INTERFEROMETRIC TECHNIQUES FOR CONSTRUCTING A TRIPLE CORRELATION FUNCTION OF A REMOTE OBJECT FROM A SINGLE EXPOSURE

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Interferometric devices are proposed to construct some two–dimensional projection of a triple correlation function of a remote object observed through the distorting medium (the atmosphere) from the data obtained from a single measurement distorted with phase perturbation. Such a measurement is shown to be sufficient for reconstructing an object image from a single exposure.

A lot of astronomic methods of observation through the Earth's atmosphere (Labeyrie and Knox–Thompson methods) are based on the correlation analysis. It is a convenient form of accumulation of radiation and statistical averaging of the atmospheric fluctuations. However, the correlation function of the second order, from which speckle interferometry started, does not allow, in the general case, reconstruction of an object image. In contrast to the second–order correlation function, the third–order one keeps the information on the phase of the spatial image spectrum, except for the linear term responsible for the shift of the object as a whole.

In recent years the third correlation function or its Fourier transform (bispectrum of the image) is successfully used in astronomy but only for processing the images with a low background.[1] It is not applied to processing the semitone images of general type because of the great computational expense. Thus according to the estimates presented in Ref. 2, bispectrum of an image with the size of 256×256 pixels, being the four–dimensional function, needs about 512 Mbytes RAM. There are only few computers in the world that are capable of processing such data arrays. For a comparison, the standard IBM PC has 1–2 Mbytes of operative memory and a hard disk of 40–80 Mbytes.

However, the four–dimensional bispectrum of a two–dimensional image is evidently very excessive. It is of great theoretical and practical interest to find such two–dimensional surfaces in four–dimensional space that could allow the reconstruction of the initial image if the bispectrum values on them are found. Solution of this problem would allow a significant reduction of the computational expenses and application of the correlation analysis of third order to semitone images. It is proved in Refs. 3 and 4 that in the vector $(\mathbf{f}_1, \mathbf{f}_2)$ space such surfaces are, in particular, the following: $f_{1x} = f_{2y}$; $f_{1y} = -f_{2x}$ (section A), and $\mathbf{f}_1 = \mathbf{f}_2$ (section B).

The Earth's atmosphere distorts the light waves, propagating through it, so the construction of the third correlation function of an astronomic object (as well as its autocorrelation) needs for averaging over the speckle image space. However, use of the interferometric observational techniques makes it possible to exclude the atmospheric fluctuations of the phase of incoming radiation even in single exposure. A technique is known for constructing the autocorrelation of an object from a single exposure when observing it with the interferometer of rotation shear at the angle π (Fig. 1).[5]

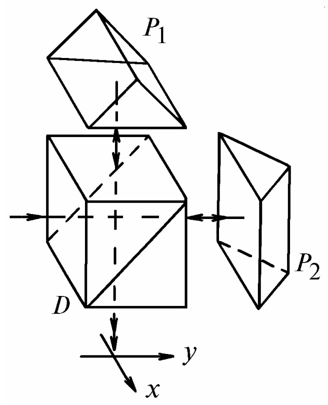

FIG. 1. Interferometer of shear at the angle π: beam–splitting cube D and right–angled prisms P_1 and P_2.

As was offered in Ref. 6, the use of two such interferometers with shears at the angles π/2 and π enables one to construct the aforementioned two–dimensional section A of the image bispectrum in a single exposure. The main elements of the interferometers are the beam–splitting cubes and two right–angled prism oriented as shown in Figs. 1 and 2.

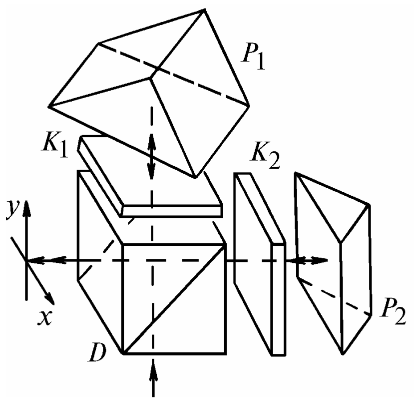

FIG. 2. Interferometer of shear at an angle π/2: K_1 and K_2 are the compensators.

