EFFECT OF DETECTION NOISE ON THE ACCURACY OF SPECKLE INTERFEROMETRY

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A comparative analysis of two variants of reconstruction of an estimate of the Fourier spectrum phase of an image from a series of its short–exposed distorted images is performed from the viewpoint of stability with respect to the detection noise. It is shown that in practice we can restrict ourselves to only one of these variants.

Post–detector processing of a series of short–exposed images (SEI's) of an object being observed with the purpose of estimating its undistorted image is one of the simplest approaches to solve the problem of vision through the Earth's turbulent atmosphere. Now the theoretical development of these methods which are generally called in astronomy as speckle interferometry has been, for the most part, finished[1] and the agenda turns to the problem of studying their stability with respect to various additional (atmospheric distortions of the SEI's.[2,3]) In this paper we consider the effect of these distortions, which arise, e.g., when recording the SEI's on negatives.

From the viewpoint of mathematics, the optimal processing of the SEI's reduces to a Fourier transform of their intensity distributions $J(x)$, a formation of the series–averaged correlation parameters of the obtained Fourier spectra $\tilde{J}(\mathrm{f})$ based on the formula

$$Z(\mathrm{f}_1, \mathrm{f}_2) = \langle J(\mathrm{f}_1)J^*(\mathrm{f}_2)\rangle - S(\mathrm{f}_1, \mathrm{f}_2) ,$$

and the estimate of the modulus and the phase $\phi(f)$ of the Fourier spectrum $\tilde{O}(f)$ of the undistorted (sought–after) image of the object $O(x)$ by solving the equations

$$Z(\mathrm{f}_0, \mathrm{f}_0)\,|O(\mathrm{f}_0)|^2\,\sigma_{\mathrm{n}}^2(\mathrm{f}_0) ,$$

$$\varphi(\mathrm{f}_0) = \arg \sum_p \mathrm{b}(\mathrm{f}_0, \mathrm{f}_p) \exp\{i\,[\Psi(\mathrm{f}_0, \mathrm{f}_p) + \varphi(\mathrm{f}_p)]\} .$$

Here, <.> denotes an averaging over the series; $S(f_1, f_2)$ is the correlation function for the Fourier spectra of the detection noise, which is well known *a priori*; $\sigma_{\Pi}^2(f)$ is the *a priori* known mean squared modulus of the optical transfer function of the system atmosphere–telescope $\tilde{q}(f) = Z(f, f)/S(f, f)$ is the mean signal–to–noise ratio being experimentally estimated at the frequency f; f_{p} are the Fourier frequencies closest to f_0 and specified, e.g., by the discrete Fourier–transform algorithm; and $\mathrm{b}(\mathrm{f}_1, \mathrm{f}_2) = \tilde{q}(\mathrm{f}_1)\tilde{q}(\mathrm{f}_2)/(1 + \tilde{q}(\mathrm{f}_1)\tilde{q}(\mathrm{f}_2))$ is the weighting factor, which characterizes the relative accuracy Ψ. As regards the experimentally obtained estimate of the phase difference $\Psi(\mathrm{f}_1, \mathrm{f}_2)$, in the case in which the signal–to–noise ratio is small, it is determined as $\arg Z(\mathrm{f}_1, \mathrm{f}_2)$ when $\tilde{q}(\mathrm{f}_1)\tilde{q}(\mathrm{f}_2) < 1$ and when this ratio is large $\tilde{q}(\mathrm{f}_1)\tilde{q}(\mathrm{f}_2) > 1$, it can be more accurately determined as[4] $\arg\langle\exp\{i\arg\tilde{J}(\mathrm{f}_1)\tilde{J}^*(\mathrm{f}_2)\}\rangle$. Unfortunately, we failed to find theoretically the precise boundary of the change over from one variant to another. Moreover, in practice it is also difficult to calculate the distribution of the ratio $q(f)$. In this connection, a working formula based on the analysis of the proper distributions $J(x)$ of the starting SEI's, which enables one to make the above–indicated choice, is of interest. In order to obtain it, numerical simulation, which consisted in playing out 40 random realizations of the SEI's (for the fixed $O(x)$) with noise superimposed on the distributions of $I(x)$ as $J(x) = I(x) + n(x)$, where $n(x)$ is the normal "white" noise with zero mean and variance ^2_n, in reconstructing from them the estimates $O_{\mathrm{r}}(x)$ of the image for both variants of determining the phases Ψ, in calculating the normalized errors of reconstruction from the formula

$$E = \frac{\int \mathrm{d}\mathbf{x}[O(\mathbf{x}) - O_{\mathrm{r}}(\mathbf{x})]^2}{\int \mathrm{d}\mathbf{x} O^2(\mathbf{x})}$$

