beam intensity one can obtain the following approximate expression:

$$r_I \approx \sqrt{2}\,\frac{z}{\kappa a} \Big/ \sqrt{1 + 2^{11/5}\left(\beta_0^2\right)^{12/5} / D^{6/5}}\,. \qquad (23)$$

One can easily see that under condition (13) formulas (23) and (18) coincide. However, if $(\beta_0^2)^{12/5} \gg D^{6/5}$, the correlation radius is

$$r_I \approx \frac{1}{2^{3/5}}\frac{z}{\kappa a}\frac{D^{3/5}}{\left(\beta_0^2\right)^{6/5}} \sim \frac{z}{z_0}\,\rho_c \qquad (24)$$

and does not depend on the initial beam radius a, being entirely determined by the radius of spatial coherence of the beam at the end $(z' = z_0)$ of the distorting layer, that is, by $\rho_c \sim [1.45\kappa^2\, C_n^2(0)\, z_0]^{-3/5}$.

Analysis of the coefficient of time correlation of the beam intensity $K(\tau)$, we have done in the region of strong intensity fluctuations, shows that in contrast to the variance σ_i^2 and the radius r_I of spatial correlation of the intensity the time scale of its correlation does not depend on the relation between β_0^2 and D parameters under condition that $(\kappa a^2/z)(1 - z/F) \ll 1$, being determined only by the time required for the turbulent inhomogeneities to travel across the beam, that is,

$$\tau_c \approx \tau_0\,. \qquad (25)$$

The matter is that at a large distance $(z \gg z_0)$ the intensity at a point ρ is determined, because of the wave diffraction on the interval $[z_0, z]$, by a superposition of the partial waves' fields from practically the entire wave surface at the exit from the distorting layer. Therefore, an essential variation of the intensity in the plane $z' = z$ occurs only if the beam field at the end of this layer is fully changed, what, in turn, happens in every τ_0 time interval.

The above analysis adapts, in certain sense, the results of classical studies[1,2,17] for application to description of laser beam propagation along real atmospheric paths what can become practicable in a number of applied problems.

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