Study of vertical transport of a buoyant admixture in water body by use of an eddy resolving model

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Vertical exchange in a water body is modeled using the equations of hydrothermodynamics on eddy resolving scales. The model under study represents an ensemble of convective structures under conditions of density instability of the upper layers of water. The exchange on a subgrid scale is described within the framework of a semi-empirical theory of turbulence. The modeling results on transport of air bubbles and oil emulsions are given as an example.

Introduction

At moderate and high winds, a lot of air bubbles come into the surface layer of water due to falling of wind-driven waves and spray drops. Relatively large bubbles having higher buoyancy leave the surface layers fast, whereas small bubbles are carried by turbulent pulsations deep into the water. The depth the bubbles can penetrate at depends on the degree of turbulence in the near-surface layer and it is usually small at moderately rough water. The transport into deeper layers sharply intensifies at density penetrative convection in water.

The development of convective processes may be connected with seasonal or diurnal cycles of surface cooling and is characterized by quick diving of cold water and slow lift of warm water masses.

The scattering and reflection properties of the bubble surface affect the optical properties of the water layer as a whole. Thus, as follows from observations, the transmission coefficient in the visible spectrum may almost halve in the zones of water mass convergence, where the increased concentration of bubbles is observed.[1]

Oil films, which break into clusters of suspended particles and solid conglomerates with positive buoyancy at the last stages of spilling, are one more example of an optically active surface admixture. In this case, the processes of vertical mixing can entrain oil emulsion down to the depth of about 15–20 m (in ocean).[2]

The mechanisms of vertical transport of a buoyant admixture are still poorly understood; theoretical studies are mostly based on one-dimensional models with the turbulence being described within the framework of diffusion K-closure.[2] A series of papers[3] is devoted to mesoscale modeling in a water body. In those papers, the Langmuir circulation is studied in a 2D formulation, but the subject for the study of admixture transport in water is not opened. The problems of generation of air bubbles at wind-induced waves were considered in Ref. 4, and their transport by ordered vertical flows was discussed in Ref. 1.

This paper is devoted to the study of penetrative convection in the upper layer of a water body and the related processes of diffusion and convective transport of admixtures having positive buoyancy.

Statement of the problem

The initial assumptions that form the basis for derivation of equations describing the studied processes in a water body are given in Ref. 5. Introducing the Cartesian coordinate system (x, y, z), in which the axis z is directed vertically upward, and denoting the components of the mean horizontal drift velocity and temperature in the water body as U, V, and T, we obtain the equations for mean horizontally homogeneous stream in the following form:

$$\frac{\partial U}{\partial t} = lV + \frac{\partial}{\partial z} K \frac{\partial U}{\partial z} - \frac{\partial}{\partial z} \overline{u}\overline{w},$$
$$\frac{\partial V}{\partial t} = -lU + \frac{\partial}{\partial z} K \frac{\partial V}{\partial z} - \frac{\partial}{\partial z} \overline{v}\overline{w}, \qquad (1)$$
$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} K_T \frac{\partial T}{\partial z} - \frac{\partial}{\partial z} \overline{\tilde{T}}\overline{w},$$

where K is the coefficient of turbulent exchange along the vertical; the overbar denotes horizontal averaging over the domain of definition; l is the Coriolis parameter; $(u, v, w, \tilde{T}, p)$ are the sought fields of velocity, temperature, and pressure of convective fluctuations.

The set of equations for describing the evolution of mesoscale convective fields has the form

$$\frac{\partial u}{\partial t} + w\frac{\partial U}{\partial z} = -\frac{1}{\rho_w}\frac{\partial p}{\partial x} + lv + D_{xy}\,u + \frac{\partial}{\partial z} K \frac{\partial u}{\partial z} + \frac{\partial}{\partial z} \overline{u}\overline{w},$$
$$\frac{\partial v}{\partial t} + w\frac{\partial V}{\partial z} = -\frac{1}{\rho_w}\frac{\partial p}{\partial y} - lu + D_{xy}\,v + \frac{\partial}{\partial z} K \frac{\partial v}{\partial z} + \frac{\partial}{\partial z} \overline{v}\overline{w}, \quad (2)$$

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$$\frac{\partial w}{\partial t} = -\frac{1}{\rho_w}\frac{\partial p}{\partial z} + g\beta_T \tilde{T} + D_{xy}\, w + \frac{\partial}{\partial z} K \frac{\partial u}{\partial w},$$

$$\frac{\partial \tilde{T}}{\partial t} + \frac{\partial T}{\partial z} w = D_{xy} \tilde{T} + \frac{\partial}{\partial z} K \frac{\partial \tilde{T}}{\partial z} + \frac{\partial}{\partial z} \overline{w\tilde{T}},$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

where

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + (U + u)\frac{\partial}{\partial x} + (V + v)\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

is the operator of individual derivative;

$$D_{xy} = \frac{\partial}{\partial x} K_x \frac{\partial}{\partial x} + \frac{\partial}{\partial y} K_y \frac{\partial}{\partial y}$$

is the operator of horizontal turbulent exchange; β_T is the coefficient of thermal expansion of water.

