LARGE BIMORPH ADAPTIVE MIRROR. 2. CALCULATION OF THE OPERATIONAL EFFICIENCY

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This paper demonstrates the efficiency of using a large bimorph adaptive mirror for correction of the large–scale low–frequency distortions of the wavefront. The correction of the low–order optical aberrations by this mirror is computer simulated. Theoretical approach to the problem of adaptive correction of distortions is described. The comparative analysis of the efficiency of bimorph mirror and adaptive mirror with discrete electromechanical actuators has been carried out. The simple and effective way of forming the mosaic bimorph structure using the multilayer piezoelectric elements is proposed.

1. INTRODUCTION

The construction of a large bimorph adaptive mirror of astronomic telescope was considered in Ref. 1. Its prototype was a mirror fabricated from glassceramic with a diameter of 3.3 m and inner aperture of 0.5 m. It was equipped with the passive vertical and horizontal unloading system and 54 electromechanical actuators for control over its shape. Without changing a construction (unloading system and geometric size), it was proposed to replace the controlling electromechanical actuators by a solid piezoelectric mosaic layer fabricated from hexagonal piezoceramic plates 1 mm thick arranged on the back side of the glassceramic mirror. The response functions of controlling electrodes, the influence of the ambient temperature on the deformed state of the reflecting surface, and the frequency responses of the mirror were calculated.

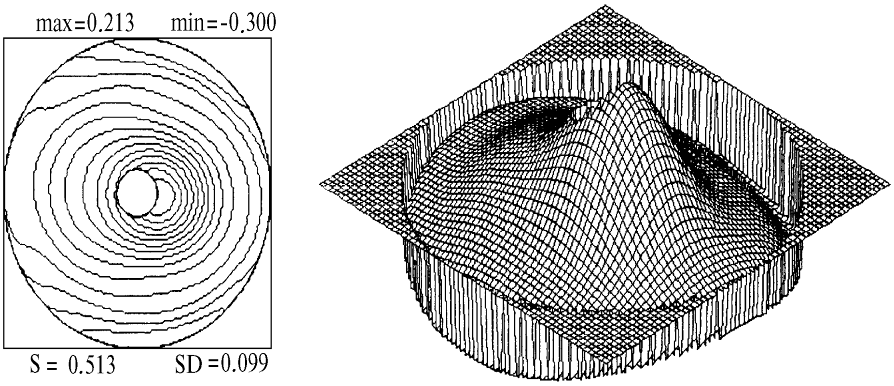

FIG. 1. Typical response function of the large bimorph adaptive mirror for a controlling voltage of 300 V *and electrode No. 1. Span (S), standard deviation (SD), and maximum and minimum values are given in micrometers.*

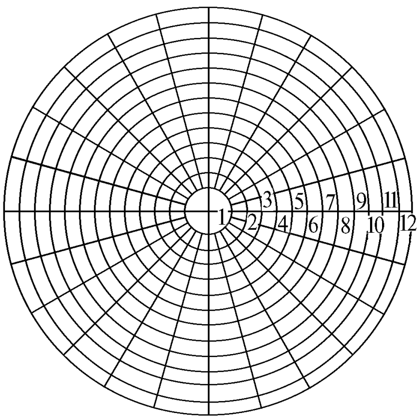

FIG. 2. Segmentation of controlling electrodes on the piezoceramic surface.

The typical response function of a large bimorph adaptive mirror is shown in Fig. 1 for a controlling voltage of 300 V in two different ways: as the isometric projection and the contours of equal response level. The span (S), standard deviation (SD), and maximum and minimum response functions were measured in the direction along the normal to the reflecting surface of the mirror. The segmentation of the controlling electrodes on the piezoceramic surface is shown in Fig. 2. The resonance frequency of the first adaptive mirror was 1 Hz. Calculations of the influence of the ambient temperature variations on the deformed state of the mirror showed that the span of the displacement of the reflecting surface reached 3.96 μm as temperature changed by 1°C.

