

# THEORETICAL INVESTIGATION OF OPTICAL RADIATION ATTENUATION BY CRYSTALLINE AEROSOL

O.V. Shefer

*Institute of Atmospheric Optics,  
Siberian Branch of the Russian Academy of Sciences, Tomsk  
Received April 3, 1995*

*Spectral dependence of the extinction coefficient of an atmospheric formation containing crystalline aerosols of coarsely and finely dispersed fractions is comprehensively investigated. The extinction coefficient of spherical ice particles is numerically evaluated. The spectral dependence of crystalline cloud extinction is shown to be primary caused by oriented ice plates.*

## 1. INTRODUCTION

Optical radiation attenuation by crystalline clouds has been insufficiently studied in the literature, as applied to the interpretation of the data of sensing of crystalline clouds, because of the specific mechanism of light scattering on nonspherical particles. At present, more widespread models, such as spherical or chaotically oriented crystals are used to interpret the data of sensing. At the same time, experimentally established is the spectral dependence of radiation attenuation in cirrus,<sup>1,2</sup> which is shown to be due to oriented plates present in a scattering volume.<sup>3</sup> Specifically, the amplitude of variations of the efficiency factor underlies the spectral behavior of the extinction coefficient.<sup>4,5</sup> Evidently, for crystals other than plates the efficiency factor undergoes much less variations with wavelength. An estimate of optical radiation attenuation by ice crystals can be found in Refs. 3 and 6; however, it was analyzed in detail only for large particles within the framework of the physical optics method.

The extinction coefficient for small particles, conventionally < 30 μm in diameter, is also known to depend on the wavelength. In this connection, experimentally observed spectral behavior of the extinction is associated with small crystals present in the scattering volume. A thorough analysis of feasibility of this assumption is made in this paper.

## 2. NUMERICAL STUDY OF RADIATION ATTENUATION BY FINELY DISPERSED FRACTION OF CRYSTALLINE AEROSOL

First let us investigate numerically the radiation attenuation by small ice crystals, most of which are known to be three-dimensional in real atmosphere and, due to their aerodynamic properties, randomly oriented.<sup>7</sup> Ensemble considerations require that the light scattering characteristics for a single ice crystal particle be averaged over particle size spectrum and orientations in space. This understandably would

smooth considerably the scattering features contributed by an individual crystal structure, thus warranting the use of the model of spherical particles as a generalized model for chaotically oriented crystalline polydispersions of arbitrary shapes. To calculate numerically the extinction coefficient, we adopt the formula:

$$\alpha = \int_0^{\infty} f(a) \sigma da, \quad (1)$$

where  $f(a)$  is the particle size distribution function, and  $\sigma$  is the extinction cross section which for spherical particles is given by the well-known formula<sup>8</sup>

$$\sigma = \frac{4\pi}{k^2} \text{Re}(S(0)), \quad (2)$$

where  $k = 2\pi/\lambda$  is the wave number, and  $\lambda$  is the wavelength. The amplitude function  $S(0)$  for particles of different size is calculated in terms of the coefficients  $a_n$  and  $b_n$  of the Mie solution.<sup>9</sup> Describing real ice cloud particle size distribution<sup>10</sup> and taken here for the function  $f(a)$  is a gamma-distribution of the form

$$f(a) = N \frac{\mu^{\mu+1}}{\Gamma(\mu+1)} \frac{1}{a_m} \left(\frac{a}{a_m}\right)^{\mu} e^{-\mu \frac{a}{a_m}}, \quad (3)$$

where  $N$  is the particle number density in a unit volume,  $a_m$  is the modal radius, and  $\mu$  is the dimensionless parameter which characterizes the steepness of the slopes of the maximum of the function  $f(a)$ . The mean particle size, related to  $a_m$  by the formula

$$\bar{a} = a_m (1 + 1/\mu),$$

was used in calculations by formula (1). Another parameter influencing the extinction cross section  $\sigma$  is the refractive index  $n$ . Figure 1 shows the plots of the

refractive index  $n$  and absorption coefficient  $\kappa$  of ice versus wavelength  $\lambda$  borrowed from Ref. 11. When going from the visible to infrared region of the spectrum, the coefficient  $\kappa$  increases by several orders of magnitude. Practically in the entire infrared region,  $\kappa$  and  $n-1$  are comparable. Therefore, the absorption coefficient  $\kappa$  is expected to affect markedly the spectral dependence of the extinction coefficient  $\alpha$ .

