

## MEASUREMENTS OF THE TENSOR OF TURBULENT DIFFUSION COEFFICIENTS IN THE ATMOSPHERE AND SOME ITS PROPERTIES

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*The paper presents some results of measurements of the turbulent diffusion coefficients in the atmospheric boundary layer. Experimental data prove the hypothesis on their proportionality to the corresponding components of the Reynolds stress tensor. Considerable deviations, in spite of general opinion, of the main tensor axes from the traditionally set coordinate system have been revealed. The effect of a step-wise change of the main axis position of the diffusion coefficients tensor in the vertical plane at some critical value of the Richardson gradient number is discussed.*

The interaction of radiation with the aerosol component of the atmosphere should be taken into account when solving some problems of atmospheric optics. Since the aerosol spread occurs in the turbulent atmosphere, its concentration at a given point of the space is subject to considerable variations. Thus, it is necessary to describe such processes quantitatively.

To simulate the aerosol dispersal in the boundary layer of the atmosphere, the semiempirical equation of turbulent diffusion<sup>1</sup> is used. The equation involves the fields of mean value of wind velocity components  $\bar{U}_i$  and the tensor of turbulent diffusion coefficients  $K_{ij}$ , where the indices  $i$  and  $j$  mean the  $x$ -,  $y$ -, and  $z$ -axes of the coordinate system. The hypothesis that  $K_{ij}$  are proportional to the corresponding components of the Reynolds viscous stress tensor<sup>2</sup> has the form

$$K_{ij} = C_\varphi \frac{b^2}{\varepsilon} \overline{\hat{U}_i \hat{U}_j} = C_\varphi \frac{b^2}{\varepsilon} \tau_{ij}, \quad (1)$$

where  $C_\varphi$  is a constant;  $b^2$  is the kinetic energy of turbulence;  $\varepsilon$  is its dissipation rate;  $\hat{U}_i$  are the components of wind velocity pulsations;  $\tau_{ij}$  are the components of the stress tensor. The overbar means averaging over the statistical ensemble. If we change the ensemble-averaging by the moving average over time with the period  $T$  much greater than the Euler characteristic time scale of wind velocity pulsations  $\tau^E$ , we can approximately assume that the ergodic theorem is valid and both ways of averaging are equivalent.<sup>1,3</sup>

The constant  $C_\varphi$  is usually supposed to be equal to 0.13 (Ref. 4). Some experiments using wind tunnels indirectly confirm the hypothesis (1) (Ref. 2). We have no data about experimental verification of the hypothesis (1) for the atmospheric boundary layer.

To substantiate the hypothesis (1), we chose the "recursion" method as an independent method for measuring  $K_{ij}$ . It was proposed and used by Galkin in order to measure  $K_{ij}$  in the water of Lake Baikal.<sup>5,6</sup> Let us write the expression for determining  $K_{ij}$  (Ref. 5):

$$K_{ij} = \frac{1}{2(T - \tau^E)} \int_t^{T - \tau^E} \left[ \hat{U}_i(t + \tau) \int_t^{t + \tau^E} \hat{U}_j(\xi) d\xi + \hat{U}_j(t + \tau) \times \int_t^{t + \tau^E} \hat{U}_i(\xi) d\xi \right] dt. \quad (2)$$

Thus it is necessary to have the measurement series of wind velocity component pulsations  $\hat{U}_i$  of duration  $T$  and know the characteristic time scale  $\tau^E$  in order to verify the hypothesis (1) using Eq. (2).

The measurements were conducted at the sites in Novosibirsk and Tomsk regions. They approximately satisfy the restrictions inherent to the similarity theory of the near-ground atmospheric layer by their orographic characteristics. An X-Y-Z acoustic anemometer and a wire resistance thermometer<sup>3</sup> were used to measure the series of instantaneous values of wind velocity and temperature.

Main characteristics of the equipment used were coordinated with the restrictions imposed on the semiempirical approach to the description of turbulent diffusion: the duration of samples of instantaneous values was  $10^{-3}$  s, the number of readings in each of them was  $10^3$ , the measurement errors of the mean values were 0.02 m/s and 0.03°C, and that of the instantaneous values were 0.2 m/s and 0.2°C, respectively (these errors provided the measurement of second moments with the error less than 5%) (Ref. 3).