Let us analyze the efficiency of the large bimorph adaptive mirror, namely, the quality of compensation for the distortions of the wavefront, using the results obtained in Ref. 1.

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2. THEORETICAL APPROACH TO AN ANALYSIS OF THE EFFICIENCY OF COMPENSATION FOR THE WAVEFRONT DISTORTIONS

Let $W(\mathbf{r})$ be the wavefront of radiation incident on an adaptive mirror, and $\Delta W(\mathbf{r})$ be the wavefront of the reflected radiation. Let us control the adaptive mirror in such a way that the residual root–mean–square error σ in the correction be minimum, i.e.,

$$\min_{\{U_i\}_{i=1}^{N}} \sigma = \min_{\{U_i\}_{i=1}^{N}} \sqrt{\frac{1}{S_\Omega}\int_{\Omega_a}\left[\Delta W(\mathbf{r})-\frac{1}{S_\Omega}\int_{\Omega_a} \mathrm{D}W(\mathbf{r})\,d\mathbf{r}\right]^2 \mathrm{d}\,\mathbf{r}}=\sigma_{\min}, \quad (1)$$

where S_Ω is the area of the adaptive mirror aperture, N is the number of the control channels of the adaptive mirror, and U_i is the controlling voltage in the ith control channel.

For the reflected radiation the following relation is valid:

$$\Delta W(\mathbf{r}) = W(\mathbf{r}) - \frac{4\pi}{\lambda}\sum_{i=1}^{N} U_i\, f_i(\mathbf{r}), \quad (2)$$

where λ is the radiation wavelength and $f_i(\mathbf{r})$ is the ith response function for maximum U_i. The normal (or almost normal) incidence of radiation on the adaptive mirror is assumed in Eq. (2), and the parameters U_i are taken to be normalized to their maximum value, so that the condition

$$-1 \le U_i \le 1,\ i = 1, \ldots, N . \quad (3)$$

is satisfied. Since the wavefront is determined to within the constant factor, without loss of generality we may assume the mean values of all the above–considered functions to be equal to zero.

In what follows that the condition of minimization of the residual root–mean–square error may be written in the form

$$\min_{\{A_i\}_{i=1}^{N}} \int_{\Omega_a} \left[W(\mathbf{r}) - \sum_{i=1}^{N} A_i\, f_i(\mathbf{r})\right]^2 \mathrm{d}\mathbf{r}, \quad (4)$$

where $A_i = \frac{4\pi}{\lambda}\cdot U_i$, $i = 1, \ldots, N$.

Let us represent $W(\mathbf{r})$ in the form of a series in Zernike polynomials[1] $Z_i(\mathbf{r})$. Practically it is suffice to take into account the first terms in the expansion. Let M be the number of Zernike polynomials sufficient for representation of the function $W(\mathbf{r})$ and $M > N$. Then

$$W(\mathbf{r}) = \sum_{i=1}^{M} \alpha_i\, Z_i(\mathbf{r}) , \quad (5)$$

$$\alpha_i = \int_{\Omega_a} W(\mathbf{r})\, Z_i(\mathbf{r})\, \mathrm{d}\mathbf{r} . \quad (6)$$

Thus we have a vector

$$\alpha = (\alpha_1, \ldots, \alpha_M) , \quad (7)$$

which specifies the incident wavefront.

Let us expand each from the N response functions $f_i(\mathbf{r})$ in an analogous series

$$f_j(\mathbf{r}) = \sum_{i=1}^{M} F_{ij}\, Z_i(\mathbf{r}),\ j = 1, \ldots, N. \quad (8)$$

Therefore, we have N vectors of $\mathbf{F}_j$:

$$\mathbf{F}_j = (\mathbf{F}_{1j}, \ldots, \mathbf{F}_{Mj}),\ j = 1, \ldots, N, \quad (9)$$

and each vector describes the individual response function.

