ON THE POWER OF THE ECHO–SIGNAL FROM A RANDOMLY ROUGH SURFACE WITH COMBINED SCATTERING PHASE FUNCTION OF LOCAL SECTIONS IN THE CASE OF NADIR SENSING THROUGH THE ATMOSPHERE

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The power of the echo–signal received from a randomly rough surface with combined scattering phase function of elementary locally flat sections is studied for the case of monostatic nadir sensing through the atmosphere. Analytical expressions are derived for the average received power and delay and width of the echo–pulse from the surface with scattering phase function comprising diffuse and quasispecular components when sensing through the optically thick aerosol atmosphere. It is shown that surface roughness can substantially distort the echo–pulse shape both in the transparent and optically dense atmosphere.

The shape of a laser echo–pulse received from a randomly rough surface with combined scattering phase function of local sections comprising quasispecular and diffuse components was considered in Ref. 1 for the case of sensing through the atmosphere along slant paths. Below we study the echo–pulse power as well as its delay and width in the case of vertical sounding of the surface with combined scattering phase function of local sections through the atmosphere.

Let us assume that every locally flat element of the randomly rough surface S has the combined scattering phase function with quasispecular and diffuse components. Let the surface S be sensed by a pulsed signal in the nadir direction through the homogeneous scattering atmosphere with strongly elongated scattering phase function.[2,3] If the angle at which the receiving aperture can be observed from the points on the scattering surface is much smaller than the angular width of the scattering phase function of radiation reflected from the surface, the characteristic scale of variation in the surface slopes, and the field–of–view angle of the receiver, the relation for the power recorded by the receiver has the form[1]

$$P(t) \simeq \frac{A}{\pi}\frac{1}{\alpha\dfrac{2}{n+2}+\beta\Delta^2}\int\limits_{S_0}\frac{d\mathbf{R}_0}{n_z}E_s(\mathbf{R}_0)\,E_r(\mathbf{R}_0)\,F_f(t',\mathbf{R}_0,\zeta)\times$$
$$\times\left[\alpha(n_z)^n+\beta\exp\left\{-\frac{\kappa_y^2}{\Delta^2}(C-\gamma_x D)^2-\right.\right.$$
$$\left.\left.-\frac{\kappa_x^2}{\Delta^2}\left[R_{0y}s-\gamma_y\kappa_y(C\gamma_x+D)\right]^2\right\}\right], \qquad (1)$$

where

$$F_f(t',\mathbf{R}_0,\zeta)\simeq f\left(t'+\frac{2\zeta}{c}-\frac{R_0^2}{cL}\right),\quad t'=t-\frac{2L}{c},$$

$$\kappa_{x,y}=\frac{n_z}{\sqrt{1-n_z^2\gamma_{x,y}^2}},\quad C=R_{0x}s,\quad s=\frac{A_s}{B_s}+\frac{A_r}{B_r},$$

$$D=\left[1-\left(\frac{A_r R_0}{B_r}\right)^2\right]^{1/2}+\left[1-\left(\frac{A_s R_0}{B_s}\right)^2\right]^{1/2},$$

$$A_{s,r}=0.5\left(\alpha_{s,r}^2+\sigma L\langle\gamma^2\rangle\right)^{1/2},$$

$$B_{s,r}=\frac{0.5L\left(\alpha_{s,r}^2+0.5\sigma\langle\gamma^2\rangle L\right)}{\alpha_{s,r}^2+\sigma L\langle\gamma^2\rangle},$$

$E_s(\mathbf{R})$ and $E_r(\mathbf{R})$ are the irradiances from the real and fictitious sources[2,4] on the surface S, L is the distance from the lidar to the surface S_0 (projection of S onto the plane $z = 0$), $2\alpha_s$ is the angular divergence of the source, $2\alpha_r$ is the field–of–view angle the receiver, σ is the scattering coefficient of the atmosphere, $\langle\gamma^2\rangle$ is the variance of the beam deflection angle arising during an elementary scattering event in the atmosphere, ζ and $\gamma = \{\gamma_x, \gamma_y\}$ are the height and the vector of the slopes of the rough surface, $\mathbf{n} = \{n_x, n_y, n_z\}$ is the unit vector of the normal to the elementary area of the surface S, A is the reflectance of the even area, $f(t)$ describes the shape of the sensing pulse, α and β are the coefficients determining the diffuse and quasispecular reflection contributions, and n and Δ are the parameters characterizing the angular width of the diffuse and quasispecular components of reflection.

