FOUR-WAVELENGTHS LIDAR SENSING OF ATMOSPHERIC AEROSOL

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We present experimental results on four-wavelength lidar sensing of urban aerosol. The measurements were calibrated at the operating wavelengths using calibrated screens. The lidar data were processed using parametric and moment models of the aerosol optical properties. Uncertainties are assessed using the reduction technique, and the validity of the mathematical models is verified using a model reliability parameter.

Introduction

Remote lidar sensing is an effective tool for gathering information on the optical and microphysical parameters of atmospheric aerosol[1,2]. A variety of models of atmospheric aerosol are used for lidar data reduction[3,4], and analyses of lidar data employ such methods as optimal parametrization[5] and linear estimation[6]. The analysis of uncertainties in the parameters thus reduced is a complicated problem, certain aspects of which have been discussed[7] as they relate to the optical characteristics of aerosol.

In general, any processing scheme assumes a certain measurement technique, including mathematical models of the aerosol and the instrumentation. For this reason, it is necessary to analyze the validity of the mathematical model of the experiment along with the processing errors. Such an analysis of validity based on measured data is a very complicated problem, and usually is not undertaken in lidar experimental studies.

This paper details studies of the optical and microphysical parameters of atmospheric haze based on four-wavelength lidar sensing. Data interpretation and reduction uses various models of aerosol microstructure. Parameter uncertainties are monitored using the reduction technique[8], and the validity of the model used is verified using the reliability parameter[9].

EXPERIMENTAL SETUP AND MEASUREMENT TECHNIQUE

The experiment was carried out using the lidar facility designed at Moscow State University, which provided radiation at 1064 nm, 532 nm, 355 nm and 694 nm. Emission at the first three wavelengths was obtained using a YAG:Nd^{3+} laser and frequency doubler (CDA crystal) or tripler (KDP crystal); the fourth was obtained with a ruby laser. The ruby laser beam was collimated using a beam expander, reducing the beam divergence to 1.5 to 2 mrad. The YAG:Nd^{3+} laser output was practically single-moded, yielding a beam divergence of 1.5 mrad with no further collimation. *A* 240-mm diameter Maksutov-Cassegrain telescope as used as the lidar receiver. Backscattered radiation was detected with FEU-84 and FEU-83 photomultiplier tubes.

Absolute calibration of the lidar measurements was performed using the calibrated screens technique[10]. Screens made of Teflon and polyurethane were placed at z_s = 750 m from the lidar; the spectral and angular characteristics of radiation reflected from the screens were determined under laboratory conditions. In the single-scattering approximation, the calibration constants are

$$\kappa_1 = P(z_s \lambda_1)\, z_s^2 \,/\, \rho(\lambda_1)\, W_0(\lambda_1) \ ,$$

where $W_0(\lambda_1)$is the lidar pulse energy of at wavelength λ_1; $P(z_s, \lambda_1)$ is the magnitude of the signal received from a screen at the distance z_s, $\rho(\lambda_1)$ is the reflection coefficient of the screen at λ_1. The uncertainty in the calibration constants is about 25 per cent.

The results of measurements made on October 16, 1987 in the Lenin Hills region of Moscow are presented in Table 1.

Table 1

wavelength λ_1, nm	κ_i,· km^2·v·sr^{-1}·mJ	τ_i, ns	energy $W_0(\lambda_i)$, mJ	$P(\lambda_1, z)$, 10^{-2}·v			
				z =0.9 km	1.05	1.2	1.35
1064	72	15–20	14.1	0.45 ± 0.23	0.45	0.2	–0.1
694	27	25–30	82.7	5.0 ± 1.0	4.0	2.6	1.0
532	2560	15–20	6.9	55 ± 10.0	32.5	17.5	10.0
355	1162	15–20	1.2	11 ± 2.0	6.0	9.0	1.6