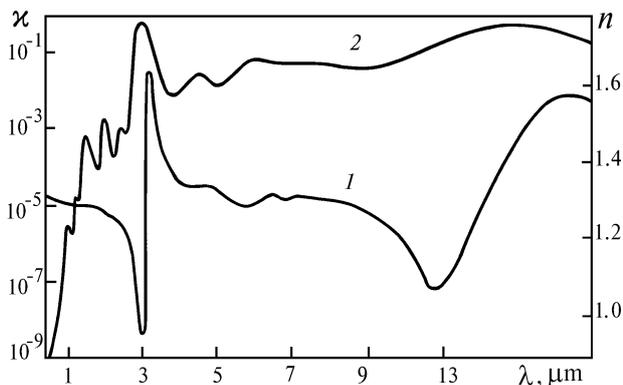


FIG. 1. Dependence of real and imaginary parts of the refractive index of ice on the wavelength: 1)  $n(\lambda)$ , 2)  $\kappa(\lambda)$ .

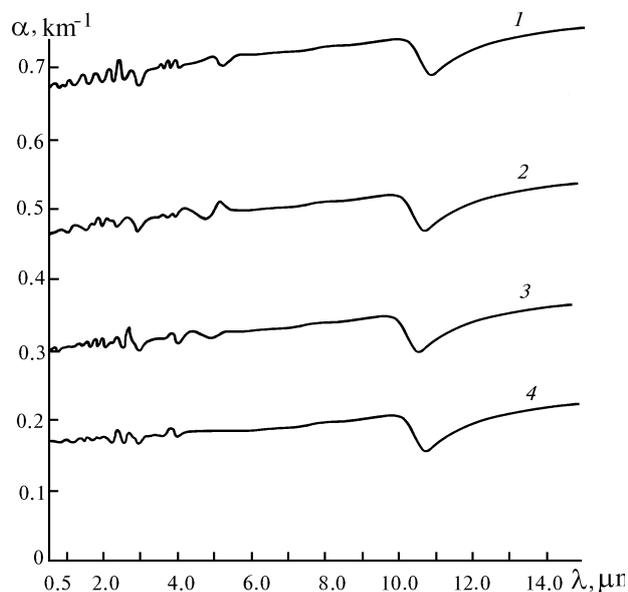


FIG. 2. Extinction coefficients for ice spheres vs. wavelength  $\lambda$ , with  $N = 100 \text{ l}^{-1}$ ,  $\mu = 5$ , and  $\bar{a} = 30$  (1), 25 (2), 20 (3), and 15  $\mu\text{m}$  (4).

Now we analyze the extinction coefficient calculated by Eq. (1). Figures 2–4 show the spectral behavior of  $\alpha(\lambda)$  in the 0.5–15  $\mu\text{m}$  wavelength range for different mean-sized particles but for the same particle number density  $N = 100 \text{ l}^{-1}$ . We note that for

large-sized crystals  $\alpha$  depends on the wavelength only slightly (Fig. 2). There are ripples on the  $\alpha(\lambda)$  curve in the 0.5–3.0  $\mu\text{m}$  wavelength region and the spike between 10.0 and 11.0  $\mu\text{m}$ . Such features of  $\alpha(\lambda)$  are connected with characteristic salient points of the dependence of the refractive index  $\tilde{n}(\lambda) = n(\lambda) - i\kappa(\lambda)$  (Fig. 1). Smaller crystals (all other factors being the same) show stronger spectral dependence of the extinction coefficient (Figs. 3 and 4). However, the changes of this light scattering characteristic are considerable only in comparison with the range of  $\alpha$  variation. From Figs. 2–4, the extinction coefficient has smaller absolute values for smaller particles. We note that ice spheres will be comparable to ice plates in the spectral dependence of  $\alpha$  only for high number density of small spherical particles which exceeds the number density of plates by several orders of magnitude.

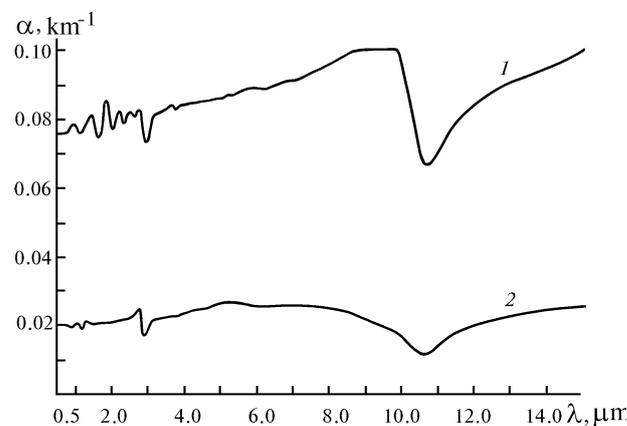


FIG. 3. Extinction coefficients for ice spheres vs. wavelength  $\lambda$ , with  $N = 100 \text{ l}^{-1}$ ,  $\mu = 5$ , and  $\bar{a} = 10$  (1) and 5  $\mu\text{m}$  (2).