As the boundary conditions along the horizontal, we accept the assumption on the process periodicity along the x and y axes. We impose the following restrictions on the equations of the mean stream (1):

$$\rho_w K \frac{\partial U}{\partial z} = \tau_x,\ \rho_w K \frac{\partial V}{\partial z} = \tau_y;\ T = T_0 \text{ at } z = 0; \qquad (3)$$

$$U = V = 0;\ \frac{\partial T}{\partial z} = \gamma_H \text{ at } z = H, \qquad (4)$$

where the level $z = 0$ is thought to be coincident with the interface surface; τ_x and τ_y are tangential stresses of the wind in the near-water air layer, they are formed in the model of atmospheric quasistationary sublayer; T_0 is the preset temperature of the water surface; H is the lower boundary; γ_H is the stable temperature stratification of the water body in depth.

For the set (2) the boundary conditions have the form

$$u = v = w = 0,\ \tilde{T} = \tilde{T}_0(t, x, y) \text{ at } z = 0; \qquad (5)$$

$$u = v = w = 0;\ \tilde{T} = 0 \text{ at } z = H, \qquad (6)$$

where $\tilde{T}_0$ are small-amplitude random temperature perturbations.

Let us calculate the parameters of the vertical turbulent exchange for mean streams using the semi-empirical theory of turbulence and writing the evolution equations for the density of turbulent energy and the rate of its dissipation. The detailed description of the model of local dynamic interaction with the allowance for coupling of the flows of turbulent energy of the near-water air layer and the water body can be found in Ref. 6.

Turbulent diffusion in two-component "water–air bubbles" medium is modeled based on the assumption of spatial continuity of both substances. We also accept the hypothesis of small ordered motions of water due to the back flow of floating-up bubbles and neglect the effect of air compressibility in them. In this case, the equation of conservation of mass for the air-bubble component has the form

$$\frac{\partial S}{\partial t} + (U + u)\frac{\partial S}{\partial x} + (V + v)\frac{\partial S}{\partial y} +$$
$$+ (w - W_g)\frac{\partial S}{\partial z} = D_{xy} S + \frac{\partial}{\partial z} K_s \frac{\partial S}{\partial z}, \qquad (7)$$

where S is the volume concentration of the admixture (m^3/m^3); W_g is the rate of the admixture floating-up caused by the balance of the buoyancy and resistance forces.

The boundary conditions for Eq. (7) are written assuming that the admixture flow on the surface is known:

$$K_s \frac{\partial S}{\partial z} = P_0 \text{ at } z = 0; \qquad (8)$$

$$S = 0 \text{ at } z = H. \qquad (9)$$

Analysis of solution and conclusions

Consider the development of convection in water for the wind speed in the near-water layer equal to 10 m/s. It is a threshold value of the wind speed, at which the income of air bubbles to the under-surface layer increases sharply.[4] Using the circumstance that the stream structure acquires, and rather quickly, (for 20–30 min) the 2D character under the effect of wind, let us pass to consideration of the problem in the (x, z) plane, assuming $\partial/\partial y = 0$.

At the start time, the water body is thought stably stratified, so mixing of the upper layer is connected with the wind-driven waves and shear turbulence (forced convection), as well as thermal convective activity. The diurnal behavior of the temperature on the water surface is taken based on the measurement results on Lake Krasnoe.[7] According to these data, the surface temperature varied from 18°C in the afternoon to 15.8°C in the morning (04:00 L.T.), i.e., night cooling of the water body surface took place.

The volume concentration $S(x, z, t)$ was calculated for the bubble size ranging as $d \geq 0.07$ mm (the bubble diameter increased successively with the step $\Delta d = 0.01$ mm). According to data from Ref. 4, such a size spectrum was obtained by measurements in a storm pool. The rate of bubbles floating-up, which is equal to 0.2 cm/s at $d = 0.07$ mm, increases roughly by 0.14 cm/s for this increment of d.

The growth of the wave height with time, as well as shear generation, causes formation of the surface turbulized layer, in which air bubbles penetrate to the depth of several tens centimeters. Cooling of the surface causes the development of instability and, as a consequence, growth of small temperature perturbations up to finite amplitudes, and this process is accompanied by intense vertical motions. The mechanism of vertical transport provides for flood of supercooled thermics and their replacement by warm air masses from the lower layers. The profile of the mean temperature transforms with formation of the mixing layer, and this leads to blocking of the unlimited growth of the perturbation amplitudes. The lower boundary of the layer involved in convection is near the surface at the initial stage, then it moves in depth; as this occurs, the value of w peaks roughly at the center of the layer.

Air bubbles reaching the levels, where the absolute value of their floating-up speed is less than the speed of downwelling water flows, are entrained to the depth. Isolines of the field of dimensionless concentration $S_n = S/S_0$, where S_0 is the concentration of bubbles at $z = 0$, are shown in Fig. 1 (unfortunately, we found no data on the absolute values of the bubble concentration). We can see that the basic mass of the admixture is concentrated in the lower 1-m layer. However, in the submersion sections, where $w < 0$ (the extreme speed in developed thermics is 2–3 cm/s), the normalized concentration is nonzero and in the interval $0.2 < S_n < 0.4$ it reaches 6–7-m depth.