Compensators in the interferometer arms equalize the phase shifts of the waves polarized in the plane of incidence on

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the reflecting faces of the prism (τ—wave) and normal to them (n—wave) that makes it possible to work with unpolarized light. The difference in optical paths is $\lambda/2$ for the x—polarized wave and $-\lambda/2$ for the y—polarized one (λ is the wavelength) in the interferometer of shear at an angle π (Fig. 1) when the geometric lengths of the interferometer arms are equal to each other, and if the absolute dielectric constant of the prism matter is $\varepsilon = 3$. This means that the superposition of the interference patterns observed in x— and y—polarized light does not lead to decrease of their contrast because these patterns are shifted with respect to each other exactly by one period of the interferogram. Since ε of glass is approximately equal to three, this interferometer does not need for polarization tools.

The radiation from two points of the input pupil is summed at the output of the first and second interferometers. Their radius—vectors form the angles $\pi/2$ and π, respectively,

$$E_1(\mathbf{r}) = (E(\mathbf{a}) + E(\mathbf{b}))/2;$$
$$E_2(\mathbf{r}) = (E(\mathbf{c}) + E(-\mathbf{c}))/2,$$

and the intensity distributions, recorded simultaneously, have the form

$$I_1(\mathbf{r}) = I_0(1 + \mu(\mathbf{a}-\mathbf{b}) \cos(\psi(\mathbf{a}-\mathbf{b}) + \kappa_1\mathbf{r})); \quad (1)$$
$$I_2(\mathbf{r}) = I_0(1 + \mu(2\mathbf{c}) \cos(\psi(2\mathbf{c}) + \kappa_2\mathbf{r}))\,. \quad (2)$$

Here the radius—vectors $\mathbf{r} = (x, y)$, $\mathbf{a} = (-y, -x)$, $\mathbf{b} = (-x, y)$, and $\mathbf{c} = (-y, x)$; $E(\mathbf{r})$ is the incident field with the intensity $2I_0$; $\mu(\mathbf{a}-\mathbf{b})$ and $\psi(\mathbf{a}-\mathbf{b})$ are the modulus and the phase of the complex function of coherence (CFC) of the incident field on the base $\mathbf{a}-\mathbf{b}$; modulation of $\kappa_1\mathbf{r}$ and $\kappa_2\mathbf{r}$ is produced by superposition of the interfering beams at small angles. Without amplitude distortions, the modulus of CFC on the base $\mathbf{a}-\mathbf{b}$ is the modulus of the object spectrum at the spatial frequency $(\mathbf{a}-\mathbf{b})/\lambda$, and the phase of CFC is the sum of the spectrum phase θ and the difference of phase distortions of the incident field at the points with coordinates $\mathbf{a}$ and $\mathbf{b}$:

$$\psi(\mathbf{a}-\mathbf{b}) = \theta((\mathbf{a}-\mathbf{b})/\lambda) + \varphi(\mathbf{a}) - \varphi(\mathbf{b}). \quad (3)$$

The object spectra $F^{(1)}$ and $F^{(2)}$ found by filtering the interferograms (1) and (2), respectively, contain the phase distortions:

$$F^{(1)}(\mathbf{f}_1) = O(\mathbf{f}_1) \exp[i(\varphi(\mathbf{a}) - \varphi(\mathbf{b}))]; \quad (4)$$

$$F^{(2)}(\mathbf{f}_1 + \mathbf{f}_2) = O(\mathbf{f}_1 + \mathbf{f}_2) \exp[i(\varphi(-\mathbf{b}) - \varphi(\mathbf{b}))], \quad (5)$$

where $\mathbf{f}_1 = (\mathbf{a}-\mathbf{b})/\lambda = (u, -v)$, $\mathbf{f}_2 = (-\mathbf{a}-\mathbf{b})/\lambda = (v, u)$, $u = (x-y)/\lambda$, $v = (x+y)/\lambda$, $O(\mathbf{f})$ is the sought spatial spectrum of the observed object. Using the spectra obtained, let us form the bispectrum of the object

$$B(\mathbf{f}_1;\mathbf{f}_2) = F^{(1)}(\mathbf{f}_1)\, F^{(1)}(\mathbf{f}_2)\, F^{(2)}(-\mathbf{f}_1 - \mathbf{f}_2) = B(u, v).$$

