INVESTIGATION OF CRYSTALLINE CLOUDS BASED ON LASER RADAR MEASUREMENTS OF BACKSCATTERING PHASE MATRICES

B.V. Kaul', L.A. Kuznetsov, E.R. Polovtseva, and I.V. Samokhvalov

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk
Received September 23, 1992

In this paper we discuss some effects of the symmetry properties of backscattering phase matrix and by an example of vertical profiles of the elements of this matrix we show a possibility of using these properties for determining preferred orientation of crystal particles.

In laser sensing of the atmosphere ensembles of aerosol particles being sounded are characterized by the backscattering coefficient. Very often, in addition to this parameter, the intensities of polarized and crosspolarized components of backscattered radiation are determined provided that linearly polarized laser radiation is used. The ratio of these components is called depolarization and it is assumed a measure of the particle nonsphericity. Use of the above characteristics is based on the concept that atmospheric aerosols are ensembles of spherical or nonspherical randomly oriented particles. An experience of optical studies has shown that such a concept is quite justifiable for the majority of atmospheric aerosols.

However, there exists quite a wide class of natural aerosols in the atmosphere, namely, the crystalline clouds, for which the lidar equation in scalar form is insufficient since such aerosol ensembles should be described with a backscattering phase matrix (BPM). Below we shall demonstrate this by an example. Of course, the necessity of using the BPM to describe such aerosols is, in certain sense, obvious because anomalous optical phenomena resulting from a pronounced anisotropy of light scattering by crystal clouds have been known long ago. Nevertheless, such phenomena are too rare and it is not *a priori* clear how often essential deviations from the scalar approximation occur. Thus, the experimental material available for our analysis at present and partially described in Refs. 1 and 2 allows us to arrive at the conclusions that in 30–40 percent lidar observations of crystalline clouds either the backscattering coefficient depends on the direction of sounding radiation polarization or the polarization of scattered light becomes elliptical, or both these effects occur simultaneously.

In our opinion three circumstances are important for explaining the fact that measurements of the BPM were not widely used in sounding of crystalline clouds so far. The first one is connected with the necessity of using much more sophisticated instrumentation than usual. The second circumstance is in a much bigger volume of the required measurements. Really, in order to determine the scattering coefficient and depolarization one has to measure intensities of only two signals while the determination of the complete scattering phase matrix would require to measure already 24 intensities. It is true, however, that the number of measurements can be reduced to 18 if one takes into account certain symmetry properties of the backscattering phase matrix. And, finally, the third circumstance is connected with the most significant difficulty of principle character caused by the fact that there occurs a change of physical volumes under study during the measurement time. We have managed to overcome this difficulty only partially. The matter is that correct measurements of the scattering phase matrix can be done only under conditions of frozen positions and orientations of aerosol particles. It is unbelievable that this condition can be satisfied in experiments, and especially impracticable it is in the optical arrangement of remote sensing.

It can surely be stated that simultaneous measurements of all the intensities necessary for determining the scattering phase matrix are impossible, in principle, for the following reasons. First, there is a need, in such measurements, for a successive change of the state of polarization of sounding radiation delivered by a transmitter. At the same time scattering properties of the aerosol ensemble vary randomly even if no macroscopic transportation of aerosol takes place. Depending on size of crystal particles of a cloud ensemble and taking into account corresponding mean angular velocities of particles Brownian rotation the freeze time may be assessed to be of 10^{-4}–10^{-1} s.

A minimum number of successive measurements are determined by the number of successively changed states of polarization of sounding radiation and is equal to four measurements. This minimum number of measurements can only be reached if all the Stokes parameters relevant to each state of polarization of sounding radiation are measured simultaneously and this is feasible, in principle. If the freeze time is assumed to be of 10^{-4} s, the needed measurement frequency must be of the order of 10^{-5} Hz what essentially exceeds admissible frequency of a transmitter operation because of inevitable overlapping of the successively coming return signals. As to the freeze time of $10^{-2} - 10^{-1}$ s one can imagine, in principle, a lidar that would allow so fast measurements of the scattering phase matrix but such a lidar would be very sophisticated and expensive.