The measurements were made at night in clear weather. The visibility S_m was 10 to 15 km, and the relative humidity was about 50 per cent. Lidar returns from the screens and their uncertainties ΔP are presented in Table 1 for different z between z_s and z_r = 1350 m. In the single-scattering approximation, and neglecting molecular scattering, one can write down the normalized lidar return from aerosol as

$$\xi(\lambda_1, z) =$$

$$= \kappa_1 \beta_\pi (z, \lambda_1, m) \exp\left[-2 \int_{z_s} \beta (z, \lambda_1, m)\, dz' \right] + \nu_1, \quad (1)$$

$$\xi (\lambda_1, z) = 2 P (\lambda_1, z) z^2 / c \tau_p (\lambda_1) W_0 (\lambda_1),$$

$$i = 1, \ldots, I,$$

where I is the number of wavelengths used; $\beta_\pi(z, \lambda_1, m)$ and $\beta(z, \lambda_1, m)$ are the volume backscattering and extinction coefficients, respectively; z is the range, and m is the complex index of refraction of the particulate matter.

MODELS OF ATMOSPHERIC AEROSOL

Mathematically, lidar sensing of aerosol can be represented by

$$\xi = A(f) + \nu, \quad f \in \tilde{R}, \quad \xi \in R,$$

$$\xi = (\xi_1, \ldots, \xi_I)^T, \quad (2)$$

where f is the vector of the parameters sought, $A(f)$ is a nonlinear operator whose form depends on the approximation and aerosol model used, $\tilde{R}$ and R are the spaces in which the vectors f and ξ (vector of lidar signals) are defined, respectively; ν is the measurement uncertainty, with mean value equal to zero. The correlation operator E for uncertainties is by a diagonal matrix whose elements are the measurement uncertainties in the presence of noise alone.

The lidar interpretation problem consists of estimating the vector f based on the observed values of the backscattered signal ξ. The parameters f might be for example, those of the aerosol size distribution function $\varphi(r)$ (r, m, N in the case of Γ-function distribution) or its moments $M_j = \pi N \int_0^\infty r^j \varphi(r) dr$.

Most models of atmospheric aerosol assume that the aerosol particles are spherical and homogeneous. Then

$$\beta (z, \lambda_1, m) = N (z) \int_0^\infty \pi r^2 K (r, \lambda_1, m)\, \varphi(r, z)\, dr \quad (3)$$

$$\beta_\pi (z, \lambda_1, m) = N (z) \int_0^\infty \pi r^2 K_\pi (\bar{r}, \lambda_1, m)\, \varphi(\bar{r}, z)\, dr, \quad (4)$$

where $N(z)$ is the number density of particles, K and K_π are the extinction and backscattering efficiency factors, respectively, for particles of radius r at wavelength λ_1.

In this paper we have used the following aerosol models:

a) the moments model[11], in which the backscattering and extinction coefficients are represented as linear sums of moments of the function $\varphi(r)$, which are generally unknown:

$$\beta(\lambda_1) = \pi \sum_{j=1}^{J} B_{1j} M_{j+1},$$

$$\beta_\pi(\lambda_1) = \pi \sum_{j=1}^{J} D_{1j} M_{j+1}. \quad (5)$$

The expansion coefficients B_{ij}, D_{ij} are in fact the expansion coefficients for the efficiency factors $K(r, \lambda_1, m)$, $K_\pi(r, \lambda_1, m)$ expressed as a power series in r:

$$K (\lambda_1) = \sum_{j=1}^{J} B_{1j} r^{j-1},$$

$$K_\pi(\lambda_1) = \sum_{j=1}^{J} D_{1j} r^{j-1} \quad (6)$$

b) the parametric model, in which the function $\varphi(r)$ is assumed to be known. Here, in this work, we use a Γ-function distribution with unknown parameters r, μ, N; i.e.,

$$\varphi(r) = N \left[(\mu+1) / \bar{r} \right]^{\mu+1} r^\mu \times$$

$$\times \exp\left[-r (\mu+1) / \bar{r} \right] / \Gamma(\mu+1) \quad (7)$$

In this model, the optical characteristics β and β_π are calculated using expressions (3) and (4), where the kernels K and K_π are calculated numerically using the Mie formulas[12].