The available samples of instantaneous values were stored and then processed on a personal computer. The data on the wind velocity were referred to the coordinate system where the  $x$ -axis is directed along the mean value of wind velocity as is accepted in the similarity theory of the near-ground atmospheric layer. Other calculations were done in accordance with the relations (1) and (2).

The estimation of  $\tau^E$  by the moment of the first zero in the autocorrelation function of wind velocity pulsations is verified in Ref. (5). This procedure is rather awkward, and so the relation  $\tau^E = (45 \pm 8)z(\bar{U}_i)^{-1}$  similar to that of Ref. 7 was used. This relation was based on the generalization of the results of the first measurement run in routine experiments. Here  $\tau^E$  is taken in seconds, the height  $z$  and the mean value of wind velocity  $\bar{U}_i$  are taken in meters and meters per second.

Let us consider some measurement results. In the first experiment (Novosibirsk region), the sensors of two equipment complexes were placed at the height of 2 m at a distance no more than 2 m from each other. The module of wind velocity decreased from 2.7 m/s to 1.7 m/s during the experiment, and its vector monotonically rotated to the north within 30° sector. The stratification of the atmosphere was determined by a turbulent flow of heat varying from -16 to -26 W/m<sup>2</sup>. The friction velocity  $U_*$  smoothly decreased from 0.3 to 0.16 m/s. The values of the components  $K_{ij}$  obtained are presented in the Table I. One can see that the first and the second devices yield practically the same values of  $K_{ij}$ . These data also enable one to estimate the achieved accuracy of measurements. The value  $K_{ij}$  considerably decreased during the experiment. It can obviously be explained by the attenuation of turbulence at nightfall.

Let us now consider the data of the experiment conducted in Tomsk in the Institute of Atmospheric Optics. The sensors were placed at the height of

12 m over a recently ploughed field. The experiment was conducted in spring at the time of sharp rise in temperature after a long cold period. The measurements were performed from 2:00 to 11:00 p.m. local time. The mean value of the temperature increased from 15 to 18°C in the beginning of the experiment and then decreased to 13°C after 6:00 p.m. The turbulent heat flow varied from -20 to -140 W/m<sup>2</sup>. The flow took positive value about 50 W/m<sup>2</sup> from 4:00 to 5:00 p.m. This corresponded to the appearance of clouds that had closed the sun and caused a weak inversion. After 7:00 p.m. the heat flow rather sharply approached the value of -5 W/m<sup>2</sup> and then it was practically constant until the end of the experiment. The wind velocity was about 6 m/s in the beginning of the experiment and lowered to 1 m/s by its end.

The values of  $K_{ij}$  obtained are presented in Fig. 1. One can see that the coefficients  $K_{ij}$  underwent considerable fluctuations during the measurement period what agrees with the observed change of turbulent flows of heat and pulse and correlate with it. One can also see that their values decreased with the weakening atmospheric turbulence. Attention is attracted to rather large absolute values of off-diagonal components of  $K_{ij}$  what must lead to deviation of the principal axes of the turbulent diffusion tensor from the coordinate system traditional for the similarity theory of the near-ground atmospheric layer.

Expressing  $\varepsilon$  by Kolmogorov's relation<sup>1</sup> and taking into account the results of the similarity theory for the estimation of the turbulent exchange coefficient<sup>1</sup> we obtain

$$K_{ij} = C_\varphi U_* z \tau_{ij} / [C_k b \varphi(z/L)] , \tag{3}$$

instead of Eq. (1). Here  $C_k = 0.046$  is the Kolmogorov constant;  $\varphi$  is the universal function of the similarity theory;  $L$  is the Monin-Obukhov scale.<sup>1</sup>

TABLE I.