It is clear therefore that time averaging of fluctuations of the elements of a BPM is, in fact, inevitable. This is especially true in relation to our measurements which normally have a duration of several minutes. As a result of a wind transportation of air each our measurement is referred to different physical volumes and horizontal length of the total volume of the atmosphere under study is several kilometers. In each measurement we observe different particles and their number varies from measurement to measurement. For these reasons all our studies of this type are based on the assumption that microphysical parameters of the general ensemble of particles responsible for formation of polarization properties of scattered radiation keep their values and only number density of particles can vary.

0235-6880/93/04 257-04 $02.00

Difficulties of data interpretation caused by these variations have been overcome by normalizing the measured BPM's of a crystal cloud to that of purely molecular atmosphere. It should be noted here that this procedure is not helpful for all measurements and we exclude such cases from the consideration.

As concerning this assumption as a whole, the answer to the question whether such large ensembles of particles we deal with in our measurements keep the peculiar features that make their scattering properties different than those in scalar approach or not can be found in our experimental results. We have mentioned above how often such deviations are observed and an example of observational data can be found in the figure.

Our measurement technique and instrumentation have been discussed earlier in Refs. 1–3, therefore below we mainly concentrate on the problems of sounding data interpretation.

Let us first consider symmetry properties of the BPM with respect to rotations. This question directly arises from the following considerations. It is clear that any backscattering phase matrix is measured in a randomly oriented polarization basis of a lidar therefore a question arises on the effect of this basis orientation on the view of the matrix. It is also interesting to elucidate what kind of information can be extracted from the analysis of the matrix view. According to the known theorem,[4] 2×2 S matrices of amplitude transportations occurring at light scattering in backward direction have the property that $S_{12} + S_{21} = 0$. A corollary of this theorem gives the following view of the backscattering phase matrix a:

$$a_{ij} = \begin{pmatrix} a & e & f & h \\ e & b & g & l \\ -f & -g & c & m \\ h & l & -m & d \end{pmatrix} . \qquad (1)$$

These relationships between elements of this matrix do not depend on the orientation of a lidar polarization basis and must hold in any case. This can easily be shown with the help of the transformation

$$a'_{ij} = \boldsymbol{R}(\varphi)\, a_{ij}\, \boldsymbol{R}(\varphi) , \qquad (2)$$

where $R(\varphi)$ is the matrix operator describing the transformation of a Stokes vector due to rotation of the coordinate system by an angle φ around the direction of sounding beam propagation. In the case of a counterclockwise rotation, if looking after the beam of radiation, the operator $R(\varphi)$ is presented by the matrix

$$\boldsymbol{R}(\varphi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & C_2 & S_2 & 0 \\ 0 & -S_2 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} , \qquad (3)$$

where $C_2 = \cos 2\varphi$ and $S_2 = \sin 2\varphi$.

Transformation (2) results in the following relations between elements of the matrices a' and a:

$$a' = a , \quad h' = h , \quad d' = d , \qquad (4)$$

that is, the corner elements of the BPM are invariant with respect to rotations. In addition, we have

$$e' = C_2\, e - S_2\, f ; \qquad f' = S_2\, e + C_2\, f ; \qquad (5)$$

$$l' = C_2\, l + S_2\, m ; \qquad m' = C_2\, m - S_2\, l ; \qquad (6)$$

$$\begin{aligned} b' &= C_2^2\, b - 2\, C_2 S_2\, q - S_2^2\, c ; \\ c' &= C_2^2\, c - 2\, C_2 S_2\, q - S_2^2\, b ; \\ q' &= C_2^2\, q + C_2 S_2 (\, b + c) - S_2^2\, q . \end{aligned} \qquad (7)$$

These relations allow the following conclusions to be drawn.

1. If only diagonal elements of a measured matrix are nonzero values and $a_{22} = -\,a_{33}$ then such a matrix is diagonal at any orientation of a lidar polarization basis (LPB).

2. If a measured matrix is diagonal but the condition $a_{22} = -\,a_{33}$ does not hold then under other orientations of the LPB the elements a_{23} and a_{32} differ from zero, the rest off–diagonal elements of the matrix being equal to zero. In this case the direction of preferred polarization of scattered light observed at irradiation of the aerosol ensemble with a linearly polarized light turns at some angle while ellipticity is absent. Such quite rare situations we have observed in our studies in 1988–1990 and included them into the developed classification[2] of crystal aerosols as the third type of scattering.