Taking into account Eq. (3) it is easy to show that the phase of the bispectrum obtained

$$\psi(\mathbf{a}-\mathbf{b}) + \psi(-\mathbf{a}-\mathbf{b}) - \psi(-2\mathbf{b}) = \theta(\mathbf{f}_1) + \theta(\mathbf{f}_2) - \theta(\mathbf{f}_1 + \mathbf{f}_2)$$

is free from all additive distortions.[3,4,6]

The bispectrum $B(u, v)$ can be found exactly from the single measured data containing the phase distortions. Algorithm for reconstructing the phase of Fourier spectrum of an object from the phase of its bispectrum $B(u, v)$ is based on the fact that each value of the bispectrum phase is the recurrent equation determining the spectrum phase $\theta(\mathbf{f}_1 + \mathbf{f}_2)$ via two phases at lower spatial frequencies $\theta(\mathbf{f}_1)$ and $\theta(\mathbf{f}_2)$. Assuming $\theta = 0$ in the vicinity of zero frequency, let us reconstruct the spectrum phase up to the frequency corresponding to the maximum base of interference.[4,5]

One can combine two shear interferometers into a single one as shown in Fig. 3. For this orientation of the coordinate axes, the field in the beam cross section in different parts of the path is insensitive to the reflections from semitransparent diagonals of the cube—prism D, D_1, and D_2, and the symmetry transformation about the $y = -x$ axis occurs as a result of the reflection from the mirror R. So the beam, passing through the loop $D - D_1 - R - D_2 - D$ in any direction, transfers the image of the plane of the input pupil with respect to the $y = -x$ axis to all three output planes A, B, and C. The beam turned by the prism P_1 transfers the image of the plane of the input pupil with respect to y axis and emerges in the plane A and C. The beam turned by the prism P_2 transfers the image of the plane of the input pupil with respect to x axis and exits through the plane A and B. Compensator in the loop $D - D_1 - R - D_2 - D$ brings about the phase shift between the x— and y—polarized waves equal to $\gamma_n - \gamma_s$, where γ_n and γ_s are the phase shifts of n— and τ—waves at the total internal reflection in prisms. As was mentioned above, for the glass with dielectric constant $\varepsilon = 3$, $\gamma_n - \gamma_s$ equals π. Then the optical lengths of the interferometer arms for the waves of x— and y—polarizations are equal to each other or differ by one wavelength that makes it possible to use unpolarized light. The radiation fields in the planes A, B, and C of the interferometer (Fig. 3) have the form

$$4\,E_A(x, y) = \sqrt{2}\,E(-y, -x) + E(-x, y)\exp(i\kappa_1 y) + E(x, -y)\exp(i\kappa_2 x);$$
$$2\sqrt{2}\,E_B(x, y) = E(-y, -x) + E(x, -y)\exp(i\kappa_2 x);$$
$$2\sqrt{2}\,E_C(x, y) = E(-y, -x) + E(-x, y)\exp(i\kappa_1 y),$$

where modulation of $\kappa_1 x$ and $\kappa_2 x$ is produced by a small tilt of the prisms to the optical axis of the system. Both Fourier spectra (4) and (5) can be determined from the interferogram recorded in the plane A.

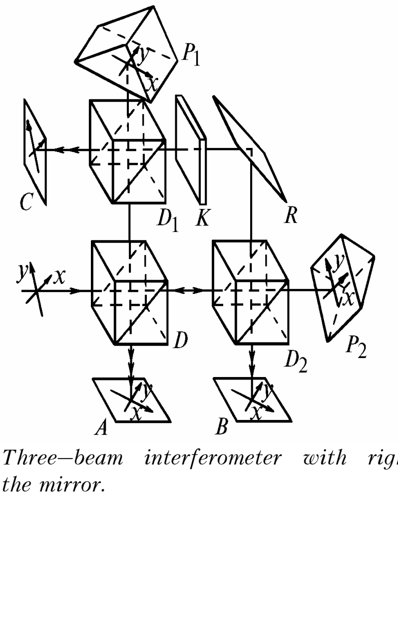

FIG. 3. Three—beam interferometer with right—angled prism, R is the mirror.

