USE OF THE PHASE FUNCTION METHOD FOR CALCULATING CHARACTERISTICS LIGHT SCATTERING BY ABSORBING PARTICLES

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The technique for calculating characteristics of light scattering by radially inhomogeneous spherical absorbing particles is proposed. The technique is based on solution of equations method of phase functions for complex phases.

The study of light scattering properties of small spherical particles is an actual problem of modern optics.[1-3] Calculation of light scattering characteristics of absorbing radially inhomogeneous particles is of particular interest.[1] This problem can analytically be solved only in some special cases. No reliable numerical methods have been developed so far for its solution in the case of a randomly radial dependence of a refractive index of particles.[1-3] In this paper we propose a simple and effective method for calculating the characteristics of light scattering by absorbing radially inhomogeneous spherical particles. This method will make it possible to study general regularities of interaction between the electromagnetic radiation and spherical particles, as well as a analyze the conditions of their resonance interaction, and to explain the experimentally observed effects, which cannot be described within the Mie theory. This method develops the ideas[4] of applying the phase functions to reduce the order of equations arising in optics of inhomogeneous spherical particles.

Nonmagnetic spherical particles with radial dependence of the complex refractive index $m(r)$ are considered. Let us assume $m(r)$ to be a continuous differentiable function. In our calculations a is the radius of a particle k is the wave number of incident radiation, and $x = ka$ is the diffraction parameter of a particle. The differential equations of optics of radially inhomogeneous particles (see Eqs. (5.1.5) and (5.1.6) in Ref. 1) can be reduced to the first order equations if one introduces the complex phase functions δ_l^{ω} and δ_l^{q} which satisfy the equations

$$\frac{d}{d\rho}\delta_l^{\omega} = (m^2(\rho) - 1)[\cos\delta_l^{\omega}(\rho)\,\psi_l(\rho) - \sin\delta_l^{\omega}(\rho)\,\chi_l(\rho)]^2 -$$
$$- [\ln(m^2(\rho))]'\,[\cos\delta_l^{\omega}(\rho)\,\psi_l(\rho) - \sin\delta_l^{\omega}(\rho)\,\chi_l(\rho)]\times$$
$$\times[\cos\delta_l^{\omega}(\rho)\,\psi_l'(\rho) - \sin\delta_l^{\omega}(\rho)\,\chi_l'(\rho)]\,; \tag{1}$$
$$\frac{d}{d\rho}\delta_l^{q} = (m^2(\rho) - 1)\,[\cos\delta_l^{q}(\rho)\,\psi_l(\rho) - \sin\delta_l^{q}(\rho)\,\chi_l(\rho)]^2$$

with the boundary conditions

$$\delta_l^{\omega}(0) = \delta_l^{q}(0) = 0\,, \tag{2}$$

where φ_1 and χ_1 are the Riccati–Bessel functions, ξ_1 is the first–order Riccati–Hankel function, $\rho = kr$, and r is the current coordinate.

The scattered field coefficients[1,2] expressed in terms of phase functions have the form

$$a_l = \frac{\psi_l(x)\,\omega_l'(x) - m^2(x)\,\psi_l'(x)\,\omega_l(x)}{\xi_l(x)\,\omega_l'(x) - m^2(x)\,\xi_l'(x)\,\omega_l(x)}\,;$$
$$b_l = \frac{\psi_l(x)\,g_l'(x) - \psi_l'(x)\,g_l(x)}{\xi_l(x)\,g_l'(x) - \xi_l'(x)\,g_l(x)}\,, \tag{3}$$

where

$$g_l(x) = \cos\delta_l^{c}(x)\,\psi_l(x) - \sin\delta_l^{c}(x)\,\chi_l(x)\,;$$
$$\omega_l(x) = \cos\delta_l^{\omega}(x)\,\psi_l(x) - \sin\delta_l^{\omega}(x)\,\chi_l(x)\,;$$
$$g_l'(x) = \cos\delta_l^{c}(x)\,\psi_l'(x) - \sin\delta_l^{c}(x)\,\chi_l'(r)\,;$$
$$\omega_l'(x) = \cos\delta_l^{\omega}(x)\,\psi_l'(x) - \sin\delta_l^{\omega}(x)\,\chi_l'(x)\,.$$

A solution of Eq. (1) under boundary conditions (2) substituted into Eq. (3) gives the solution of the problem of light scattering by a particle which is characterized by a complicated radial dependence of the complex refractive index.

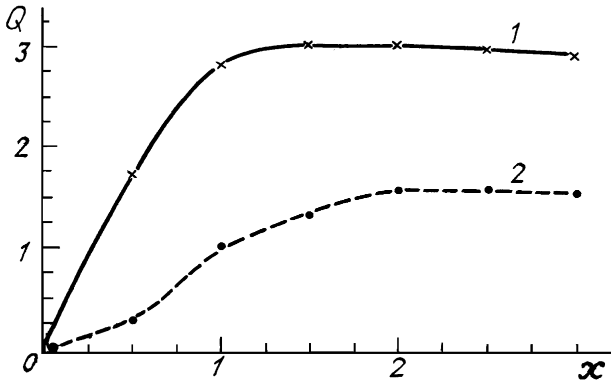

FIG. 1. Extinction Q_{ex} and scattering Q_{sc} efficiency factors as functions of the diffraction parameter x: solidcurve is for Q_{ex} calculated by formula (1) and crosses are the data taken from Ref. 3; and dashed curve is for Q_{sc} calculated by formula (1) and dots are the data taken from Ref. 3.

The calculations have been made[3] for iron spheres with the refractive index $n(r) = 1.27 + i1.37$ at the wavelength $\lambda = 4200$ Å (see Ref. 3). The extinction Q_{ext} and scattering Q_{sc} efficiency factors as functions of the diffraction parameter x obtained using the method of phase functions and taken from Ref. 3 are shown in the figure. The agreement between the results reveals a satisfactory accuracy of the method of phase functions in application to calculations of optical characteristics of the absorbing particles.

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Thus the algorithm has been developed, which allows one, due to the introduction of complex phase functions, to reduce the order of radially inhomogeneous particles arising in optics. The problem on determining the optical characteristics of such particles is reduced to the solution of the first–order differential equations (1) under initial conditions (2).

The method provides for the possibility of rapid and accurate calculation of optical characteristics of spherical particles with a sufficiently general form of radial dependence of the complex refractive index. This makes the method an efficient tool for solving some applied problems and for further development of optics of small particles.

REFERENCES

1. A.P. Prishivalko, V. A. Babenko, and V.N. Kuz'min, *Light Scattering and Absorption by Inhomogeneous and Anisotropic Spherical Particles* (Nauka i Tekhnika, Minsk, 1984).
2. M. Kerker, *The Scattering of Light and Other Electromagnetic Radiation* (Acad. Press, N.Y. and London, 1969).
3. M. Born and E. Wolf, *Principles of Optics, 4th ed.,* (Pergamon Press, Oxford, 1970).
4. L. Shafai, Can. J. Phys. **50**, No. 8, 743–749 (1972).