ESTIMATION OF TIME LAGS IN LOCATION OF RADIATION SOURCES ON THE EARTH'S SURFACE FROM SATELLITE: THE PSEUDOZERO POINT METHOD

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The method of time delay estimation is described as applied to the problem on detecting ground-based pulsed sources from satellite. The method provides for separate processing of signals coming to different spacecrafts comprising the satellite network. The method is based on detecting a conditional bench-mark (pseudozero point), which is artificially constructed by a computer from the finite-size set of noisy sampled values of the signal function and uniquely determined by these values. Theoretical analysis and the results of numerical experiments aimed at checking the accuracy characteristics of this method show that the method may be useful in the applications where tough restrictions are imposed on the bulk of information transmitted via the communication channels.

1. The detection of radiation sources using a satellite information and measurement system, such as NAVSTAR or GLONASS[1] is usually made by the range difference method. This method in its version adapted to satellite observations is well known in radionavigation[2]; recently it was applied to detection of optical sources too (see, for example, Ref. 3). It is based on processing and comparison of data recorded with several receivers placed at different points of space aboard spacecrafts that comprise a system. Time delays in this method serve as input values for calculating the source coordinates. In the simplest case, time delay is defined as a difference between the start times of recording at different spacecrafts.

The efficiency of the method depends, to a great extent, on the accuracy of estimating these values from observations. In the case of signals with a gradual front and observed against additive noise, the concept "start of recording" is ambiguous and the algorithm of time lag estimation based on it becomes inaccurate. The problem arises on development of precise algorithms for the time lag estimation, in which the above ambiguity is excluded. This problem is of great importance not only for satellite ranging but for many other practical applications; and a lot of works are devoted to its solution.[4]

There are two possible approaches to estimation of signal time lags in satellite ranging of radiation sources. They provide for simultaneous or separate processing of data from different spacecrafts. Both approaches use the concept that the time series formed of these data are samples of the same signal, but changed in scale, shifted in time, and perturbed by noise.

In the first approach, the time series are compared by pairs by varying time lag and thus the value of time lag is sought, which gives, after corresponding scaling, their best coincidence by least-squares or some other criterion. It is this value that serves as the estimate of the time of relative delay of signals in the pair under consideration. Corresponding methods are performed using correlometers[5] or adaptive filters operating in the regime of identification of an unknown system related to it[6] (see also Ref. 7). Unknown system here is the operator of transformation of one of the data series compared into another one. These methods give good results, but their performance is connected with the overload upon the communication channels between a spacecraft and a ground-based computer center. At the limited capacity of informational channels, interesting are the methods of bench-mark point with separate processing of information from each spacecraft that corresponds to the second of the above approaches.

In the bench-mark methods, the time moments t_i, $i = 1,\ldots,N$ are found, at which each of the N spacecrafts observing the source has recorded some characteristic point (structure element of vanishingly small extent in time) of a signal. In this case the delay time estimates are the time differences $\Delta t_{ij} = t_i - t_j$.

Similar approach is widely used in radionavigation, where signals at the outputs of transducers have a well pronounced maximum. For signals with a complex shape, unknown *a priori*, such an approach is much more complicated. The bench-mark point for such signals is to be artificially constructed from the samples recorded against the noise. But one can achieve good results in this direction too, if the position of artificially constructed (conditional) bench mark on the time axis is uniquely determined by a set of sampled data and is low-sensitive to random shifts of readout points within the

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observation interval common for all spacecrafts. It is just this approach to estimation of signal delays employed in the method of pseudozero point considered below. This method allows simple algorithm to be used for its performance at a relatively high accuracy.

2. Any method based on artificially constructed bench-mark point includes two different operations: smoothing and extrapolation of noisy data sampled from a signal at a finite number of points within the observation interval. Both operations can be done simultaneously using the same mathematical formalism which is based on the extrapolation method for segments of frequency-limited functions and a series of data sampled at the Nyquist frequency.[8] Such an approach is elegant and, in some cases, efficient; however, as far as its computer realization is concerned, all known algorithms implementing it (see, for example, Ref. 9) are rather complicated. Such a complication is justifiable when the problem is to completely reconstruct the signal shape. But to construct the bench-mark point with the accuracy, dictated by needs of one or other applied problem, it is excessive. Computations can be not so bulky, if these two operations are separated and the extrapolation of already smoothed data is done by fitting the function modeling a signal. These principles form the basis for the pseudozero point method. This name originates from the fact that the bench-mark point selected is zero of a model function, which approximates the signal within the observation interval but probably disagrees with it beyond this interval.