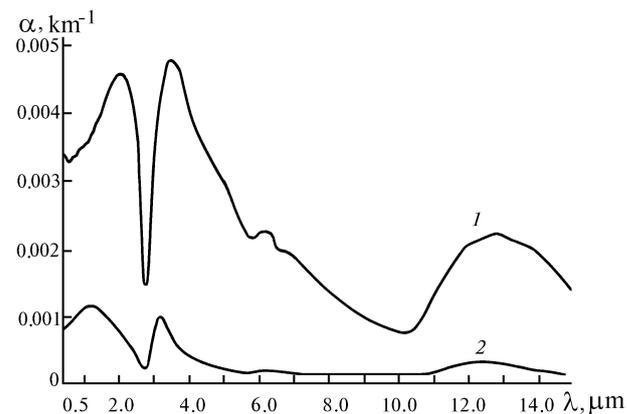


FIG. 4 Extinction coefficients for ice spheres vs. wavelength  $\lambda$ , with  $N = 100 \text{ l}^{-1}$ ,  $\mu = 5$ , and  $\bar{a} = 2 \mu\text{m}$  (1) and 1  $\mu\text{m}$  (2).

### 3. MICROPHYSICAL PROPERTIES OF ATMOSPHERIC CRYSTALLINE FORMATIONS

Below we review the literature in order to (a) identify most representative microstructural parameters of real crystalline clouds and (b) use them as *a priori* information in comparing the extinction coefficient for small crystals to that of large plates, both calculated from Eq. (1).

Typically, atmospheric crystalline formations contain particles of different size, with most particles having mean radii of several hundreds of microns. This is explained by evaporating and freezing out of finely dispersed fraction. Hobbs and Rangno<sup>12</sup> pointed out that the number density of large crystals in lower and middle clouds, in particular, is approximately by three (in some cases by five) orders of magnitude higher than that of small particles. Different-type clouds such as altostratus (As), nimbostratus (Ns), and cumulus (Cu) extending over the ocean and the ground of different types were the subject of investigation. Small and large particles were in the same proportion in upper clouds, mainly cirrus. As for very high clouds such as mother-of-pearl and noctilucent, they are mainly comprised of finely dispersed crystalline aerosol fraction. Naturally, each cloud type has distinctive microstructural properties. In crystalline clouds whose centers were located between 4 and 20 km, ice particles were sampled with size varying in the range from 1 to 8000  $\mu\text{m}$ , and particle number density changed within a wide limits from  $10^{-4}$  to  $10^4 \text{ l}^{-1}$ ; however, their most typical values were 250 m and  $30 \text{ l}^{-1}$ , respectively.<sup>13</sup>

In dry atmosphere, dominating crystal shapes are hexagonal due to the specific mechanisms of crystal growth. Accordingly, these particles get a certain preferred orientation in space, most stable for plate crystals. It is well known<sup>13</sup> that ice plates enter into the composition of crystalline clouds in one or another proportion, and their number density is higher than that of small particles.

### 4. COMPARATIVE ANALYSIS OF SPECTRAL DEPENDENCE OF EXTINCTION BY SMALL CRYSTALS AND LARGE PLATES

Above we have identified two types of crystals, namely, small crystals and large plates with preferred orientation, each with distinct spectral dependence of the extinction. Let us now perform a comparative analysis of the absolute values of the extinction coefficients for such particles. Ice plates with number density  $N$  close to that shown in Figs. 3-4 will have the extinction coefficient being several orders of magnitude higher than that of small crystals.<sup>3</sup> The difference will be the greater, the larger will be the

difference between the mean radii of two classes. For instance, the extinction coefficient  $\alpha$  ranges from  $10^0$  to  $10^{-1} \text{ km}^{-1}$  (Ref. 2) for ice plates with a mean radius of 250  $\mu\text{m}$  at  $N = 1 \text{ l}^{-1}$ , being of the order of  $10^{-4} \text{ km}^{-1}$  for small particles with radius  $a = 5 \mu\text{m}$ . Moreover, small values of the extinction coefficient would be for small crystals with number density occurring naturally (Figs. 2-4). So, at  $N = 0.03 \text{ l}^{-1}$ ,  $\alpha$  will be within  $10^{-8}$ – $10^{-5} \text{ km}^{-1}$ .

Thus we conclude that in most real crystalline clouds, absolute values of the extinction coefficient of finely dispersed aerosol fraction are much less than that of large oriented plates.

### 5. CONCLUSION

Spectral dependence of the extinction coefficient, found from sounding of real crystalline clouds, can be attributed to small particles only when their number density  $N$  is huge, say  $10^5 \text{ l}^{-1}$ . This number density, however, is hardly reached naturally and, moreover, a great number of large crystals is formed in clouds. In addition, oriented ice plates present in the volume under study will cause the spectral dependence of the extinction to which small particles contribute only slightly. Hence, the wavelength dependence found experimentally should be primarily attributed to oriented ice plates present in crystalline cloud.

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