EFFECT OF QUANTIZATION NOISE ON THE ACCURACY OF SPECKLE INTERFEROMETRY

P.A. Bakut, V.N. Leksina, and A.D. Ryakhin

Scientific-Production Company "Astrofizika", Moscow
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We discuss the effect of noise due to finite number of image quantization levels on the accuracy of reconstruct ion using a series of images distorted by the atmosphere. We obtain a working formula relating the minimum number of quantization levels to the contrast of the distorted images. We present the results of a computer simulation.

In recent years, much attention has been devoted to research on the capabilities of speckle interferometry methods based on post-detector processing of a series of images of an astronomical object distorted by the atmosphere in order to obtain an estimate of the undistorted image. Optimal algorithms for this data-reduction procedure have now been synthesized and the potential capabilities of this method have been determined.[1] The agenda now turns to the problem of studying the stability of these methods with respect to various additional distortions (that is, in addition to the atmospheric distortions) to the images recorded in this fashion. This paper discusses the effect of noise introduced by the finite number of quantization levels in the images.

In systems where monochromatic images are recorded digitally using pulse-coded modulation, a binary code combination is assigned to each reading, with the number of quantization levels L being determined by the condition $L = 2^b$, where b is the number of binary digits during the coding process.

In the most common quantization method (uniform quantization), this corresponds to replacing the real intensity reading J by the value $l\Delta J$, where $\Delta J = J_{\max}/L$ is the quantization interval; $J_{\max}$ is the maximum intensity to be quantized, $l = [J/\Delta J + 0.5)$ is the level assigned to J; and, [...] is used to indicate the integer part of a number. The dispersion of the error produced by this substitution[2] is $(\Delta J)^2/12$. Our practical interest lies in determining the smallest number of digits b for which quantization error may be neglected.

A detailed analysis indicates that the information on the fine structure of the image is coded in the distribution of the intensity fluctuations in the distorted images about their mean value $\langle J\rangle$.

The contrast in the fluctuations K, defined as the ratio of their dispersion in the fluctuations to $\langle J\rangle^2$, can be approximated by the expression[3]

$$K = K_\lambda \cdot K_{tel} \cdot K_0,$$

where

$$K_\lambda = \frac{\Delta\lambda_c}{\Delta\lambda_c + \Delta\lambda}, \quad K_T = \frac{T_c}{T_c + T}.$$

$$K_0 = \frac{S_{tel}}{S_o + S_{tel}} \cdot \frac{S_0}{S_{atma} + S_o}$$

are the spectral time, and spatial components of the contrast, respectively, and $\Delta\lambda$ is the spectral range used for the light; T is the exposure time per image; $\Delta\lambda_c$ and T_c are the coherence wavelength difference and the coherence time for atmospheric distortions (the optimum conditions for recording Images by speckle interferometry assume $\Delta\lambda \le \Delta\lambda_c$, and $T \le T_c$); S_0 is the angular surface area of the object being observed; $S_{tel} = (\lambda/D)^2$ is resolution of the telescope (of diameter D) being used; $S_{atm} = (\lambda/r_0)^Z$ is the mean resolution of the atmosphere; λ is the wavelength; r_Q is the Fried parameter. Since the methods under consideration are based on extracting information on an object by determining the statistical characteristics of these fluctuations,[1] it is qualitatively clear that the dispersion of quantization error must be much smaller (a factor of y) than that of fluctuations, i.e.,

$$(\gamma/12) \cdot (J_{\max}/L)^2 \le K \langle J^2\rangle.$$

which in turn implies that the possible number of quantization levels is given by

$$L_{\min} = (J_{\max}/\langle J\rangle) \cdot \sqrt{\gamma/12K} \qquad (1)$$

Experiments indicate that the portion of the image is important with $\langle J\rangle \ge 0.1\langle J\rangle_{\max}$, is the most important from the point of view of processing the image, and we can therefore use

$$\langle J\rangle = 0.1 \cdot J_{\max},$$

to determine the effective number of levels in Eq. (1).

In order to test this qualitative conclusion and estimate the value y we produced a model of the image-formation process. The initial image used consisted of the letter "F". We then modeled random distortions of this image due to atmospheric turbulence with contrast values $K_T = K_\lambda = 1$ and $K_0 \approx 1/14$ ($S_0 = 13 \cdot S_{tel}$, $S_{atm} = 120 \cdot S_{tel}$). The resulting image were then quantized into 4, 8, 16, 64, and 128 levels. Series of 20, 40, 60, and 80 quantized levels were processed using the algorithm described by Bakut et al. The most typical of these reconstructed images are shown in Fig. 1.