When developing the method, two versions of data smoothing were tested. One uses the discrete Legendre polynomials and another one uses splines. The approximation errors for both versions, in our numerical experiment, proved to be close to each other.

Theoretical analysis of the errors is developed much better for the process of polynomial smoothing[10]; that is why we consider it below in more detail.

3. The problem of sampled data smoothing is formulated in the following way. There are $L + 1$ numbers: $y_{n-L}, \ldots, y_n$, which are the results of observations of the signal $y(t)$ at the moment t_n and L previous moments. Given is the class of admissible (smooth) functions $f_n(t)$, from which the function $\hat{f}_n(t) \equiv \hat{y}_n(t)$ should be selected that most closely fit the data (u_k, t_k), $k = n - L,\ldots,n - L + 1,\ldots,n$ according to the least-squares criterion.

We refer to this function as an approximating one and consider it as a signal representation within the interval (t_{n-L}, t_n). Subscript n in the designations $\hat{y}_n(t)$ is indicative of the dependence of the approximating function on the data sample set by the time moment t_n. Samples from $\hat{y}_n(t)$ within this window will be referred to as smoothed data.

Let us consider that readings are equally biased, assuming $t_k = k\tau$, $k = n - L,\ldots,n - L + 1,\ldots,n$, where τ is the step of observation, and restrict ourselves by the polynomial functions f_n. The latter can be presented as linear combinations,

$$f_n = \sum_{j=0}^{m} (\beta_j)_n \, \varphi_j(t) \tag{1}$$

of the normalized discrete Legendre polynomials $\varphi_i(t)$, orthogonal at a finite set of equally biased points

$$\sum_{k=0}^{L} \varphi_i(k)\varphi_j(k) = \delta_{ij} \ , \quad \varphi_i(k) \equiv \varphi_i(t_k) \ , \quad t_k = k\tau \ . \tag{2}$$

Let us select the sum of square discrepancies

$$e_n = \sum_{r=0}^{L} \left[y_{n-L+r} - \sum_{j=0}^{m} (\beta_j)_n \, \varphi_j(r) \right]^2 \tag{3}$$

as the cost function.

In fitting the approximating function $\hat{y}_n(t)$ to the values y_k, the parameters varied are the coefficients $(\beta_j)_n$ from Eq. (1). They are chosen from the minimum condition for the functional (3). Taking the derivatives $\partial e_n / \partial \beta_j$, $j = 0, 1,\ldots,m$, and equalizing them to zero, we obtain the system of $m + 1$ equations for $(\beta_j)_n$. From this system and taking into account the conditions of orthonormality (2), we can find

$$(\hat{\beta}_i)_n = \sum_{k=0}^{L} y_{n-L+k} \, \beta_j(k) \ , \quad i = 0, 1, \ldots , m. \tag{4}$$

In this case

$$D^i \, \hat{y}_n(t) = \sum_{j=0}^{m} (\hat{\beta}_j)_n \frac{\partial^i}{\partial t^i} \varphi_i(t) \ , \quad i = 0, 1, 2, \ldots \ . \tag{5}$$

Thus we derived the approximating dependences for the signal $y(t)$ and its derivatives $D^i y(t) \equiv d^i y(t)/dt^i$, $i = 1, 2,\ldots$. Let us call the set of numbers $D^i \hat{y}_n(t_n + h) \equiv D^i \hat{y}_{n,n+h}$, $i = 0, 1, 2,\ldots$, resulting from equations (5) as the estimate of the system state at the time moment t_{n+h}.