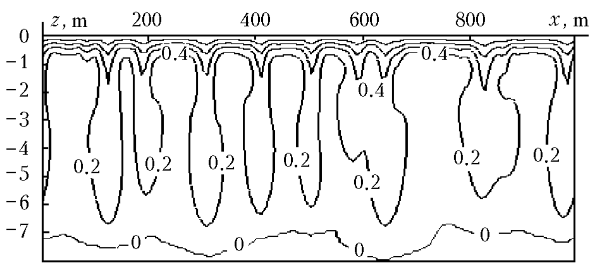

Fig. 1. The field of normalized volume concentration of air bubbles in the plane xz.

Figure 2a illustrates the vertical distribution of the $\bar{S}_n$ values averaged over x for the bubble fractions with d from 0.07 to 0.1 mm (curves *1–4*).

As the bubble diameters increase, the buoyancy force increases and penetration of bubbles into the depth is hampered. Largely, this is connected with the decrease of the admixture flow from the surface due to turbulent diffusion. In Fig. 2b, one can see the contributions of the convective $\overline{S_n w}$, ordered $\bar{S}_n W_g$, and subgrid-scale turbulent $K_s(\bar{S}_n)_z$ flows of the admixture (curves *1*, *2*, and *3*).

In Fig. 2b, the positive values of the flows correspond to the transport of the admixture in depth (note that the flow $\bar{S}_n W_g$ is negative). Analysis of Fig. 2b shows that the dominating processes immediately under the surface are turbulent transport and extrusion due to Archimedean force (curves *2* and *3*). These flows are of the same order of magnitude; the increase of the floating-up speed disturbs this balance and decreases the income of the admixture into the convective zone, where the flow $\overline{S_n w}$ (curve *1*) dominates.

Curve *1′* in Fig. 2a shows the concentration profile obtained neglecting the convection ($u = v = w = 0$ in Eq. (7)). Comparing curves *1* and *1′*, we have drawn the conclusion on the efficiency of the convective mechanism of admixture transport: the depth-summed volume concentration differs by more than 20 times.

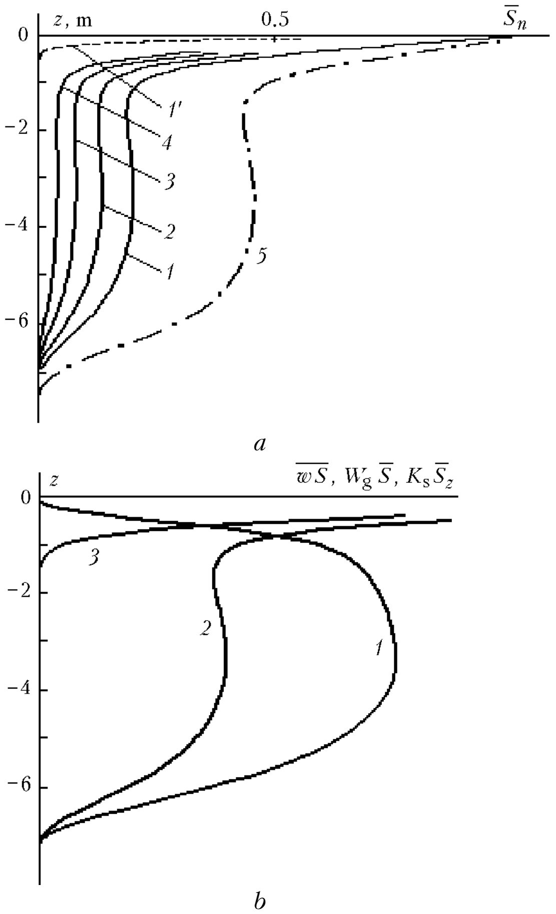

Fig. 2. Vertical distribution of the mean concentration of bubbles with the diameter of 0.07, 0.08, 0.09, and 0.1 mm with regard (curves *1–4*) and without regard (curve *1′*) for convective mixing (a). Curve *5* corresponds to the oil fraction. Vertical distribution of convective, ordered, and turbulent flows (curves *1–3*) (b).

Curve *5* in Fig. 2a illustrates spreading of the oil admixture. The size and density of the oil emulsion vary widely; for a convenient comparison, in this calculation we take $d = 0.07$ mm, $\rho_n = 0.8\,\rho_w$, where ρ_n is the density of oil particles. The accepted ρ_w value close to the water density decreases the role of the buoyancy force in the flow balance. Consequently, the contribution of the surface turbulent diffusion and convective flow increases, and this leads to the total increase of the rate of admixture transport to the depth and formation of a slight peak of the concentration in the central part of the mixing layer ($z = 4$ m, curve *5*).

Summarizing, let us note that penetrative convection plays an important role in formation of turbulent conditions in a water body and can change considerably the hydrooptical characteristics of the upper layer due to the transport of the buoyant admixture to the deep layers. The presented model of the convective ensemble gives the qualitatively correct pattern of the fine turbulent structure and can serve a convenient instrument for theoretical study of natural phenomena caused by convection in water bodies.

Acknowledgments

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