3. If only the off–diagonal elements a_{23} and a_{32} of a measured matrix are nonzero, then based on Eqs. (7) one can find such an orientation of the LPB which provides for a diagonal form of the backscattering phase matrix and determine the elements a_{23} and a_{32} in this new basis.

4. The same can be shown to be valid for the pairs of elements a_{24}, a_{34} and a_{42}, a_{43}.

As concerning the classification of the types of light scattering[2] it can be revised a little bit. Thus the fourth type of scattering in this classification can be shown based on relations (4) and (6), to be a particular case of the fifth type, though it also can be an independent type when all the off–diagonal elements of the matrix are equal to zero, except for a_{14} and a_{41}.

In a practically important case of a BPM, because of its high frequency of occurrence, when we have $a_{11} = -a_{33}$ and $a_{22} = -\,a_{44}$, it is possible to predict the view of the forward scattering phase matrix. Assumption on the symmetry properties of an aerosol ensemble resulting in such a BPM view are similar to the symmetry properties resulting in the diagonal forward scattering phase matrix with the elements $A_{11} = -\,A_{33}$ and $A_{22} = -\,A_{44}$. As a consequence, in the case of forward light scattering by an ensemble of aerosol particles with such a BPM one could expect only depolarization of scattered radiation. Other transformations of the polarization are merely absent. Of course some conclusions can definitely be drawn in the cases of other BPM views but here we will not consider them. It is important that in the case of the general form of a BPM there exists a possibility of finding such an orientation of a lidar polarization basis when one or another element of the BPM becomes zero or reaches its maximum. These angles can surely be used as reference points for making comparisons between measured BPM's and their theoretical models for one or another type of a crystal cloud. The information important for a comparison can be about the direction of preferred orientation of particles. Let us show this using the profiles of elements of a BPM depicted in the figure.