Since such an approach is computationally expensive, an approximate analytical parametric model[13] is also used which gives functional forms for β and β_π

$$\beta = F_1(\bar{r}, \mu, N), \quad \beta_\pi = F_2(\bar{r}, \mu, N).$$

For the approximation (6) in the moment model, prescribing $\varphi(r)$ enables one to write down the moments M_j in terms of r, μ, N:

$$M_j = N \left(\bar{r} / (\mu+1) \right)^j (\mu+1) \times \ldots \times (\mu+j). \quad (8)$$

The problem then reduces to finding the parameters r, μ, N, which according to (5) determine the optical characteristics of the aerosol. Such a model could be called the parametric moments model.

THE REDUCTION TECHNIQUE IN APPLICATION TO THE EXPERIMENTAL DATA INTERPRETATION AND THE RELIABILITY OF THE MATHEMATICAL MODEL

The problem of estimation errors is of paramount importance in the process of data interpretation. The reduction technique[8] enables one to interpret the experimental data with minimum uncertainties within the framework of a linear model of the measurement scheme. In this model, one can introduce the sensitivity η_f of the desired parameters to errors in the experimental data ξ:

$$\eta_f = (\sigma_f^2 / \sigma_{\xi_1}^2)^{1/2},$$

$$\sigma_\xi^2 = (1/I) \sum_{1=1}^{I} \sigma_{\xi_1}^2, \qquad (9)$$

where $\sigma_{\xi_1}^2$ is the normalized variance of the back-scattered signal at wavelength λ_1. In the parametric models, σ_f^2 is the mean variance of the parameters $\bar{r}$, μ, and N of the size-distribution function $\varphi(\bar{r})$. In the case of identical Mj+1 or optical parameters $\beta(\lambda_1)$, $\beta_\pi(\lambda_1)$, σ_f^2 can be defined as

$$\sigma_f^2 = (1/J) \sum_{j=1}^{J} \sigma_{M_{j+1}}^2,$$

or $$\sigma_f^2 = (1/I) \sum_{1=1}^{I} \sigma_{\beta_\pi}^2 \qquad (10)$$

Consider now a linear approximation to the nonlinear operator $A(f)$:

$$A(f) \simeq A(\bar{f}) + A'(f-\bar{f}),$$

where f is the desired parameter vector for which the linearization of operator A is being performed. The linear operator A' is a matrix whose elements are the derivatives $A_{ij} = \partial A_i \left|\partial f_j\right|_{f=\bar{f}}$.

In this work the linearization was carried out using the least squares method:

$$\| \Sigma^{-1/2} (\xi - A(\bar{f})) \|^2 = \inf_f \| \Sigma^{-1/2} (\xi - A(f)) \|^2$$

As a result, after linearization, we have transformed from (2) to a linear model $[A', \Sigma]$,in which the measured lidar return $\tilde{\xi}$ is

$$\tilde{\xi} = A' f + \nu, \qquad \tilde{\xi} = \xi - A(\bar{f}) + A' \bar{f}$$

The basic idea of the reduction technique as applied to lidar sensing is to transform the experimental data $\tilde{\xi}$ into the form they would have if the vector of aerosol parameters were measured directly. In other words, we seek a transformation T of the backscattered signal $\tilde{\xi}$ that could be interpreted as a direct measurement of f, distorted by a minimum possible error[8] h. The transformation T is chosen according the condition

$$E \| T \tilde{\xi} - f \|^2 =$$

$$= \inf_{T'} \left\{ E \| T' \tilde{\xi} - f \|^2 \,\middle|\, T' \in R \quad \tilde{R},\ T' A' = I \right\} \qquad (11)$$

The error $h = E \|T\nu\|^2 = E\|T\tilde{\xi} - f\|^2$ is called the reduction error. Thus, using the reduction technique for linearized lidar models, one can estimate the aerosol parameters f with known uncertainty, which is minimed within the framework of the model used.

The parameter estimates $T\tilde{\xi}$ and reduction error h depend on the measurement model used. The validity of the model can be assessed In terms of its reliability $\alpha(\xi)$, which in fact is the probability of erroneously rejecting the model, based on the measured data ξ and additional information. For the model $[A', \Sigma]$ and assuming normally distributed measurements errors, one obtains[9]

$$\alpha(\xi) = \int_X^\infty P_{\chi^2}(t)dt \quad X = \| \Sigma^{-1/2} (\tilde{\xi} - A' \hat{f}) \|^2 \qquad (12)$$

were $\hat{f} = T\tilde{\xi}$ is the solution of the reduction problem (11), and $P_{\chi^2}(t)$ is χ^2 distribution with $k = \dim R - \dim R(A')$ degrees of freedom.

