ADAPTIVE POSTDETECTOR PROCESSING OF A LIDAR SIGNAL

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An adaptive postdetector processing of signals is considered in this paper. Tfiis approach allows one to detect a weak signal by increasing the time of accumulation and to reduce the dynamic error in the presence of a strong signal when the time of accumulation is being decreased. It is shown that the process at the output of the lidar system photodetector is a Gaussian–Markovian one, for which the mean integral estimate of the mathematical expectation converges to the optimal one. TIte optimal observation time is found where the sum of the fluctuation and dynamic errors appears to be minimum. For the given maximum relative error of the signal recovering the threshold energy is determined that could provide for this error.

INTRODUCTION

In spaceborne lidar systems for a global ecological control there is a problem on extraction of a weak signal from noise. The simplest postdetector processing of signals used in optical detectors does not meet current requirement for precision and in some cases it is impossible to detect the signal at all.

An increase of the mean laser power due to an increase of pulse repetition frequency results in undesirable growth of weight, overall dimensions, and power consumption as well as cost of the lidar system. Moreover the laser service life and reliability become lower.

The most promising way of extracting a signal from noise is the use of Kalman-Byusi nonstationary optimal filter or adaptive systems of signal processing.

The adaptive postdetector processing of a lidar signal (APPLS), in contrast to the Kalman digital filter, does not employ an analog-to-digital converter (ADC) and hence can acquire input signals of a wider dynamic range. The latter is much promising for use of adaptive postdetector processing of signals in ground-based lidars. The APPLS enables one to detect a weak signal by automatically increasing in the time of accumulation and to reduce the dynamic error in the case of a strong signal when the time of accumulation decreases.

STATISTICS OF AN OUTPUT SIGNAL

In a photodetector there occurs conversion of the detected field into a signal current (or voltage) which then undergoes postdetector processing. Choice of an optimal signal processing depends on statistics of a signal at the photodetector output. Since the conversion of an optical field into an electron flux and the electron multiplication (for a PMT or an avalanche diode) are intrinsically random, the photodetector output current is also random. The output voltage of a PMT may be regarded as a filtered Poisson process[1]

$$U(t) = \sum_{j=1}^{N(0,\,t)} R_{\mathrm{L}}\, G\, q\, h\,(t - t_j), \qquad (1)$$

where G is the PMT gain, R_{L} is the PMT load resistance, q is the electron charge, $N(0, t)$ is the simple Poisson process, $h(t)$ is the photodetector current response to a single electron, and t_j is the moment of the jth electron release.

The response function $h(t)$ satisfies the normalization condition $\int\limits_{-\infty}^{\infty} h(t)dt = 1$. Duration of the $h(t)$ response function of a PMT is set by a transit time τ_h, which is inversely proportional to a photodetector transmission band. When the photodetector transmission band is unlimited ($\tau_h \to 0$) the $h(t)$ function can be treated as the Dirac δ–function.

The random process (1) may be interpreted mathematically assuming that a δ–pulse train acts on an input of an inertia) section having a pulse characteristic $h(t)$

$$\xi(t) = R_{\mathrm{L}}\, G\, q \sum_{j=1}^{N(0,\,t)} \delta(t - t_j).$$

Then a random process at the output of the inertial section is written as the convolution integral

$$U(t) = \int\limits_0^t h(t - \tau)\, \xi(\tau)\, d\tau\,, \qquad (2)$$

what corresponds to expression (1).

The process $\xi(\tau)$is stationary in a wide sense and has the following statistical characteristics:

the mathematical expectation

$$m_\xi(t) = R_{\mathrm{L}}\, G\, q\, n_0 = m_\xi\,,$$

where n_0 is the mean photoelectron count rate and F is the PMT's noise coefficient;

the covariance function

$$K_\xi(\tau) = m_\xi^2 + C_1\, \delta(\tau)\,, \qquad (3)$$

where $C_1 = (R_{\mathrm{L}}\, G\, q)^2\, n_0 F$.

As is seen from expression (3) the Poisson process is a purely "white" process (a process with an unlimited frequency band and infinite variance).

Let an integrating RC–chain with the parameter $a_1 = 1/RC$ be an inertial section then the output process $U(t)$ is described with a fluctuation linear differential equation of the first order.[2] From its solution it is possible to find the mathematical expectation of the output process

$$m_U(t) = m_\xi \left\{1 - \exp\left(-\alpha_1 t\right)\right\},$$

and the variance

$$D_U(t) = (C_1 \alpha_1 / 2) \left\{1 - \exp\left(-\alpha_1 t\right)\right\}.$$

These statistical characteristics are time dependent, therefore the output process in the general case is nonstationary despite the stationarity of the input process, but for $t_1 = 1.5/\alpha_1$ the variance $D_U(t)$ is $0.475\,\alpha_1 C_1$ and for $t_2 = 3/\alpha_1$ the mathematical expectation $m_U(t) = 0.995\ m_s$ is independent of time.

Hence, in a time $t = 3/\alpha_1$ the variance D_U and the mathematical expectation m_U of the output process can be considered independent of time t and the output process is stationary in a wide sense.

In a steady mode

$$m_U = m_\xi\ ;\ D_U = C_1 \alpha_1 / 2\ , \tag{4}$$

and the correlation function

$$R_U(\tau) = D_U \exp\left(-\alpha_1 |\tau|\right). \tag{5}$$

A Gaussian process with correlation function (5) has a Markovian property.