and in constructing the dependences of E on the signal–to–noise ratio $Q = \langle I\rangle/\sigma_{\mathrm{n}}$. Some values of the obtained dependences are presented in Table I, in which the subscripts 1 and 2 at E correspond to the first and second variants of estimating Ψ (in terms of arg Z). Note that by virtue of randomness of samplings of the 40 SEI's from the statistical ensemble of possible realizations these values are also random. However, an additional simulation has shown that the spread of these values does not exceed 10%. When analyzing these dependences, we must take into account that the information about the fine structure of the image is coded in the distributions of the intensity fluctuations of the SEI's above and below the mean distribution <I>, while the speckle interferometry methods reduce to extracting data from the average parameters of these fluctuations. The contrast of the fluctuations K defined as the ratio of their variance σ_{I}^2 to $\langle I\rangle^2$ is approximated by the relation[5] $K = K_\lambda K_{\mathrm{t}} K_0$, where

$$K_\lambda = \frac{\Delta\lambda_{\mathrm{c}}}{\Delta\lambda_{\mathrm{c}} + \Delta\lambda} ,\; K_{\mathrm{t}} = \frac{T_{\mathrm{c}}}{T_{\mathrm{c}} + T} , \text{ and}$$

$$K_0 = \frac{S_{\mathrm{t}}}{S_0 + S_{\mathrm{t}}}\left(\frac{S_{\mathrm{A}}}{S_{\mathrm{A}} + S_0}\right)^{-1}$$

are the spectral, temporal, and spatial components of the contrast. Here $\Delta\lambda$ is the width of the spectral range of

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optical radiation used when recording the SEI's, T is the exposure time per the SEI's, $\Delta\lambda_c$ and T_c are the wavelength correlation interval and the correlation time for atmospheric distortions, S is the angular surface area of the object being observed, $S_t = (\lambda/D)^2$ is the resolution of the employed telescope of diameter D, $S_A = (\lambda/r_0)^2$ is the mean resolution of the atmosphere, λ is the wavelength, and r_0 is Fried's parameter. From the viewpoint of speckle interferometry, it is natural to define the signal–to–noise ratio as $q = \sigma_1^2/\sigma_n^2 = KQ^2$. It should be noted that the image– and spectrum–averaged values q and $\tilde{q}$ are comparable. Simulation was carried out for $K_t = K_\lambda = 1$ and $K = K_0 \approx 1/70$. The first variant has proved to be more preferable than the second one for $q \le 3$ alone.

TABLE I. Normalized error variance E of image reconstruction based on 40 distorted images $J(x) = I(x) + n(x)$ for different signal–to–noise ratio Q ($Q = \langle I\rangle/\sigma_n$).

q	∞	100	20	10	5
E_1	0.088	0.094	0.165	0.197	0.279
E_2	0.088	0.094	0.151	0.209	0.281