The reliability (12) characterizes the validity of the linear approximation.

Interpretation and discussion of the experimental results

For the conditions under which the experiment was carried out (relative humidity 50 per cent) we took[3] m = 1.5 – *i* 0 3.

Since the number of wavelengths is insufficient for a reconsruction of the function $\varphi(r)$ from the sensing data, using only Eqs. (1), (3), and (4) with no a priori assumptions[2], one can only determine the gross characteristics of the size-distribution function.

An analysis of these data shows that the experimental results are in good agreement with a gamma distribution of the probability for $\tilde{r}_0$ over the full range of possible D/r_0 values. In addition, we see from these data that at large values of ε for H_2O were taken to be 82 for the static field and 1.77 for the optical wave. The expression used for calculations is valid only for low concentrations. The results obtained are presented in Table 2.

Table 2

Moments	Models			
	Momenta (j = 4)	Parametric	Parametric	
			Analytical	Momenta (j = 8)
M_2, $\mu m^2 cm^{-3}$	8.0 ± 10	14.0	15.8	15.6
M_3, $\mu m^2 cm^{-3}$	5.0 ± 1.0	5.0	5.3	5.2
M_4, $\mu m^2 cm^{-3}$	1.4 ± 0.3	2.0	2.0	1.0
M_5, $\mu m^2 cm^{-3}$	0,05 ± 0.03	0.9	0.8	0.8

We therefore the first discuss the data processed via the moment model, which does not require an a priori form for $\varphi(r)$. The moments obtained for $J = 4$ are given in the first column of Table 2 for the first layer of the sounding path. The backscattering β_π and extinction β coefficients calculated using Eq. (5) are presented in Fig. 1. The extinction coefficient has a maximum in the vicinity of $\lambda = 700$ nm, while the backscattering coefficient $\beta_\pi(\lambda_i)$ decreases monotonically with wavelength.

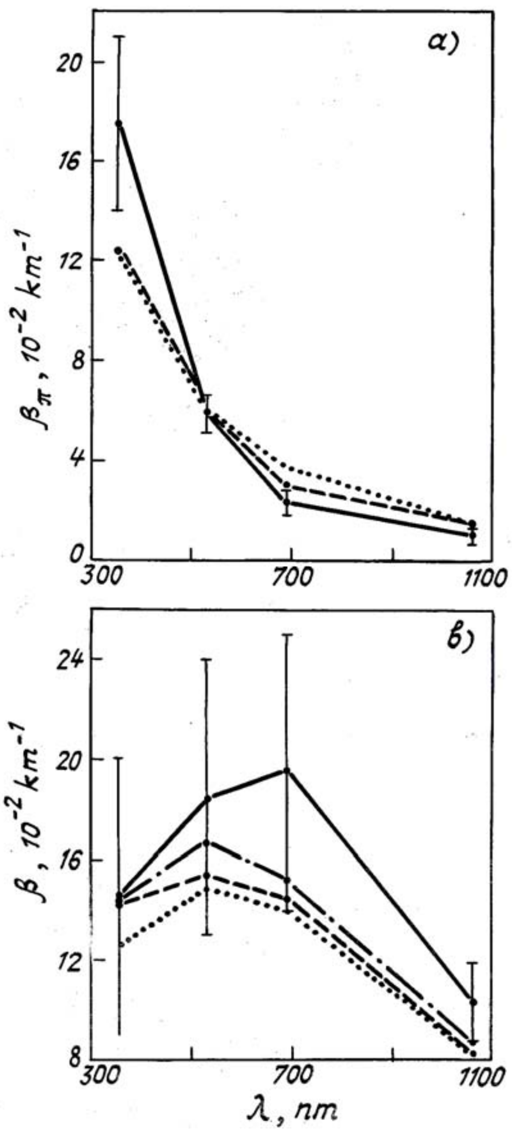

Fig. 1. Spectral behavior of the backscattering coefficient (a) and extintion coefficient (b), calculated using moment (solid), parametric (dotted), parametric moment (dashed), and analytical parametric (dash-dot) models for the nearest atmospheric layer $z = 0.75$ to 0.9 km. The uncertainties of the model are shown in the figure. The dash-dot and dotted lines coincide for β_π.

