PROPAGATION OF LIMITED LASER BEAMS IN THE INFLAMMABLE AEROSOL

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Propagation of a laser beam limited in its diameter through a reactive aerosol under thermal blooming conditions is considered. Decrease of optical cross section of particles due to burning out is taken into account within the threshold approximation. The expression is obtained to describe the velocity of the aerosol ignition wave which allows for the refraction and diffraction distortions of the beam.

When a high–power optical radiation propagates through an inflammable aerosol, clearing up of the medium connected with a decrease of the aerosol particles' size due to burning out can occur.[1] At present nonlinear effects of clearing up have been studied in detail within the framework of Bouguer law approach.[2] Actually, it was a question of a wide beam propagation when the diffraction divergence could be neglected. In Ref. 3 the case of propagation of a diverging beam far from caustic was considered but the divergence angle was a preset parameter. Propagation of limited beams was analyzed numerically in Ref. 4 for the case of sufficiently small particles of soot burning under kinetic regime.

It is of interest to analyze qualitatively the influence of diffraction divergence caused by perturbation of the refractive index of a medium in the beam channel on the process of clearing up of the inflammable aerosol. In this paper the analysis is carried out within the approximation of quasioptics. It is shown below that within the parabolic equation approximation for the light wave amplitude one can separate the effects of Bouguer extinction and beam broadening due to diffraction and refraction.

The light wave amplitude satisfies parabolic equation

$$2\,i\,\kappa\frac{\partial A}{\partial z} = \Delta_\perp A + \kappa^2\left(\Delta\varepsilon - i\frac{\alpha}{\kappa}\right)A\,, \tag{1}$$

where κ is the wave number, z is the coordinate along the beam, $\Delta_\perp = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$, $\Delta\varepsilon = n^2(r) - n_\infty^2$, $n(r)$ is the medium refractive index, $\alpha = \int\limits_0^\infty \kappa_0 \pi a^2 f(a, r, t)\,\mathrm{d}a$, κ_0 is the extinction efficiency factor, a is the radius of an aerosol particle, and $f(a, r, t)$ is the particle size distribution function which depends on time owing to nonlinear interaction. Equation (1) is not closed. It has to be supplemented with equations defining the dynamics of the volume aerosol extinction coefficient and field of the refractive index perturbations.

Equation (1) is of Schrdinger equation structure. Therefore, the equation for the radiation intensity can be obtained in a way similar to derivation of the equation for probability density. Having taken a complex conjugation of Eq.(1) we obtain

$$2\,i\,\kappa\frac{\partial A^*}{\partial z} = \Delta_\perp A^* + \kappa^2\left(\Delta\varepsilon + i\frac{\alpha}{\kappa}\right)A^*. \tag{2}$$

By multiplying Eq. (1) by A^* and Eq. (2) by A and summing them up we find

$$A\frac{\partial A^*}{\partial z} + A^*\frac{\partial A}{\partial z} = \frac{1}{2\,i\,\kappa}(A^*\Delta_\perp A - A\,\Delta_\perp A^*) - \alpha A A^*.$$

Hence,

$$\frac{\partial I}{\partial z} = \frac{1}{2\,i\,\kappa}(A^*\Delta_\perp A - A\,\Delta_\perp A^*) - \alpha I\,. \tag{3}$$

Here $I = AA^*$ is the radiation intensity. The first term in the right–hand side of Eq. (3) can be rearranged in the form

$$\frac{1}{2\,i\,\kappa}(A^*\Delta_\perp A - A\,\Delta_\perp A^*) = \mathrm{div}_\perp \mathbf{J}\,, \tag{4}$$

where

$$\mathbf{J} = \frac{1}{2\,i\,\kappa}\left(A\left(\mathbf{e}_x\frac{\partial A^*}{\partial x} + \mathbf{e}_y\frac{\partial A^*}{\partial y}\right) - A^*\left(\mathbf{e}_x\frac{\partial A}{\partial x} + \mathbf{e}_y\frac{\partial A}{\partial y}\right)\right). \tag{5}$$

In relation (5) $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors along the transverse coordinate axes. Using Eq.(4) we obtain

$$\frac{\partial I}{\partial z} = \mathrm{div}_\perp \mathbf{J} - \alpha I\,. \tag{6}$$

If the transverse energy flux across the beam axis can be neglected, the relation (6) is reduced to Bouguer law

$$\frac{\partial I}{\partial z} = -\,\alpha I\,. \tag{7}$$

Just within this approximation the main results for radiation interaction with the inflammable aerosol were obtained. Refraction and diffraction effects are described by the term $\mathrm{div}_\perp \mathrm{J}$ which was earlier ignored.