The vector of observations $\mathbf{y}_n$ and the vector of system state estimates $\mathbf{y}_{(n,\, n+h)}$ are related by the linear relationship

$$\mathbf{y}_{(n,\, n+h)} = W(h;\tau)\mathbf{y}_{(n)} \ . \tag{6}$$

The matrix operator $W(h; \tau)$ can be factorized; it can be presented as a product of several simpler matrices, that can readily be calculated based on the methods of discrete functional analysis.[10] Having $W(h; \tau)$ known, one can calculate the covariance matrix, $\hat{S}_{(n)}(h; \tau)$, for the errors of estimation. If the measurement noise is uncorrelated and has zero mean and a variance σ_v^2, it is calculated by the expression

$$\hat{S}_n(h; \tau) = \sigma_v^2 \, W(h; \tau) \, W^T(h; \tau) \ . \tag{7}$$

For large samples $L \gg m$, the elements of the matrix (7) can be presented as functions of τ, L, and σ_v^2 as follows

$$[\hat{S}_{(n)}(h;\tau)]_{ij} \approx \{\alpha_{ij} / [\tau^{i+j} L^{i+j+1}]\} \sigma_v^2 , \quad (8)$$

where α_{ij} are constants depending on h and m. Using the approximate equality (8), let us consider the diagonal elements of the matrix $\hat{S}_n$ with indices (0, 0) and (1, 1) in more detail. They correspond to the variances of errors of estimating the signal σ_v^2 and its first derivative σ_v^2. At $h = -L/2$, i.e. at the center of window, where the accuracy of estimating $y(t)$ and $y'(t)$ is the best, the coefficients entering into Eq. (13) take the following values:

$$\begin{aligned} &\alpha_{00} = 1.00 , \quad \alpha_{11} = 12 \quad \text{for} \quad m = 1; \\ &\alpha_{00} = 2.25 , \quad \alpha_{11} = 12 \quad \text{for} \quad m = 2; \\ &\alpha_{00} = 2.25 , \quad \alpha_{11} = 75 \quad \text{for} \quad m = 3; \end{aligned} \quad (9)$$

Thus for $m = 2$ the variances of the corresponding estimates are equal to

$$\left. \begin{aligned} \sigma_{\hat{y}}^2 &\approx \frac{2.25}{L} \sigma_v^2 ; \\ \sigma_{\hat{y}'}^2 &\approx \frac{12}{\tau^2 L^3} \sigma_v^2 . \end{aligned} \right\} \quad (10)$$

The values of α_{ij} and consequently the variances σ_v^2 and σ_v^2 increase as the sampled point moves outward from the window center. This decrease is initially slow and then, beyond the observation interval, it becomes faster and faster. The estimates of the system state (values of the signal and its derivatives) lose their meaning at polynomial smoothing for values $h > 1$ and $h < -(L + 1)$. The systematic errors of estimation, determined by the value of the first neglected term in $y(t)$ expansion into a power series, also increase outward from the window center. But their dependence on m and L is opposite to that characteristic of the random errors: the estimates biases decrease with increasing m and increase with increasing L. In the case, when polynomial model

$$\hat{y}(t) = \sum_{k=0}^{m} a_k t^k \quad (11)$$

with h-independent coefficients a_k describes the signal front rather well in the region of observations spanning all possible estimation windows in processing the data from different spacecrafts, the systematic errors of smoothing can be neglected in the pseudozero point method.

4. Quick increase of the errors in polynomial approximation of the signal beyond the estimation window makes the use of this approximation worthwhile only for smoothing, but not for extrapolation of the data. In all cases, when the *a priori* information needed is available, the use of the structure model of the signal with its fitting to smoothed data turns out to be more efficient for extrapolation purposes.