| Local time,<br>h:min | Coefficient $K_{xx}$ , m <sup>2</sup> /s |      | Coefficient $K_{yy}$ , m <sup>2</sup> /s |      | Coefficient $K_{zz}$ , m <sup>2</sup> /s |      |
|----------------------|--|------|--|------|--|------|
|                      | 1  | 2    | 1  | 2    | 1  | 2    |
| 21:00                | 2.60                                     | 2.81 | 0.52                                     | 0.48 | 0.13                                     | 0.17 |
| 21:20                | 2.15                                     | 1.92 | 0.22                                     | 0.12 | 0.16                                     | 0.09 |
| 21:40                | 0.39                                     | 0.40 | 0.13                                     | 0.12 | 0.10                                     | 0.08 |
| 22:00                | 0.29                                     | 0.24 | 0.05                                     | 0.06 | 0.08                                     | 0.06 |
| 22:20                | 1.19                                     | 1.41 | 0.63                                     | 0.74 | 0.04                                     | 0.03 |
| 22:40                | 0.76                                     | 0.79 | 0.25                                     | 0.25 | 0.03                                     | 0.04 |
| 23:00                | 0.32                                     | 0.37 | 0.09                                     | 0.07 | 0.05                                     | 0.05 |
| 23:20                | 0.21                                     | 0.19 | 0.06                                     | 0.05 | 0.04                                     | 0.04 |
| 23:40                | 0.17                                     | 0.14 | 0.06                                     | 0.04 | 0.03                                     | 0.04 |
| 24:00                | 0.22                                     | 0.20 | 0.07                                     | 0.05 | 0.04                                     | 0.06 |
| 00:20                | 1.66                                     | 1.45 | 0.15                                     | 0.08 | 0.04                                     | 0.03 |
| 00:40                | 0.47                                     | 0.31 | 0.14                                     | 0.12 | 0.04                                     | 0.02 |
| 01:00                | 0.98                                     | 0.86 | 0.06                                     | 0.05 | 0.02                                     | 0.04 |

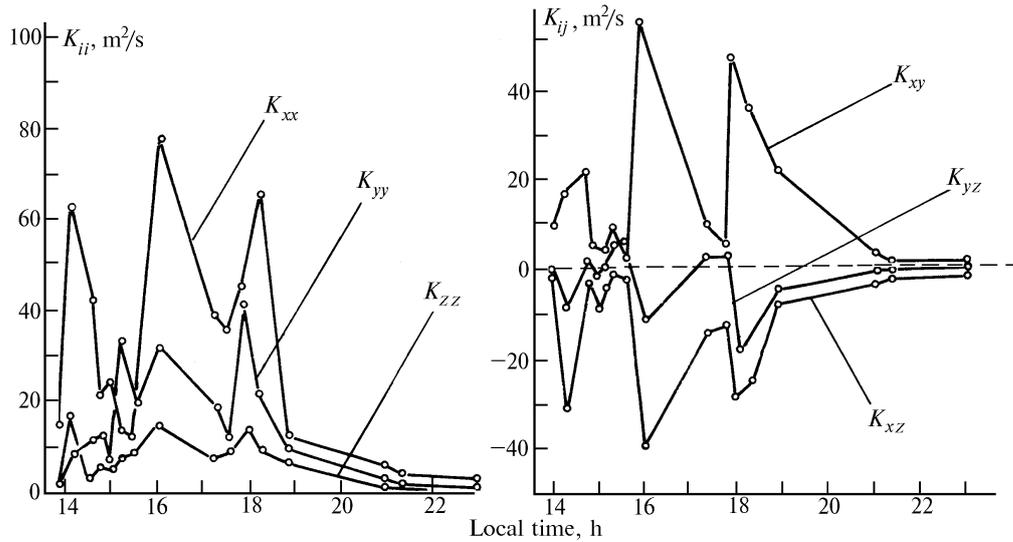


FIG. 1. An example of measured turbulent diffusion coefficients as a function of time.

An example of verification of the assumption (1) accounting for formula (3) is presented in Fig. 2. The values  $K_{ij}$  calculated by the relation (2) make the ordinate axis; the values calculated by the expression (3) from the series of instantaneous values and conclusions of the similarity theory of the near-ground atmospheric layer are on the abscissa axis. The spread of points is typical for data obtained within the frames of the similarity theory of the near-ground atmospheric layer. Linear regression of the obtained results leads to the value  $C_\varphi = 0.12$ . Thus, the validity of hypothesis (1) is also verified in the field experiments.