Let us rewrite expression (2) in the form

$$\Delta = \alpha - \hat{F}\,\mathbf{A}, \quad (10)$$

where

$$\Delta = (\Delta_1, \ldots, \Delta_M) \quad (11)$$

is the vector describing the reflected radiation with the wavefront $\Delta W(\mathbf{r})$; in addition,

$$\Delta W(\mathbf{r}) = \sum_{i=1}^{M} \Delta_i\, Z_i(\mathbf{r}), \quad (12)$$

$$\mathbf{A} = (A_1, \ldots, A_N) \quad (13)$$

is the vector describing the controlling forces, and

$$\hat{F} = \begin{pmatrix} \mathrm{F}_1 & \ldots & \mathrm{F}_{1N} \\ \ldots & \ldots & \ldots \\ \mathrm{F}_{M1} & \ldots & \mathrm{F}_{MN} \end{pmatrix} \quad (14)$$

is the matrix whose columns are the vectors $\mathbf{F}_j$.

Condition (4) can be rewritten in the form

$$\min \sum_{i=1}^{M} \Delta_i^2, \quad (15)$$

where the minimum is meant with respect to all possible values of the vector $\mathbf{A}$. Obviously, condition (15) can be written in the form

$$\frac{\partial}{\partial \mathrm{A}_\kappa} \sum_{i=1}^{M} \Delta_i^2 = 0,\ \kappa = 1, \ldots, N . \quad (16)$$

Taking into account Eq. (10), we derive the system of differential equations

$$\frac{\partial}{\partial \mathrm{A}_\kappa} \sum_{i=1}^{M}\left(\alpha_i - \sum_{j=1}^{N} F_{ij}\, A_j\right)^2 = 0,\ \kappa = 1, \ldots, N. \quad (17)$$

After differentiation, we derive the linear system of equations

$$\sum_{i=1}^{M}\left(\alpha_i - \sum_{j=1}^{N} F_{ij}\, A_j\right) F_{i\kappa} = 0,\ \kappa = 1, \ldots, N \quad (18)$$

for the unknown values A_j, $j = 1, \ldots, N$. Having solved system (18), we obviously find the vector $\mathbf{A}_0$:

$$\mathbf{A}_0 = (A_{01}, \ldots, A_{0N}), \quad (19)$$

for which condition (15) is satisfied.

We note that a set of the solution of system (18) depends on the relationship between the number of the response functions N and that of Zernike polynomials M. If $N > M$, then Eq. (18) has no unique solution. Let us show it. Let us write Eq. (18) in the form

$$\sum_{i=1}^{M}\sum_{j=1}^{N} F_{ij}\, A_j\, F_{i\kappa} = \sum_{i=1}^{M} \alpha_i\, F_{i\kappa},\ \kappa = 1, \ldots, N. \quad (20)$$

Changing the summation order in the left side, we derive

$$\sum_{j=1}^{N} A_j \sum_{i=1}^{M} F_{i\kappa}\, F_{ij} = \sum_{i=1}^{M} \alpha_i\, F_{i\kappa},\ \kappa = 1, \ldots, N. \quad (21)$$

Recall that $\mathbf{F}_\kappa$ is the M–dimensional vector with the coordinates on the orthonormal basis Z_j, $j = 1, \ldots, M$ [see Eq. (19)]. Then the scalar product of two vectors can be written as

$$(\mathbf{F}_\kappa, \mathbf{F}_j) = \sum_{i=1}^{M} F_{i\kappa}\, F_{ij}\ . \quad (22)$$

Therefore, we derive from Eq. (20)

$$\sum_{j=1}^{N} (\mathbf{F}_\kappa, \mathbf{F}_j)\, A_j = \sum_{i=1}^{M} \alpha_i\, F_{i\kappa},\ \kappa = 1, \ldots, N. \quad (23)$$

The principal matrix of the linear system of equations (23) has the form

$$\begin{pmatrix} (\mathbf{F}_1, \mathbf{F}_1) & \ldots & (\mathbf{F}_1, \mathbf{F}^M) \\ \ldots & \ldots & \ldots \\ (\mathbf{F}^M, \mathbf{F}_1) & \ldots & (\mathbf{F}^M, \mathbf{F}^M) \end{pmatrix}. \quad (24)$$