In the class of problems considered on detecting sources of optical radiation, the model of signal as a γ-distribution function in the form

$$\begin{aligned} &\hat{y}(t) = C(t - \tilde{t}) \exp[-(t - \tilde{t}) / T] \, U(t - \tilde{t}) , \\ &(t - \tilde{t}) \ll T , \\ &U(t - \tilde{t}) = \begin{cases} 0, & t \geq \tilde{t}, \\ 1, & t < \tilde{t}; \end{cases} \end{aligned} \quad (12)$$

can be adequate in many practical applications. This model can be also considered as a particular case of the Neubool distribution. The parameters of this model are the signal $\tilde{t}$ zero and the constants C and T. This model can readily be fitted to the experimental data in both time and frequency regions. Let us take the frequency region and estimate the expected errors of data extrapolation to zero for signals allowing their presentation by the model (12). As the initial values, we use the above-obtained estimates of $\hat{y}_k$ and $\hat{y}'_k$ for the signal and its derivative, or, to be more correct, their ratios

$$A_k = \hat{y}'_k / \hat{y}_k , \quad k = k = 0, 1, \ldots , n - 1 , \quad (13)$$

found for the center points of the smoothing windows (Fig. 1).

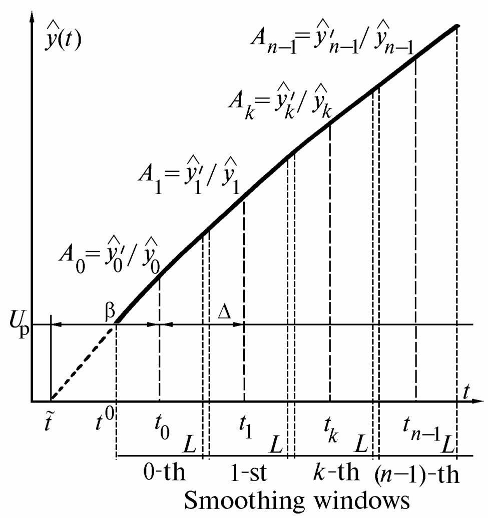

FIG. 1. Determination of the bench-mark point – pseudozero point of a signal.

The substitution of $\hat{y}'_k$ and $\hat{y}_k$ by their ratios A_k allows us, with the help of equality

$$\hat{y}'(t) / \hat{y}(t) = [1 / (t - \tilde{t})] - 1/T \quad (14)$$

following from Eq. (13), to exclude the constant C from our consideration. The calculational relationships to fit the parameters $\tilde{t}$ and T are

$$[1/(t_i - \tilde{t})] - 1/T = A_i ;$$

$$[1/(t_j - \tilde{t})] - 1/T = A_j , \quad (15)$$

where t_i and t_j are the time moments, corresponding to the arbitrarily selected pair of windows i and j. Let $\Delta = t_j - t_i$ and $\beta = t_0 - \tilde{t}$. Then it immediately follows from Eq. (15) that

$$\tilde{t} = t_0 - \beta , \quad T = (\beta + i\Delta)/[1 - A_i(\beta + i\Delta)] ;$$

$$C = \hat{y}_i / \{(t_i - \tilde{t}) \exp [- (t_i - \tilde{t})/T]\} , \quad (16)$$

where

$$\beta = \Delta\left\{-\frac{i+j}{2} + \frac{1}{2}\left[(i+j)^2 + \frac{4(i-j)}{(A_i - A_j)\Delta} - 4ij\right]^{1/2}\right\}. \quad (17)$$

The errors of estimating β and, consequently, $t_{con} \equiv \tilde{t}$ are determined by the errors of estimating A_i and A_j entering into Eq. (17) that depend on $\hat{y}_i$, $\hat{y}'_i$ and $\hat{y}_j$, $\hat{y}'_j$. The values A_i and A_j are statistically independent and there is no correlation between them. If estimation is done by points situated at the centers of smoothing windows i and j, there is no correlation between $\tilde{y}'_i$ and $\tilde{y}_i$ too, what can be seen from the dependences shown in Fig. 2. Under these conditions, the variance of errors of t_{con} estimation can readily be calculated.