There are three characteristic dimensions along the beam path in this problem: $l_\partial = R_0^2/\lambda$, $l_n = 1/\alpha$, $l_p = R_0/(\delta\, n)^{1/2}$. Let us estimate, by order of magnitude, characteristic values of these parameters for a carbon aerosol and a Gaussian beam of radius R_0. For the aerosol concentration $n \sim 10^9\ \mathrm{m}^{-3}$ and characteristic size of particles ~ 1 mm we have $\alpha \sim 10^{-2}\ \mathrm{m}^{-1}$, $l_n \sim 100$ m. The diffraction length is $l_\partial \sim 100$ m for a laser emitting at $\lambda = 1.06\ \mu\mathrm{m}$ for the beam radius $R_0 \simeq 1$ cm. Estimating δn as $\delta n \sim (n-1)(\Delta T/T)$, where ΔT is the mean

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perturbation of the medium temperature within the beam, we obtain $\delta n \sim 10^{-8}$ and then $l_p \sim 100$ m. For the above parameters the effects of refraction and diffraction distortions of the beam have the same order and become comparable with the Bouguer extinction effect at the paths about 100 m long. Hence, it is clear that the results of Refs. 2 and 3 are only applicable to the paths of a smaller length.

It is of interest to try to separate out the extinction effects from the refraction and diffraction ones. Let us find the solution of Eq. (1) in the form

$$A(r, z) = \varphi(r, z) \exp\left(- \int_0^z \Phi(r, \xi)\, d\xi \right), \tag{8}$$

where

$$\Phi(r, z) = \frac{\alpha(r, z)}{2} + i\, \kappa \frac{\Delta\varepsilon\,(r, z)}{2}.$$

After substitution of Eq. (8) into Eq. (1), for the function $\varphi(r, z)$ we obtain the equation

$$2\, i\, \kappa \frac{\partial \varphi}{\partial z} = \Delta_\perp \varphi + 2 \frac{\partial \varphi}{\partial r} \int_0^z \frac{\partial \Phi}{\partial r}\, d\xi + \varphi \int_0^z \Delta_\perp \Phi\, d\xi. \tag{9}$$

For linear interaction of radiation with a homogeneous aerosol when the state of the aerosol medium does not change, the last two terms in the right–hand side of Eq. (9) vanish. In this case the function $\varphi(r, z)$ satisfies the diffraction equation in vacuum

$$2\, i\, \kappa \frac{\partial \varphi}{\partial z} = \Delta_\perp \varphi ,$$

which has an analytical solution

$$\varphi(r,z) = \int_0^\infty \frac{\kappa}{2iz} \varphi(R, 0) \exp\left(- \kappa \frac{r^2 + R^2}{2iz} \right) J_0\left(\frac{\kappa R r}{z} \right) R\, dR. \tag{10}$$

Here $\varphi(R, 0)$ is the distribution of light wave amplitude at the beginning of the path. For a Gaussian beam the integral (10) can be calculated

$$\varphi(r, z) = \varphi_0 \frac{R_0}{R(z)} \exp\left(- \frac{r^2}{R^2(z)} \right) \exp\left(ik\left(z + \frac{r^2}{2l\,(z)} \right) + i\phi \right), \tag{11}$$

where

$$R^2(z) = R_0^2 \left(1 + \left(\frac{2\,z}{\kappa R_0^2} \right)^2 \right); \quad l(z) = z\left(1 + \left(\frac{\kappa R_0^2}{2\,z} \right)^2 \right);$$

$$\tan \phi = \frac{\kappa R_0^2}{2\,z}.$$

One can see that for a linear problem the extinction effect is completely separated from the diffraction one. The diffraction in this approximation gives a contribution to a phase change and does not affect the beam intensity. This conclusion is rather obvious because in the absence of the refractive index perturbations the beam refractive distortions do not arise in a homogeneous aerosol.