Obviously, in this case ($N > M$) a set of vectors $\mathbf{F}_j$, $j = 1, \ldots, N$ is linearly dependent since the dimension of the orthonormal basis Z_i, $i = 1, \ldots, M$ is equal to M. This means that at least one vector $\mathbf{F}_j$ can be represented in the form of linear combination of the others. Let it be the vector $\mathbf{F}_n$

$$\mathbf{F}_n = p_1\mathbf{F}_1 + \ldots + p_{n-1}\mathbf{F}_{n-1} + p_{n+1}\mathbf{F}_{n+1} + \ldots + p_N\mathbf{F}_N = \sum_{\substack{i=1\\ i\neq n}}^{N} p_i\mathbf{F}_i\ , \quad (25)$$

where p_i, $i = 1, \ldots, N$, $i \neq n$ are the coefficients of the linear combination. Then matrix (24) assumes the form

$$\begin{pmatrix} (\mathrm{F}_1, \mathrm{F}_1) & \ldots & \left(\mathrm{F}_1, \sum\limits_{\substack{i=1\\ i\neq n}}^{N} p_i\mathrm{F}_i\right) & \ldots & (\mathrm{F}_1, \mathrm{F}^M) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \left(\left[\sum\limits_{\substack{i=1\\ i\neq n}}^{N} p_i\mathrm{F}_i\right], \mathrm{F}_1\right) & \ldots & \left(\left[\sum\limits_{\substack{i=1\\ i\neq n}}^{N} p_i\mathrm{F}_i\right], \left[\sum\limits_{\substack{i=1\\ i\neq n}}^{N} p_i\mathrm{F}_i\right]\right) & \ldots & \left(\left[\sum\limits_{\substack{i=1\\ i\neq n}}^{N} p_i\mathrm{F}_i\right], \mathrm{F}^M\right) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ (\mathrm{F}^M, \mathrm{F}_1) & \ldots & \left(\mathrm{F}^M, \sum\limits_{\substack{i=1\\ i\neq n}}^{N} p_i\mathrm{F}_i\right) & \ldots & (\mathrm{F}^M, \mathrm{F}^M) \end{pmatrix} \quad (26)$$

Taking p_i out of the scalar product, we reduce the nth row of matrix (26) to the form

$$\sum_{\substack{i=1\\ i\neq n}}^{N} p_i(\mathbf{F}_i, \mathrm{F}_1) \ldots \sum_{\substack{i=1\\ i\neq n}}^{N} p_i(\mathbf{F}_i, [\sum_{\substack{i=1\\ i\neq n}}^{N} p_i\, \mathbf{F}_i]) \ldots \sum_{\substack{i=1\\ i\neq n}}^{N} p_i(\mathbf{F}_i, \mathbf{F}^M). \quad (27)$$

Expression (27) is nothing but the linear combination of the rows of matrix (26). Therefore, the determinant of matrix (24) is equal to zero for $N > M$, and in this case system (18) has no unique solution. In addition,

$$\min_{\mathbf{A}} \sum_{i=1}^{M} \Delta_i^2 = 0. \quad (28)$$

In this case we find the vector–solution $\mathbf{A}_0$, but cannot say a word about the accuracy of correction.

For $N < M$ system (18) has a unique solution for which we can calculate Eq. (15) and determine the minimum error in correction.

3. COMPUTER SIMULATION

In computer simulation the distortions of the radiation incident on the adaptive mirror were assigned in the form of model wavefronts described by Zernike polynomials, namely,

– defocusing $C_4\sqrt{3}(2\,r^2 - 1)$,

– astigmatism $C_6\sqrt{6}\,r^2\cos 2\theta$,

– coma $C_8\sqrt{8}(3\,r^2 - 2\,r)\cos\theta$,

– trefoil $C_{10}\sqrt{8}\,r^3\cos 3\theta$, and

– spherical aberration $C_{11}\sqrt{5}(6\,r^4 - 6\,r^2 + 1)$.