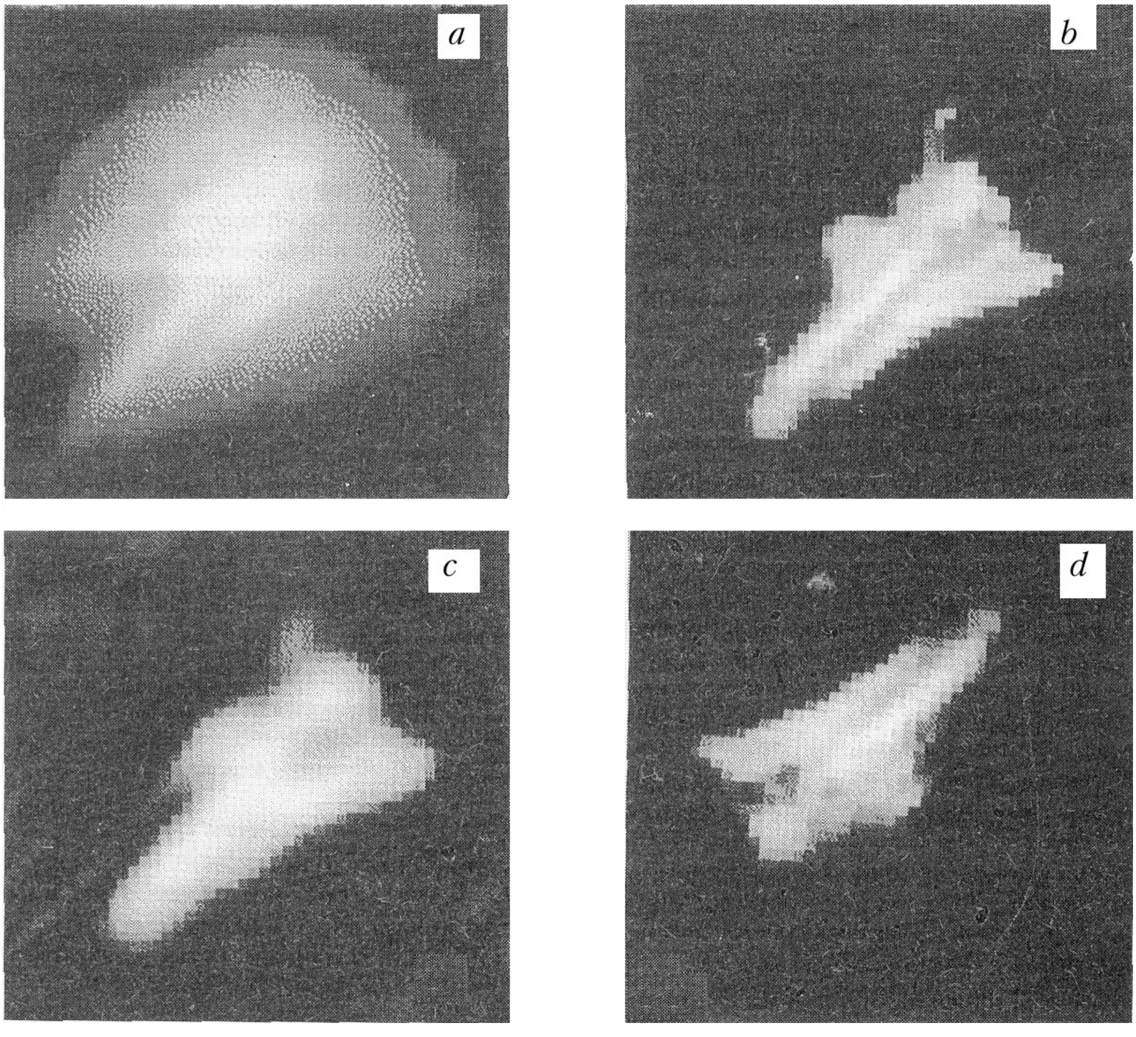

FIG. 1. Reconstraction results from 40 distorted ($J(x) = I^C(x) + n(x)$) short–exposed images $I(x)$: the characteristic original image $I(x)$ (a) and the images reconstructed for $Q = 100$ and $C = I$, $Q = 10$ and $C = 1$, and $Q = 100$ and $C = 0.7$ (b, c, and d), where $Q = \langle I\rangle/\sigma_n$.

TABLE II. Normalized error variance E of image reconstruction based on 40 distorted images $J(x) = I^C(x)$ for different exponent C.

C	0.5	0.7	0.9	1.0	1.1	1.3	1.5	2.0
E_1	0.441	0.240	0.116	0.088	0.098	0.118	0.169	0.842
E_2	0.534	0.162	0.103	0.088	0.089	0.118	0.258	0.906

We have additionally investigated the sensitivity of both variants to nonlinear distortions of the form $J(x) = I^{C}(x)$. These distortions may arise when transforming the distributions of optical densities from negatives of the SEI's into the corresponding intensity distributions owing to an inaccurate knowledge of the contrast ratio. The results given in Table II show that when $1.3 \ge C \ge 0.7$ the second variant is more preferable than the first. For illustration, the Fig. 1 shows the typical reconstruction results.

In conclusion it should be emphasized that in practice usually $Q \le 30$, $K_t \approx K_\lambda \approx 0.5$, $S_0 > S_A \gg S_T$, and $K_0 \approx (r_0/D)^2 \le 0.01$. As a result, it turns out that $q \le 3$, which leads to an inference that it is necessary to use the conventional estimate of the phase difference from the mean correlation products of the Fourier spectra of images.

REFERENCES

1. P.A. Bakut, A.D. Ryakhin, and K.N. Sviridov, Radiotekhn. Elektron. **33**, No. 7, 1446–1452 (1988).
2. P.A. Bakut, A.D. Ryakhin, and K.N. Sviridov, Opt. Spektrosk. **63**, No. 5, 1159–1162 (1987).
3. P.A. Bakut, V.N. Leksina, and A.D. Ryakhin, Atm. Opt. **3**, No. 2, 181–183 (1990).
4. P.A. Bakut, E.N. Kuklin, A.D. Ryakhin, et al. , Opt. Spertrosk. **58**, No. 6, 1314–1317 (1985).
5. P.A. Bakut, A.D. Ryakhin, K.N. Sviridov, and N.D. Ustinov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **29**, No. 3, 274–280 (1986).