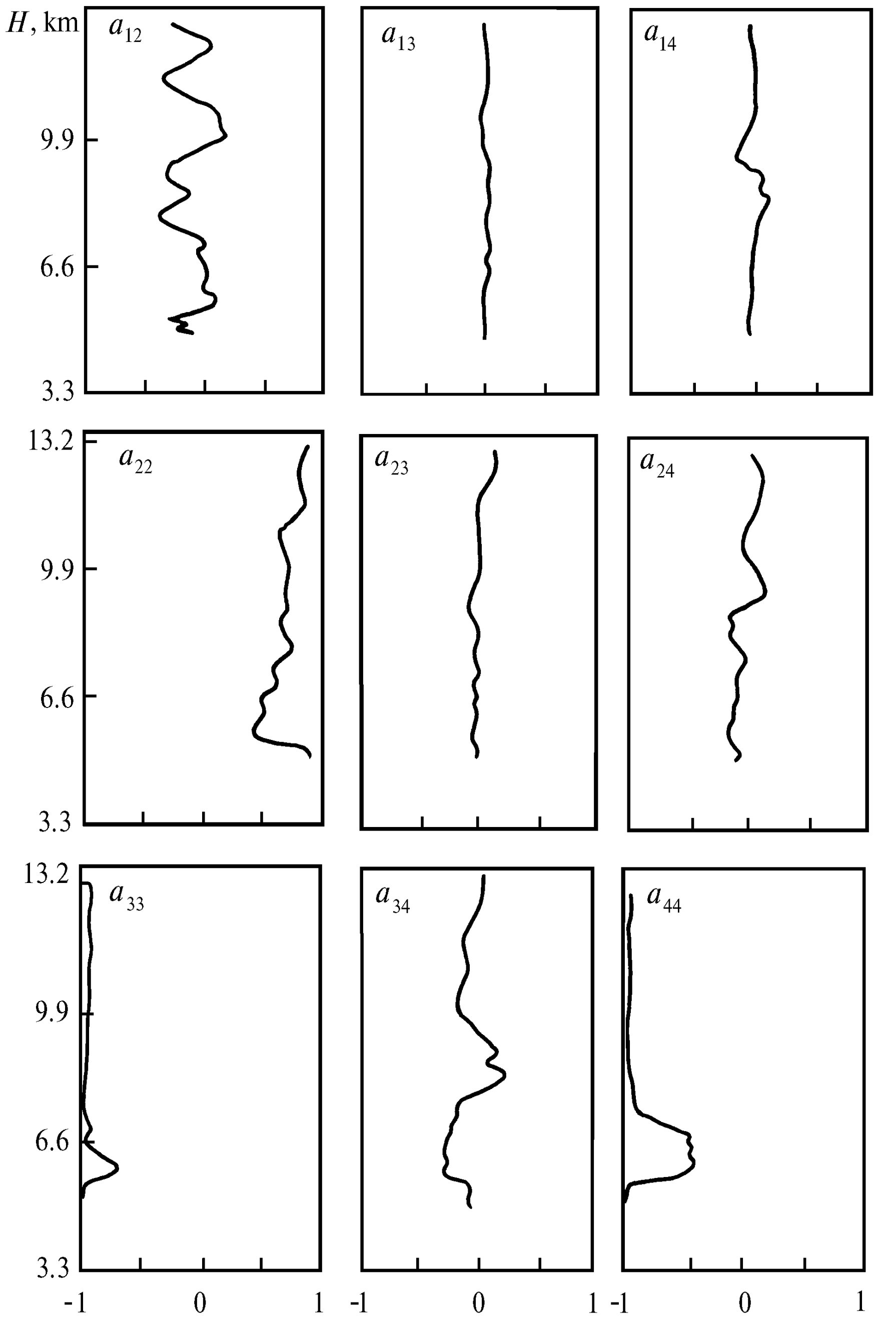

FIG. 1. An example of altitude behaviors of elements of the normalized backscattering phase matrix a_{ij} *observed in the layers of crystalline cloudiness. The elements not shown in the figure can be found from the following relations:* $a_{11} = 1$, $a_{21} = a_{12}$, $a_{31} = -a_{13}$, $a_{32} = -a_{23}$, $a_{43} = -a_{34}$, $a_{42} = -a_{24}$, $a_{41} = -a_{14}$. *Absolute values of the matrix elements are obtained by multiplying* a_{ij} *elements by the value* $[R(h) - 1]\beta_m(h)$, *where* $\beta_m(h)$ *is the backscattering coefficient of the molecular atmosphere and* $R(h)$ *is the scattering ratio.*

When choosing a model of an aerosol ensemble we take into account the following considerations. In spite of a great variety of shapes of ice particles two most widely spread shapes can be separated out. The first class involves hexagonal plates and dendrites (asterisks). The second class of shapes involves the hexagonal prisms and needles, the latter being, in fact, the same prisms but having not so distinct sides.

Because of obvious axial symmetry and a known property of orienting their axes along vertical direction, the particles of the first kind make up an ensemble characterized by a diagonal BPM. Here and below we assume that laser sounding is performed along vertical direction.

It is characteristic property of the particles of the second class of shapes that their symmetry axis is oriented horizontally and only due to either aerodynamic or electrostatic forces they can take a preferred orientation along horizontal or slant directions. In analysis of our data we assume that the profiles of the BPM elements refer just to this class of particles since the elements a_{12}, a_{24}, and a_{34} differ from zero. For the model of particles we take, in this case, an ensemble of cylinders of finite lengths (CFL). In doing so we keep in mind that at least for cylinders with the dimeters comparable to a light wavelength one can evidently consider them to be electrodynamically identical to the hexagonal prisms. Probably the same can be proved valid for larger crystals of this type too, if one takes into account the averaging effect of their rotations around their long axes. It is important, in this connection, that one can find in the literature[5] a solution of the problem on light scattering by an ensemble of CFL's whose axes lie in the reference plane and, moreover, in Ref. 6 one finds a solution of the problem in backscattering of light by an ensemble of randomly oriented CFL's. In this case (see Ref. 6) the matrix of amplitude transformations is as follows:

$$\begin{pmatrix} S_2 & S_3 \\ S_4 & S_1 \end{pmatrix} = \begin{pmatrix} A_2\cos^2\alpha - A_1\sin^2\alpha\ , & -(A_1 + A_2)\sin\alpha\cos\alpha \\ (A_1 + A_2)\sin\alpha\cos\alpha\ , & A_1\cos^2\alpha - A_2\sin^2\alpha \end{pmatrix}, \quad (8)$$

where A_1 and A_2 are the corresponding elements of the transformation matrix S for the CFL's whose axes are oriented in the reference plane, α is the angle at which the projection of a cylinder axis onto the horizontal plane is turned with respect to this reference plane. Once the matrix S is determined, the 4×4 scattering matrix, in terms of intensities,[7] can be found using the transformation

$$a = M\,(S \times S^*)\,M^{-1}\ . \quad (9)$$

The sign × in expression (9) denotes the Kronecker product of matrices, the asterisk denotes a complex conjugate, and the matrix operator M takes the form

$$M = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -i & i & 0 \end{pmatrix}.$$

Matrix M is a unitary matrix, that means that $M^{-1} = M^{+}$, where M^{+} is a Hermitian conjugate matrix.