Computer simulations of the error obtained with the moment model and reduction technique have shown that the errors are overestimated compared to those obtained from statistical sampling. Thus, for example, one obtains sensitivity values $\eta_M = 64.0$, $\eta_{\beta\pi} = 3.5$, $\eta_\beta = 33$, using linear reduction, while statistical sampling gives $\eta_m = 1.6 - 2$, $\eta_{\beta\pi} = 9 - 1$, $\eta_\beta = 1 - 1.3$ for $\sigma_\xi = 10$–20%. It is clear that the backscattering coefficients are determined somewhat more accurately than the extinction coefficients, and the reconstruction uncertainty for the optical characteristics is smaller than that for M_{j+1}. This conclusion is confirmed by experiments in the field (see Table 1), for which the sensitivity values are $\eta_M = 58$, $\eta_{\beta\pi} = 1.1$, and $\eta_\beta = 5.5$.

Relative errors of ξ_1 measurements depend on the wavelength λ_1 and sounding distance. Is seen from Table 1, the lowest-accuracy lidar return measurements are at $\lambda_1 = 1064$ nm; errors at the other wavelengths are lower, and approximately the same. This level of uncertainty (see Table 1) does not provide one with satisfactory estimates of the Γ-function distribution parameters $\bar{r}$, lg$N$, μ. For example, data obtained using the analytical parametric model, with experimental errors, gave $r = 20 \pm .07$, lg$N = 3.0 \pm 8$, $\mu = 20 \pm 50$ for the nearest atmospheric layer. The reconstruction errors in this scheme rapidly increase with sounding distance. This means that the experimental data do not carry enough information about the desired parameters, and as a consequence the use of parametric models becomes problematic with no additional assumptions involved.

On the other hand, there is presently an enormous amount of data available on the properties of the most typical aerosols. The use of such data in the inversion scheme allows one to supplement the definition of the problem. For example, we processed the experimental data assuming that the aerosol is a random sample of the accumulative fraction of urban aerosol, whose microstructure is described by a Γ-function distribution with parameters[7] $\bar{r}_0 = 3 \pm 2$ μm, lg$N_0 = 1.5 \pm 1$, $\mu_0 = 5 \pm 4$. This information can be represented more conveniently. Let the vector of aerosol parameters f be a random vector whose expectation value E is $f = f_0$, $f_0 = (\bar{r}_0,\ \lg N_0,\ \mu_0)$. Its correlation operator F is a diagonal matrix whose elements are the variances of $\bar{r}_0$, lgN_0 and μ_0. The use of additional information in the data interpretation leads to a two-measurement scheme within the framework of the $[A, \Sigma]$ model:

$$\begin{cases} \vec{\xi} = A' f + \nu_1 \\ f_0 = I f + \nu_2 \end{cases} \qquad A = \begin{pmatrix} A' \\ I \end{pmatrix}, \qquad \bar{\Sigma} = \begin{pmatrix} \Sigma & 0 \\ 0 & F \end{pmatrix} \qquad (13)$$

This scheme was used to process of the field data, based on parametric models. It was found that μ does not depend on the parametric model, being almost entirely determined by the additional information.