Assuming the distribution of heights and slopes of the surface S to be normal and averaging relation (1) over ζ and γ, we can derive the following relation for average (over the ensemble of rough surfaces) power $\overline{P}(t)$ of the echo–signal recorded by the receiver from the randomly rough surface with combined scattering phase function of local sections for the case of vertical sensing through the atmosphere (assuming that the shape of the sounding pulse is Gaussian, i.e., $f(t) = 2\pi^{-1/2}\exp\{-4t^2/\tau_s^2\}$, and the surface is smoothly rough, i.e., $\gamma_x^2, \gamma_y^2 \ll 1$):

$$\overline{P}(t) = \frac{1}{\alpha\frac{2}{n+2} + \beta\Delta^2}\,\frac{A P_0 r_r^2 \alpha_r^2 \tau_s c L}{32 B_r^2 B_s^2}\exp\left\{-(\varepsilon-\sigma)\,2L - \frac{4(t')^2}{\tau_s^2\,\overline{\nu}_f}\right\}\times$$

$$\times\left\{\alpha F_f(\gamma_0)\exp(E_1)\left[1-\Phi(E_1^{1/2})\right] + \right.$$

$$\left. + \beta\mu_f^{-1}\exp(E_2)\left[1-\Phi(E_2^{1/2})\right]\right\}, \qquad (2)$$

where

$$E_1 = E_0 p_1^2,\ p_1 = p - \frac{8t'}{\tau_s^2 c L \overline{\nu}_f},\ p = \frac{1}{4B_s^2} + \frac{1}{4B_r^2},$$

$$\overline{\nu}_f = 1 + \frac{32\sigma^2}{\tau_s^2 c^2},\ E_0 = \frac{\tau_s^2 c^2 L^2 \overline{\nu}_f}{16},\ E_2 = E_0 p_2^2,$$

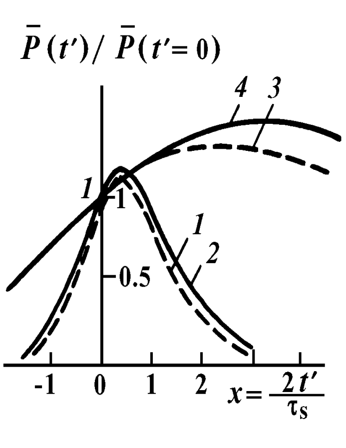
$$p_2 = p_1 + \frac{s^2}{\Delta^2\mu_f},\ \mu_f = 1 + \frac{8\gamma_0^2}{\Delta^2},$$

$$F_f(\gamma_0) = \left(\frac{1}{2\gamma_0^2}\right)^{(n+1)/4}\exp\left(\frac{1}{4\gamma_0^2}\right) W_{-\frac{(n+1)}{4};-\frac{(n-1)}{4}}\left(\frac{1}{2\gamma_0^2}\right),$$

σ_0^2 and γ_0^2 are the variances of heights and slopes of the randomly rough surface S, $W_{n,m}(x)$ is the Whittaker function, r_r is the effective radius of the receiving aperture, P_0 is the power emitted by the source, ε is the extinction coefficient of the atmosphere, and τ_s is the width of the sensing pulse.

As σ_0 and $\gamma_0 \to 0$ formula (2) coincides with the relation for the echo–signal power received from the flat surface with combined scattering phase function when sounding through the atmosphere.[5] For $\beta = 0$, $n = 0$, and $\sigma = 0$ formula (2) transforms into the relation for the average power of the echo–signal received from the randomly rough locally Lambertian surface through the transparent atmosphere. For α and $\sigma = 0$ as $\Delta \to 0$ formula (2) transforms into the relation for the average power of the echo–signal received from the randomly rough locally specular surface through the transparent atmosphere.

Figures 1 and 2 show the calculated results of the shape of the echo–pulse received from the randomly rough surface with combined scattering phase function of local sections for different values of the parameter $\gamma = \beta/\alpha$ (for different relative contributions of diffuse and quasispecular components of the scattering phase function of the surface). The quantities $\overline{P}(t')/\overline{P}(t'=0)$ were calculated from formula (2) with the following values of the parameters: $\alpha_s = 10^{-2}$, $\alpha_r = 10^{-1}$, $\tau_s = 10^{-9}$ s, $\Delta = 10^{-2}$, $L = 10^3$ m, $n = 0$, $\sigma\langle\gamma^2\rangle = 0$ (Fig. 1), $\sigma\langle\gamma^2\rangle = 10^{-5}\ \mathrm{m}^{-1}$ (Fig. 2), $\gamma = 0.1$ (curves *2* and *4*), $\gamma = 0.9$ (curves *1* and *3*), $\sigma_0 = 0$ and $\gamma_0 = 0$ (curves *1* and *2*), and $\sigma_0^2 = 3\cdot10^{-2}\ \mathrm{m}^2$ and $\gamma_0^2 = 10^{-8}$ (curves *3* and *4*).



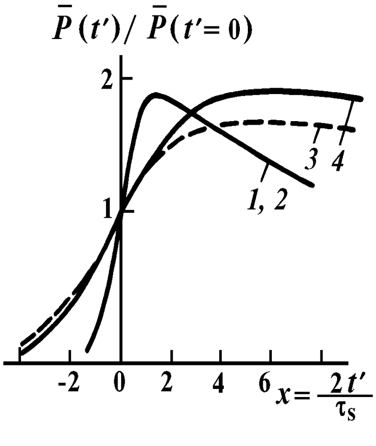


FIG. 1. Echo–pulse received from the randomly rough surface with combined scattering phase function through the transparent atmosphere.