Let us now consider nonlinear interaction of radiation with an inflammable aerosol. Let the threshold approach[1] be taken as a model of the burning process. In the threshold approximation the aerosol particles are assumed to be inflammable when intensity exceeds the threshold value I_{thr}. After the inflammation of a particle the dynamics of the volume extinction coefficient is proposed to be independent of the intensity and determined by the processes of heat exchange and mass transfer.

First, some intensity distribution $I(r, z)$ is formed in the space. The particles start to burn when r and z take such values that $I(r, z) \ge I_{\mathrm{thr}}$. The area occupied by burning particles is a body of rotation stretched along the direction of the beam propagation. For the Gaussian beam we find the equation of surface limiting the area of burning particles from the relation (11)

$$\frac{I_{\mathrm{thr}}}{I_0} = \frac{\exp\left(-2 \dfrac{r^2}{R_0^2\,(1 + (2\,z/\kappa R_0^2)^2)} - \alpha\, z \right)}{1 + \left(2\,z/\kappa R_0^2 \right)^2}. \tag{12}$$

Here I_0 is the beam intensity on its axis at the beginning of the path. From relation (12) the surface equation can be presented as a function r of z

$$r^2 = R_0^2 \left[1 + \left(\frac{2\,z}{kR_0^2} \right)^2 \right] \left[\ln \frac{I_0}{I_{\mathrm{thr}}} - \ln\left(1 + \left(\frac{2z}{\kappa R_0^2} \right)^2 \right) - \alpha\, z \right]. \tag{13}$$

During the burn–out of particles in the region where $I(r, z) \ge I_{\mathrm{thr}}$ the extinction cross section decreases and the inflammation front moves forward along the path. Let us derive the equation determining the velocity of the inflammation front motion. From the condition of inflammation within the threshold approximation we have

$$I_{\mathrm{thr}} = \varphi(r, z_{\mathrm{thr}})\, \varphi^*(r, z_{\mathrm{thr}}) \exp\left(- \int_0^{z_{\mathrm{thr}}} \alpha(r, z)\, dz \right). \tag{14}$$

Here $z_{\mathrm{thr}} = z_{\mathrm{thr}}(t)$ is the coordinate of the inflammation front. Let us introduce the velocity of the inflammation front $v(t) = \dfrac{dz_{\mathrm{thr}}}{dt}$. It is clear that a change of the function $\alpha(r, z)$ at the point z is determined by the time of radiation action at this point with the intensity above the threshold one. Therefore, the function $\alpha(r, z)$ can be presented as the function $\alpha(r, t - \tau(z))$, where t is the time from the beginning of the action and $\tau(z)$ is the time of the threshold intensity I_{thr} appearance at the point z. By substituting the variable $z = z(\tau)$ $dz = v\,(t)d\tau$, we find from Eq. (14)

$$\frac{\varphi(r, z_{\mathrm{thr}})\ \varphi^*(r, z_{\mathrm{thr}})}{I_{\mathrm{thr}}} = \int_0^t \alpha\,(r, t - \tau)\ v\,(\tau)\ d\tau\,. \tag{15}$$

Let us consider propagation of the inflammation front along the beam axis. In this case, according to Eq. (11), Eq. (15) takes the form

$$\ln\left(\frac{I_0}{I_{\rm thr}[1 + (2\, z_{\rm thr}/\kappa\, R_0^2)^2]}\right) = \int\limits_0^t \alpha\,(0,\, t - \tau)\ \nu(\tau)\ \mathrm{d}\tau\ , \qquad (16)$$

where the designation $\varphi^*(0,\ 0)\ \varphi(0,\ 0) = I_0$ is introduced. Equation (16) has a singularity at $t = 0$ (see also Refs. 2 and 3). This singularity occurs because the light speed is assumed to be infinitely large in this approximation. Therefore, a certain layer of aerosol, z_0, flashes simultaneously.