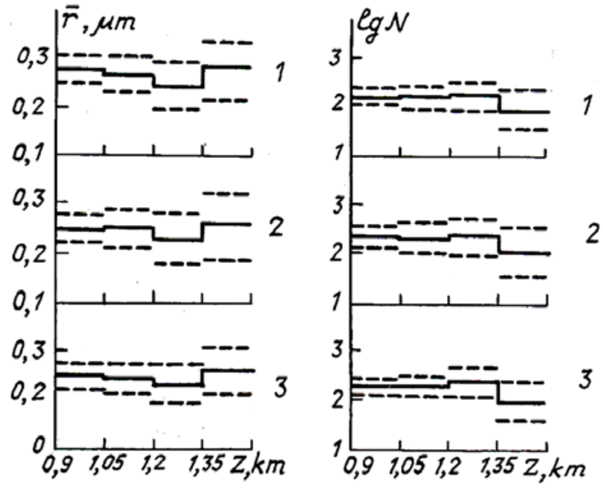

Fig. 2. The range behavior of the Γ-function distribution parameters $\bar{r}$, lgN, restored within the framework of the parametric model (1), analytical parametric (2), and parametric moments model (3). Dashed curves show the error margin.

Figure 2 illustrates the dependence of $\bar{r}$ and lg$N$ on the sensing range when they are reconstructed via parametric models. The results do not differ by more than 10%, in spite of the fact that the models are constructed using different approximations. This demonstrates that a priori specification of the class of size-distribution functions and additional information enforce quite rigorous constraints. The moments of the size-distribution function in Table 2 confirm this statement. These moments, calculated using $\bar{r}$, lg$N$ and μ reconstructed via parametric models, are practically identical.

At the same time, the moment calculated on the basis of the moment model differ from those obtained using parametric models. However, it should be noted that the difference between aerosol mass density values, which is determined by M_3, does not exceed 6 per cent among all the models used.

The $\beta(\lambda_1)$ and $\beta_\pi(\lambda_1)$ values recovered within the framework of parametric models differ by at most 15%, while they may differ -from those calculated using the moments model by 40% (see Fig. 1). This also bears out the conclusion that a priori determination of the class of size-distribution functions can influence the final results more strongly than the type of the model used. At the same time it is seen from the data presented here that spectral behavior of the optical characteristics obtained with either the moment or parametric models is qualitatively the same, regardless of the model used. Note that the parametric models are inherently aimed at the Interpretation of data in terms of $\bar{r}$, lg$N$ and μ, being thus optimal, in the sense of minimum uncertainties, for this purpose. The use of additional information in the moment model also reduces the uncertainties to be achieved (see Table 2 and Fig. 1).

Investigation of the validity of this models has shown that the use of additional information in all the models considered enables one to achieve a reliability $\alpha(\xi)$ between 0.98 and 0.99. This means that based on the experimental data and a priori information, one can not reliably reject from any of the models. It is interesting, finally, to investigate the influence of m on the data processing results. To illustrate this, we have processed experimental data within a parametric model with $m = 1.33 - i0$, the additional information being the same as before. As a result, we obtained for the nearest atmospheric layer the values $\bar{r} = 0.50 \pm 0.07$ μm, Ig$N = 2.0 + 0.2$, $\mu = 4.8 \pm 3.7$. The reliability parameter decreased from 0.98 to .0.85. The meteorological conditions of the experiment indicate that the value of $m = 1.5 - i \cdot 0$ is more likely than $m = 1.3 - i \cdot 0$, which probably accounts for the decrease in reliability.

Conclusion

Field experiments on four-wavelength sensing of atmospheric aerosols have been carried out using the first, second and third harmonics of a Nd^{3+}YAG laser and ruby laser.

The experimental data enable one to reconstruct the spectral behavior of the extinction and backscattering coefficients of an aerosol haze. The spectra obtained using moment and parametric models in the data processing procedure are similar.

A priori assumptions about the range of aerosol parameter variations enable one to interpret experimental data of modest accuracy. The a priori specification of $\varphi(r)$ in parametric models sets stringent limits on lidar data interpretation. For that reason the moment model, in wich the form of $\varphi(r)$ is not specified a priori, is preferable when such assumptions cannot be made.

The reduction technique can be considered an efficient algorithm for assessing the uncertainties of parameter recovery. Numerical simulations show that the technique provides an upper limit on the uncertainties In the moments obtained from laser sensing data.

As far as the validity of the models examined is concerned, they are indistinguishable within the context of the experimental studies discussed in this paper.

The analytical parametric, parametric moment, and moment models require fewer computations than the parametric ones. Therefore these models can be recommended for practical use in lidar data processing.

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