FIG. 2. Echo–pulse received from the randomly rough surface with combined scattering phase function through the optically thick atmosphere.

It can be seen from the figures that the echo–signal shape depends weakly on the relative contributions of diffuse and quasispecular components of the scattering phase function of the surface when sensing in the nadir. The randomly rough surface and turbidity of the atmosphere strongly distort the echo–signal shape. This is physically connected with the increase of the spot size on the reflecting surface from which the echo–signal arrives to the receiver (due to spreading of the quasispecular component of the scattering phase function of the surface in the atmosphere and of directions of normals to the local reflecting sections).

The delay and width of the echo–signal are the most important parameters determining the temporal behavior of its power. Let us define the delay T and width τ of the echo–pulse as follows:

$$T = \frac{\int_{-\infty}^{\infty} dt\,\overline{P}(t)\,t}{\int_{-\infty}^{\infty} dt\,\overline{P}(t)},\quad \tau^2 = \frac{\int_{-\infty}^{\infty} dt\,\overline{P}(t)\,(t-T)^2}{\int_{-\infty}^{\infty} dt\,\overline{P}(t)}. \qquad (3)$$

Then using formula (1) and performing the calculations, we obtain

$$T = K\,T_L + (1 - K)\,T_{qs}\,, \qquad (4)$$

$$\tau^2 = K\,\tau_L^2 + (1 - K)\,\tau_{qs}^2 + K\,(1 - K)\,(T_L - T_{qs})^2\,, \qquad (5)$$

where

$$T_L = \frac{2L}{c} + \frac{1}{p\,c\,L}\,,\quad T_{qs} = \frac{2L}{c} + \frac{1}{c\,L}\left(p + \frac{s^2}{\Delta^2\,\mu_f}\right)^{-1},$$

$$\tau_L^2 = \frac{\tau_s^2}{8\,\hat{\omega}_f} + \frac{1}{p^2\,c^2\,L^2}\,,\quad \tau_{qs}^2 = \frac{\tau_s^2}{8\,\hat{\omega}_f} + \frac{1}{c^2\,L^2}\left(p + \frac{s^2}{\Delta^2\,\mu_f}\right)^{-2},$$

$$\hat{\omega}_f = \left(1 + \frac{32\,s^2}{c^2\,\tau_s^2}\right)^{-1},\quad \gamma = \frac{\beta}{\alpha},\quad K = \frac{F_f(\gamma_0)\,p^{-1}}{F_f(\gamma_0)\,p^{-1} + \gamma\mu_f^{-1}\left(p + \dfrac{s^2}{\Delta^2\,\mu_f}\right)^{-1}},$$

(T_L, τ_L) and (T_{qs}, τ_{qs}) are the delay and width of the echo–pulse received from the randomly rough locally Lambertian and locally quasispecular surfaces, respectively, when sounding in the nadir through the atmosphere.

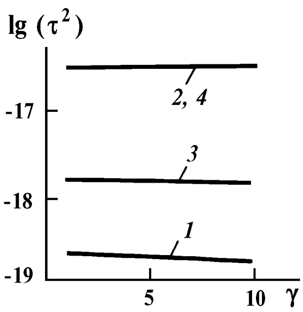


FIG. 3. Echo–pulse width vs the relative contribution of diffuse and quasispecular components of the scattering phase function of the surface.

Figure 3 shows the calculated results of the shape of the echo–pulse received from the randomly rough surface when sensing in the nadir through the atmosphere as a function of the parameter characterizing the relative contributions of diffuse and quasispecular components of the scattering phase function of the surface.

Calculations were performed using formula (5) with the following values of the parameters: $\alpha_s = 10^{-2}$, $\alpha_r = 10^{-1}$, $\tau_s = 10^{-9}$ s, $\Delta = 10^{-2}$, $L = 10^3$ m, $n = 0$, $\sigma\langle\gamma^2\rangle = 0$ (curves *1* and *3*), $\sigma\langle\gamma^2\rangle = 10^{-5}$ m^{-1} (curves *2* and *4*), $\sigma_0 = 0$ and $\gamma_0 = 0$ (curves *1* and *2*), and $\sigma_0^2 = 3\cdot10^{-2}$ m^2 and $\gamma_0^2 = 10^{-8}$ (curves *3* and *4*).

It can be seen from the figure that the echo–pulse width depends weakly on the relative contributions of diffuse and quasispecular components of the scattering phase function of the surface when sensing in the nadir. The randomly rough surface results in the increase of the echo–pulse width. This is associated with a spread of directions of normals (resulting in the increase of the spot size on the reflecting surface from which the signal is received) as well as of heights (resulting in the additional spread of widths of elementary echo–signals detected by the receiver). The atmospheric turbidity increases strongly the echo–pulse width and substantially weakens its dependence on the type of a reflecting surface.

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