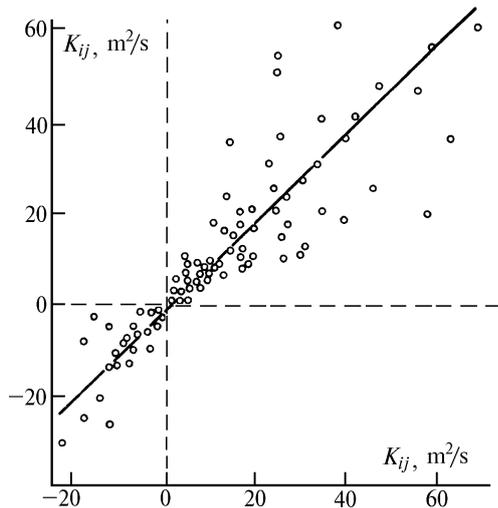


FIG. 2. The result of verification of the hypothesis (1):  $K_{ij}$  measured in accordance with Eq. (2) and calculated by Eqs. (1) and (3),  $m^2/s$ . Solid line is the result of linear regression of the data.

Let us consider the question on whether do or not the principal axes of the measured tensor of turbulent diffusion coefficients correspond to the coordinate

system usual for the similarity theory of the near-ground atmospheric layer. After the tensor was reduced to the diagonal form, the angles between the coordinate system traditionally used in the theory of the near-ground atmospheric layer and the chosen one were obtained. The deviation angle of the principal axis of the tensor  $K_{ij}$  in the vertical plane proved to be, on the average, about  $15^\circ$  and varied within  $\pm 40^\circ$  in the horizontal plane.

The cause of the rotation of the principal axis of the tensor  $K_{ij}$  in the vertical plane in accordance with the hypothesis (1) is obviously connected with the presence of vertical turbulent flow of momentum. Theoretical estimations of the deviation angle in the vertical plane with allowance for Eq. (1) and the data typical for the near-ground atmospheric layer about the characteristics of turbulent pulsations lead to the value about  $18^\circ$  for the rotation angle.<sup>3</sup> The deviations of the principal axis in the horizontal plane demonstrate the presence of correlation between  $x$  and  $y$  components of the wind velocity pulsations. Off-diagonal components of the tensor  $K_{ij}$  are usually neglected.<sup>1</sup> But we are convinced that they are large in real atmosphere and, obviously, can appreciably influence the solution of the semiempirical equation.

The results obtained make it possible to come to some very interesting conclusions. Let us consider real boundary layer of the atmosphere and assume simplified, but very close to real, estimations of turbulent flows, stress and other necessary parameters based on the algebraic model<sup>8,2</sup> as estimations of the values in the right-hand side of Eq. (1). Using this model, all the values needed for simulation of the turbulence of the atmospheric boundary layer are expressed by universal functions depending only on the gradient Richardson number  $Ri$  characterizing the degree of thermal stability of the atmosphere. The Richardson number is available in simulation of the turbulent diffusion process because one has to solve the

full system of dynamics equations of the turbulent atmospheric boundary layer including the equations for  $\bar{U}_i$ , potential temperature, air humidity, and other parameters in order to obtain the components of the mean value of wind velocity. So it is easy to obtain the expressions for  $K_{ij}$  as functions of the Richardson number  $Ri$  and calculate the rotation angle  $\varphi_0$  of the principal axis of the tensor  $K_{ij}$  in the vertical plane by the use of Eq. (1). Calculations result in the expression

$$\varphi_0 = 1/2 \arctan [2K_{xz}/(K_{xx} - K_{zz})].$$

According to the algebraic model, it turns out that the angle  $\varphi_0$  changes by  $90^\circ$  in the atmospheric boundary layer for  $Ri = -0.37$ . At this moment  $K_{xx}$  becomes equal to  $K_{zz}$ . For the near-ground layer, the jump of the rotation angle of the principal axis of the tensor occurs for the value of the dimensionless stability parameter  $z/L = -3.04$ . We also see that the step-wise change of the inclination angle of the principal axis of the tensor  $K_{ij}$  occurs only in unstable stratification of the atmosphere and changes the sign from minus to plus when instability increases.

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