We have considered the aberrations only in cosine representation since the results for the sine representation are analogous for our segmentation of controlling electrodes due to the symmetry of the problem.

The number of the response functions N is equal to 288 for the above–considered large bimorph adaptive mirror. We used 55 Zernike polynomials for the expansion of functions. Symmetry of the problem was used in the numerical solution of system (18) to decrease the number of the response functions (i.e., to satisfy the condition $N < M$). It is clear that due to axial symmetry of defocusing and spherical aberration the controlling voltages on the electrodes arranged in one circle are equal in optimal correcting. For this reason these aberrations were controlled by 12 electrodes, i.e., electrodes arranged in one circle were interconnected forming one electrode. Therefore, the condition $N < M$ was satisfied and the residual error in the correction could be determined.

The fourth–order symmetry of coma was used for its correction taking into account the sign of deformation of the reflecting mirror surface (Fig. 3*a*). It is seen from Fig. 3 that when the deformation due to coma is equal to $+S$ at a given point of the surface, deformation in the other three points, which are conjugate to the first point, is also known. Therefore, the number of controlling electrodes is 4 times less. In this case it is suffice to interconnect the electrodes as shown in Fig. 3*b* in order to satisfy the condition $N < M$, and, most likely, the quality of correction will deteriorate. The number of the controlling electrodes in the case of coma was 36.

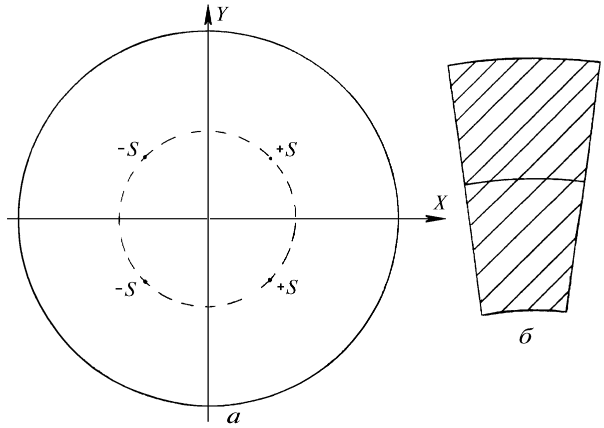

FIG. 3. Symmetry of coma (a) and the assembly of controlling electrode used for correction (b).

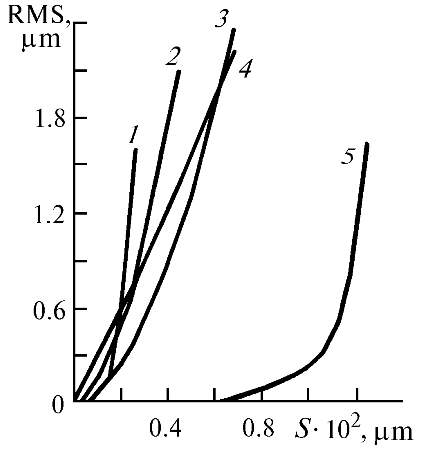

FIG. 4. Residual root–mean–square error in correcting the third–order aberrations by means of the large bimorph adaptive mirror vs. the span of aberrations: 1) spherical aberration, 2) trefoil, 3) astigmatism, 4) coma, and 5) defocusing.

The symmetry of astigmatism was also taken into account together with the sign of deformation in correcting the astigmatism. The number of electrodes was 36, and the condition $N < M$ was satisfied.

The symmetry of trefoil with allowance for its sign made it possible to decrease the number of electrodes to 24, which also provided the condition $N < M$.

The results of correction of the above–enumerated aberrations with the large bimorph adaptive mirror are shown in Fig. 4 for normal incidence of radiation. The maximum controlling voltage used for correction was equal to ±300 V.