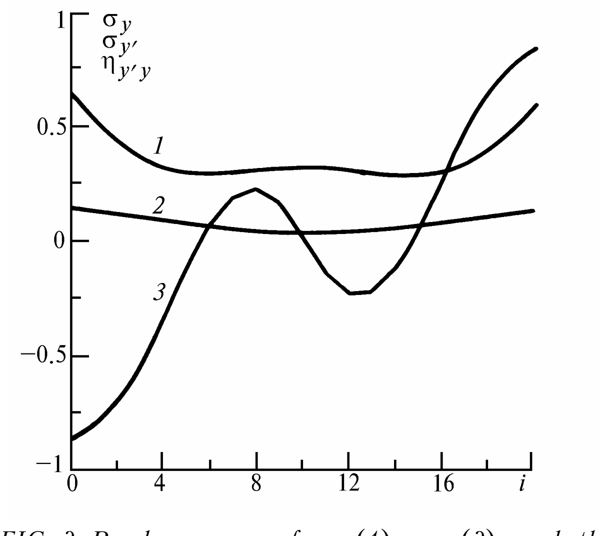

FIG. 2. Random errors of $\sigma_{\hat{y}}$ (1), $\sigma_{\hat{y}'}$ (2), and the correlation coefficient $\eta_{\hat{y}'\hat{y}}$ (3) vs. the number of the grid point i of the window of smoothing (L = 21) by the discrete orthogonal Legendre polynomials.

Let $m = 2$, $\Delta = L$, $i = 0$ and $j = 1$. Taking into account that signals have a considerable portion of leading edge close to linear one, we assume

$$\bar{\hat{y}}_0 \simeq C\bar{\beta} , \quad \bar{\hat{y}}'_0 \simeq C, \quad \bar{\hat{y}}_1 \simeq C(\bar{\beta} + L), \quad \bar{\hat{y}}'_1 \simeq C ,$$

where the bar over $\hat{y}_i$ and $\hat{y}'_i$ denotes that these values correspond to the points situated at the centers of smoothing windows. Then, with regard for above-obtained estimates (10) for errors of the smoothed data, we have

$$\sigma_{con} \simeq \frac{\sigma_v}{C}\frac{[\bar{\beta}(\bar{\beta} + L\tau)]^2}{L(L\tau + 2\bar{\beta})} \times$$
$$\times\left[\left(\frac{1}{\bar{\beta}^2} + \frac{1}{(\bar{\beta} + L\tau)^2}\right)\frac{12}{L^3} + \left(\frac{1}{\bar{\beta}^4} + \frac{1}{(\bar{\beta} + L\tau)^4}\frac{2.25}{L}\right)\right]. \quad (18)$$

The parameter σ_v/C entering into Eq. (18) corresponds to the time t_v during which the model signal (12) increases from zero level to the level of noise. This time is related to the signal duration T and the signal-to-noise ratio at maximum, S, through the following relationship: $t_v = T/Se$, where $e \cong 2.7$ is the base of natural logarithm. For different values close to unity at $L = 21$ we obtain $\sigma_{con} \simeq 1.5$, 0.87, 0.35 μs for $t_v = 1.4$, 1.0, 0.5 μs.

These values of t_v correspond to signals with a sufficiently large signal-to-noise ratio in maximum – about 20 dB or greater. For example, at $T = 500$ μs, the value $t_v = 0.5$ μs corresponds to $3S \approx 370$. For small S the error of t_{con} estimation grows.

5. Delay times are determined as differences between two independent random values: t_{con} for the corresponding pair of signals. Therefore, for the variance of delay time estimation we can take

$$\sigma^2_{\Delta t} \approx 2\sigma^2_{t_{con}} . \quad (19)$$

At $\sigma_{t_{con}} = 0.35$ μs the uncertainty in Δt values is about 0.5 μs, that makes it possible to estimate the source coordinates accurate to 100–150 m. Such an accuracy is sufficient for a number of practical applications.

The above values of errors of signals delay time estimation by the method of pseudozero point were checked in numerical experiment with 600 series of model data using 256 samples in each series for both polynomial and spline smoothing. The results were close in both cases and they differed from the theoretical estimates by no more than 15–20%. Thus, for sufficiently high-power signals the accuracy characteristics of the method meet medium-stringent requirements to the resolution of a ranging system. The method is easy in realization and may be useful for applications connected with the problem of satellite detection of radiation sources, when it is necessary to minimize the bulk of information transmitted via communication channels to the Earth. An example of such an application is the problem of prompt estimation of parameters of light emitting objects at the Earth's surface.

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