Two rectangle prisms can be replaced by mirrors and Dove prism in the loop $D - D_1 - R - D_2 - D$ (Ref. 3) (Fig. 4). The beams reflected from mirrors R_1 and R_2 transfer the identical projections of the input field $E(x, y)$ to the output planes A, B, and C. The fields of the beams passing the loop with the mirror R in counter directions undergo a number of transformations. First, the Dove prism specularly reflects the plane of the input pupil about its reflecting face. If the reflecting face is in the plane $x = y$ (axes orientation is shown in Fig. 4) then the field $E(x, y)$ at the input of the prism is transformed to the field $E(y, x)$ at the output. Let us introduce the operator

$$(\arg_1 \Leftrightarrow \arg_2)$$

that makes permutation of coordinates:

$$(\arg_1 \Leftrightarrow \arg_2)\, E(x, y) = E(y, x) .$$

Second, at the reflection from mirror R the mirror transformation of the field occurs in the cross section of the beam about the x axis. For designation of this transformation let us introduce the operator

$$(\arg_2 \Rightarrow - \arg_2)$$

that changes the sign of the second coordinate. Third, the air wedge that provides the phase shift κy is produced by small tilt of the mirror R.

The sequence of the field transformations in the cross section of the beam passing the loop with the Dove prism, mirror, and air wedge in the clockwise and counter clockwise directions takes the form

$$E_1(x, y) = \exp(i\kappa y)\,(\arg_2 \Rightarrow - \arg_2)\,(\arg_1 \Leftrightarrow \arg_2)\, E(x, y) =$$
$$= \exp(i\kappa y)\, E(y, - x);$$
$$E_2(x, y) = (\arg_1 \Leftrightarrow \arg_2)\,(\arg_2 \Rightarrow - \arg_2)\, \exp(i\kappa y)\, E(x, y) =$$
$$= \exp(- i\kappa x)\, E(- y, x).$$

The first beam passing the loop in the clockwise direction emerges through the planes A and B, and the second through the planes A and C. As a result, the following fields are transferred to the planes A, B, and C:

$$4 E_A(x, y) = \sqrt{2}\, E(x, y) + \exp(i\kappa y)\, E(y, - x) +$$
$$+ \exp(- i\kappa x)\, E(- y, x);$$
$$2\sqrt{2}\, E_B(x, y) = E(x, y) + \exp(i\kappa y)\, E(y, - x);$$
$$2\sqrt{2}\, E_C(x, y) = E(x, y) + \exp(- i\kappa x)\, E(- y, x).$$

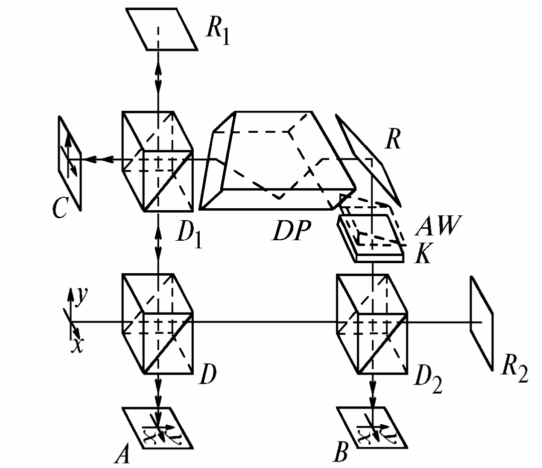

FIG. 4. Three–beam interferometer with Dove prism (DP). AW is the air wedge.

To reconstruct the image, single interference pattern recorded in the plane A of any of two interferometers is quite sufficient. In order to collect more power (up to 75% instead of 50% in operation with two interferometers) one may additionally record the interferograms in the planes B and C.

Let us note again, for the conclusion, that the proposed interferometric techniques make it possible to exclude both atmospheric fluctuations and instrumentation distortions of the phase of incident radiation in single exposure. This is the advantage of the proposed interferometric techniques over the traditional speckle interferometry based on statistical averaging of fluctuations over many exposures.

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