Let us look at the figure once more and try to estimate the direction of preferred orientation of particles in the layer between 7.5 and 10 km altitude.

Average over heights values of the BPM elements make up the following matrix:

$$\begin{matrix} 1 & -0.34 & 0.02 & -0.08 \\ -0.34 & 0.75 & -0.01 & -0.11 \\ -0.02 & 0.01 & -0.93 & 0.20 \\ -0.08 & -0.11 & -0.20 & -0.93 \end{matrix}\ .$$

The most important peculiarity of this matrix is a large absolute value of the elements a_{12} and a_{21} while, at the same time, the elements a_{13} and a_{31} are close to zero. This means, first of all, that there is a significant anisotropy in the backscattering coefficient for sounding radiations of two mutually orthogonal linear polarizations. Second, this shows that the orientation of a LPB is close to a specific one, in the sense of the above–mentioned conclusions 4 and 5.

On the other hand, according to Eqs. (8) and (9) we have that

$$a_{12} = a_{21} = (1/2)\,(A_2 A_2^* - A_1 A_1^*)\cos 2\alpha\ . \quad (10)$$

It is clearly seen from Eq. (10) that at $\alpha = \pm\pi/4$ the element a_{12} becomes zero while at $\alpha = 0$ and $\pi/2$ it has extremums. It depends on the sign of the value $(A_2 A_2^* - A_1 A_1^*)$ which of these extremums we have, a minimum or a maximum.

In Ref. 6 it is shown that cylinders are most efficient light scatterers if their axes are orthogonal to the polarization plane of linearly polarized incident radiation. In the case we discuss now the polarization basis of the lidar is oriented so that light scattering is minimum for the radiation with the Stokes parameters (1, 1, 0, 0), in other words, for linearly polarized radiation whose electric field vector lies in the vertical plane that involves the x axis of the lidar polarization basis. As a consequence, this means that the direction of preferred orientation also lies in this plane and the value $(A_2 A_2^* - A_1 A_1^*)$ is negative.

Similar analysis of the characteristics of a layer at altitudes from 5.5 to 7 km reveals that there is also a preferred orientation of particles in it, but its direction makes an angle of 45° with that in the above–considered layer. In addition, small absolute values of the elements a_{44} and a_{22} observed in this layer well agree with the results from Ref. 6 that show a strong depolarization of backscattered radiation at α values close to $\pm\pi/4$. Unfortunately, the volume of one article does not permit a more detailed discussion of these results therefore we hope to continue the discussion of the subject under study in the nearest future.

REFERENCES

1. B.V. Kaul', O.A. Krasnov, A.L. Kuznetsov, and I.V. Samokhvalov, Opt. Atm. **4**, No. 4, 303–308 (1991).
2. B.V. Kaul', A.L. Kuznetsov, and E.R. Polovtseva, Atmos. Oceanic Opt. **5**, No. 6, 381–383 (1992).
3. *Second International Lidar Researches Directory,* Comp. by M.P. McCormick (Langley Research Center, Hampton, 1989), 64. pp.
4. H.C. van de Hulst, *Light Scattering by Small Particles* (Wiley, New York, 1957), 536 pp.
5. O.A. Vokovitskii, L.N. Pavlov, and A.G. Petrushin, *Optical Properties of Crystalline Clouds* (Gidrometeoizdat, Leningrad, 1984), 200 pp.
6. R.F. Rakhimov and D.N. Romashov, Atm. Opt. **4**, No. 10, 707–710 (1991).
7. V.A. Potekhin and V.N. Tatarinov, *Theory of an Electromagnetic Field Coherence* (Svyaz', Moscow, 1978), 208 pp.