The maximum values of the span (S) of the third–order aberrations are presented in Table I. The root–mean–square error in correcting these aberrations by means of the above–considered mirror does not exceed λ/20 (λ = 0.55 μm). The number of controlling electrodes for each type of aberration is also presented. The analogous data for the adaptive mirror with 54 electromechanical electrodes are also given in Table I for comparison.

TABLE I.

Type of aberration	Representation form	Span of aberrations of the adaptive mirror with 54 actuators, μm	Bimorph adaptive mirror	
			S, μm	Number of electrodes for correction
Defocusing	$C_4\sqrt{3}(2r^2 - 1)$	2.4	69	12
Astigmatism	$C_6\sqrt{6}\,r^2\cos 2\theta$	9.1	7.2	36
Coma	$C_8\sqrt{8}(3r^2 - 2r)\cos\theta$	1.1	1	36
Trefoil	$C_{10}\sqrt{8}\,r^3\cos 3\theta$		4.8	24
Spherical aberration	$C_{11}\sqrt{5}(6r^4 - 2r^2 + 1)$	0.3	8.5	12

4. DISCUSSION OF RESULTS

It is well known that the large bimorph mirror has the highest quality of correction of the axially symmetric aberrations, i.e., defocusing and spherical aberration. Correction of trefoil, astigmatism, and coma is less effective and is a few micrometers. We also note that maximum voltages on the controlling electrodes did not exceed ±300 V in our investigations, though it is not a limit for the given piezoceramics 1 mm thick. Deformation of the reflected surface of large bimorph adaptive mirror is greater for larger controlling voltage; therefore, the correction of distortions is more effective. We used the voltage up to 300 V for control over the mirror only because, as it seems to us, difficulties arise in the construction of the control system for larger voltage.

It is interesting to compare the results obtained with the aforementioned data for the analogous mirror with 54 electromechanical actuators. The results of correction with these two mirrors are comparable only in the case of astigmatism and coma. The advantage of the bimorph mirror is very essential in correction of defocusing and spherical aberration.

Recall that the results given in this paper and in Ref. 1 confirm the high efficiency of the large bimorph adaptive mirror in correcting the large–scale low–frequency distortions of the wavefront. These results were obtained for the controlling mosaic piezoelectric layer 1 mm thick (thickness of the mirror plate was 78 mm). Such a relationship between the thicknesses was obviously not optimal and was selected only to demonstrate the capabilities of the large bimorph adaptive mirror. A multilayer mosaic piezoelectric structure was proposed in Ref. 1 to decrease the controlling voltage. Evidently, analogous approach can be applied to obtain the optimal thickness of the piezoelectric structure.

Here we would like to discuss another variant of the piezoelectric structure of the large mirror of the telescope. It is the following. As before, one mosaic piezoelectric layer is used. Its thickness is preliminary optimized for the given thickness of the mirror plate against the criterion of the maximum of controllable deformations of the reflecting surface. However, the elements are fabricated in the form of multilayer composition with as small thickness of piezoceramic film as possible and the optimal total thickness. The authors are familiar with some experiments in which the films with the thickness of the order of 20–40 μm were used. The element can be pressed into plastics similar to the above–described construction of the controlling piezoelectric actuators.[2] Such elements are very convenient in operation and it is not difficult to form the mosaic piezoelectric structure of the large mirror of the telescope on their basis. The transverse size of such a multilayer element can likely be of the order of 50 mm (corresponding to the typical diameter of commercial piezoelectric plates), while the limiting controlling voltage can be a few tens of volts. It is not of necessity to interconnect the Individual elements when forming the mosaic piezoelectric structure on the back side of the large mirror, only minimum gaps must be provided between neighbouring elements for best filling of the back surface of the mirror.

It seems to us that analogous construction of the large bimorph mirror will provide a very high amplitude of the controllable deformations of the reflecting surface. However, additional computer calculations are needed to obtain the numerical estimates.

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2. A.V. Ikramov, S.V. Romanov, I.M. Roshchupkin, et al., Opt. Mekh. Prom., No. 5, 60–63 (1992).