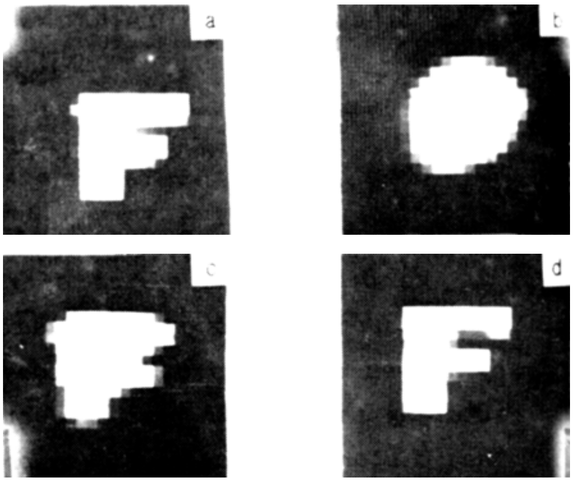

FIG. 1. Reconstruction results from 80 images: from unquantized images (a); from images quantized into 4, 8, and 16 levels, respectively, (b), (c), and (d).

The accuracy of reconstruction was quantitatively determined as a function of the number of quantization levels by comparing these estimates with the original images. We used the degree of correlation Q and the normalized rms error P, as measures of the similarity between the images:

$$Q = C \sum_{kl} O_{kl} \cdot I_{kl} / \sum_{kl} O_{kl}^2;$$

$$P = \left[\sum_{kl} (O_{kl} - CI_{kl}) \right]^2 / \sum_{kl} O_{kl},$$

where O_{kl} and I_{kl} are, respectively, the intensities of the original image and reconstructed image, in a given discrete element kl. Summation was carried out over all elements of the image, and the value of coefficient C was calculated from the condition of equality that the mean intensities be equal:

$$C = \sum_{kl} O_{kl} / \sum_{kl} I_{kl}.$$

The quantities Q and P are shown as a number of quantization levels in Fig. 2, where we have compared the results of reconstruction from various quantized series of images against similar obtained from the same images, but without quantization, on order to obtain the "pure" function, free of inaccuracies in the algorithm itself.

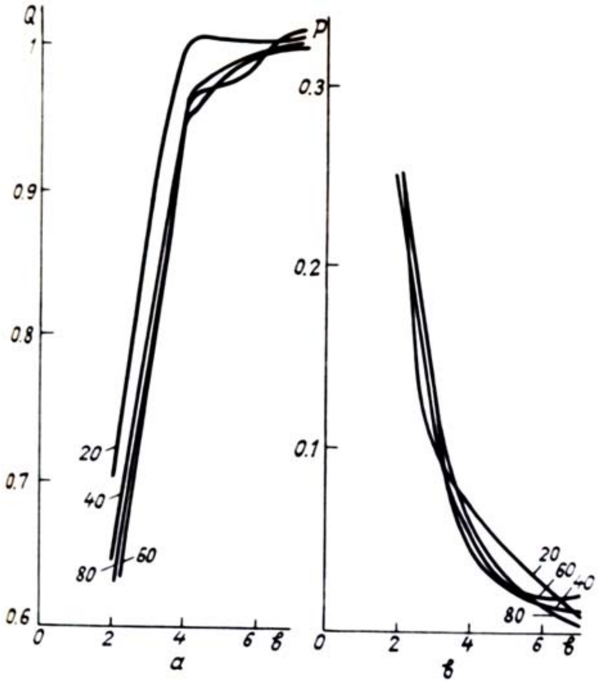

FIG. 2. Accuracy on the number of quantization levels: a) correlation degree; b) rms error.

This research indicates that satisfactory image reconstruction can be achieved using 16-level quantization, which corresponds to $\gamma = 2$.

This leads to a working formula of the following form for the calculation:

$$L_{min} = 4/\sqrt{K},$$

where K is the contrast of the images to be recorded. In particular, this implies that at least 128 levels (7 bits) are needed under typical observing conditions for extended objects: $S_0 \gg S_{atm}$, $D/r_0 \sim 10$, $K_0 \approx 0.01$, and $K_T \cdot K_\lambda \sim 0.2$.

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