Separating this singularity by the substitution

$$\nu(t) = z_0\ \delta(t) + \nu_0\,(t)\ ,$$

we find the regular integral equation to determine the velocity of the inflammation front

$$\ln\left(\frac{I_0}{I_{\rm thr}[1+ (2\, z_{\rm thr}/\kappa R_0^2)^2]}\right) = \alpha(0,\, t)\ z_0 + \int\limits_0^t \alpha(0,t-\tau)\ \nu_0(\tau)\mathrm{d}\tau\ ,\qquad (17)$$

where z_0 is determined from the condition

$$\ln\left(\frac{I_0}{I_{\rm thr}[1 + (2\, z_0/\kappa R_0^2)^2]}\right) = \alpha\ (0,\ 0)\ z_0\ .$$

When particles burn in the diffusion regime, $\alpha(t) = \alpha_0\ (1 - t/t_0)$, $\alpha = 0$ for $t > t_0$. In this case, assuming that the function $\nu_0(t)$ slightly changes on the interval $[t - t_0,\ t]$, when $t > t_0$ we find

$$\nu_0 = \frac{\ln\left(\dfrac{I_0}{I_{\rm thr}[1 + (2\, z/\kappa R_0^2)^2]}\right)}{\displaystyle\int\limits_0^{t_0} \alpha\ (t)\ \mathrm{d}\ t}\ . \qquad (18)$$

In Eq.(18) the velocity of the inflammation front has been found as a function of a coordinate along the path. A dependence of the velocity ν_0 on time can be obtained now by an ordinary numerical integration. It is clear from Eq.(18) that the velocity $\nu_0(z)$ monotonically decreases when z increases and vanishes at the point

$$z_{\max} = \frac{\kappa\, R_0^2}{2} \sqrt{\frac{I_0}{I_{\rm thr}} - 1}\ .$$

Now let us consider propagation of the inflammation front out of the beam axis. In this case Eq. (17) takes the form

$$\ln\left[\frac{I_0\ \exp\left(-\dfrac{r^2}{R_0^2(1 + (2z/\kappa\, R_0^2)^2)}\right)}{I_{\rm thr}\left(1 + \left(\dfrac{2\, z}{\kappa\, R_0^2}\right)^2\right)}\right] =$$

$$= \alpha(r,\ t)\ z_0\ (r) + \int\limits_0^t \alpha(r,\ t)\ v_0(r\ ,\tau)\ \mathrm{d}\tau\ . \qquad (19)$$

When $t > t_0$ under the same assumptions on a slow change of the velocity $\nu_0(t,\ r)$, we have

$$\nu_0(z,\ r) = \frac{\ln\left(\dfrac{I_0}{I_{\rm thr}}\right) + \ln\left(\dfrac{\exp\left(-\dfrac{r^2}{R_0^2(1+(2z/\kappa R_0^2)^2)}\right)}{1 + (2z/\kappa R_0^2)^2}\right)}{\displaystyle\int\limits_0^{t_0} \alpha(r,\ t)\mathrm{d}t}\ . \qquad (20)$$

In this case the velocity ν_0 does not decrease monotonically along the axis z since a diffraction in a certain region causes an increase in the radiation intensity. But for sufficiently long z the velocity begins to decrease and finally vanishes.

REFERENCES

1. V.I. Bukatyi, A.M. Sagalakov, A.A. Tel'nikhin, and A.M. Shaiduk, Fiz. Goreniya Vzryva **46**, No. 6, 46–50 (1979).
2. V.I. Bukatyi and A.M. Shaiduk, in: *Propagation of High–Power Laser Radiation through the Light Absorptive Medium*, Institute of Atmospheric Optics of the Siberian Branch of the Academy of Sciences of the SSSR, Tomsk (1982), pp. 40–48.
3. V.I. Bukatyi, A.A. Tel'nikhin, and A.M. Shaiduk, Zh. Prikl. Spektrosk. **36**, No. 4, 557–562 (1982).
4. V.S. Loskutov and G.M. Strelkov, in: *Propagation of High–Power Optical Radiation through the Solid Inflammable Aerosol* (State University, Barnaul, 1982), pp. 28–42.