Consider now the effect of the Markovian–Gaussian process with the correlation function $R_1(\tau) = D_U \exp(-|\tau|/\tau_0)$ on the integrating RC–chain when the time constant T of the integrating chain has three different values: $T \ll \tau_0$, $T = \tau_0$, and $T \gg \tau_0$.

In this case let us generalize the above result to a signal passing through an amplifier or an integrator. Obtaining first the energy spectrum of the output process and then using the Wiener–Khinchin transform we have a correlation function of the output process

$$R_2(\tau) = D_U \frac{\tau_0}{\tau_0^2 - T^2}\left[\tau_0 \exp\left\{-\frac{|\tau|}{\tau_0}\right\} - T \exp\left\{-\frac{|\tau|}{T}\right\}\right]. \tag{6}$$

The variance of the output process D_{out} is found from Eq. (6) at $\tau = 0$:

$$D_{\text{out}} = D_U \tau_0 / (\tau_0 + T)\ . \tag{7}$$

Depending on the proportion between τ_0 and T the correlation function $R_2(\tau)$ has the form

1) $\tau_0 \gg T$

$$R_2(\tau) = D_U \exp\left\{-\frac{|\tau|}{\tau_0}\right\},\quad D_{\text{out}} = D_U\ ;$$

2) $\tau_0 = T$

$$R_2(\tau) = (D_U / 2) \exp\left\{-\frac{|\tau|}{\tau_0}\right\},\quad D_{\text{out}} = D_U / 2\ ;$$

3) $\tau_0 \ll T$

$$R_2(\tau) = (D_U\,\tau_0 / T) \exp\left\{-\frac{|\tau|}{\tau_0}\right\},\ D_{\text{out}} = D_U \tau_0 / T\ .$$

Irrespective of the proportion between τ_0 and T the output process is Gaussian and Markovian.

ESTIMATION OF THE MATHEMATICAL EXPECTATION

Let the integrated mean value of the weighted process on the interval $[0,\ T]$ be an estimate of the mathematical expectation of the input process

$$\hat{m}_T = \int_0^T h_1(t)\, U(t)\, dt\ ,$$

where $U(t)$ is a random and stationary in a wide sense process; $h_1(t)$ is the determinate weighting function which is interpreted as a pulse characteristic of the filter.

The weighting function $h_1(t)$ must satisfy the normalization condition

$$\int_0^T h_1(t)\, dt = 1.$$

In a particular case for an ideal integrator $h_1(t) = 1/T$ and the normalization condition holds for any T.

As shown in Ref. 3, for a stationary random process with the correlation function of the type of Eq. (5) an optimal linear estimate of the mathematical expectation is

$$\hat{m}_T = \frac{1}{2 + \alpha_1 T}\left[U(0) + U(T) + \alpha_1 \int_0^T U(t)\, dt\right].$$

The variance of the optimal estimate is determined as

$$D\left\{\hat{m}_T\right\} = 2D_U / (2 + \alpha_1 T)\ .$$

The integrated over the interval $[0,\ T]$ mean value of the process

$$m_T = \frac{1}{T}\int_0^T U(t)\, dt\ , \tag{8}$$

can be taken as an estimate of the mathematical expectation. Here m_{T} is a random value since it varies from realization to realization of the same stationary process $U(t)$ of duration T. The variance of the estimate of the type of Eq. (8) is

$$D\left\{\hat{m}_T\right\} = \left[2D_U / (\alpha_1 T)\right]\left[\alpha_1 T - 1 + \exp\left(-\alpha_1 T\right)\right].$$

Under condition $\alpha_1 T \gg 1$ the variance of the integrated mean estimate is equal to the variance of the optimal estimate, i.e.,

$$D\left\{m_T\right\} = D\left\{\hat{m}_T\right\} = 2D_U / (\alpha_1 T)\ ,\ \alpha_1 T \gg 1\ . \tag{9}$$

Thus, the integrated mean estimate can be used for a Gaussian–Markovian process with correlation function (5) instead of optimal linear estimate of the mathematical expectation, provided that the observation time of the random process is much longer than the interval of correlation of the input process.

MINIMIZATION OF THE VARIANCE OF A LIDAR SIGNAL ESTIMATE

An estimate of a time–dependent mathematical expectation of a random process based on a single observation is hindered with the necessary of determinating optimal time of averaging (integrating). Optimal time of integration provides for reaching minimum of the mean squared deviation (dispersion) of the estimate of the mathematical expectation. The dispersion of the estimate $\hat{D}\{m(t_j, T)\}$ is composed of two components which are the fluctuation and dynamic errors

$$\hat{D}\{m(t_j, T)\} = B^2\{m(t_j, T)\} + D\{m(t_j, T)\}\ , \qquad (10)$$

where $B\{m(t_j, T)\}$ is the bias of the estimate at the time t_j (dynamic error); $\hat{D}\{m(t_j, T)\}$ is the variance of the mathematical expectation at the time t_j (fluctuation error) determined by expression (9).

By restricting the dynamic error to a term of its series expansion with the second derivative of the signal we can find the optimal time of integration

$$T_{\rm opt} = 2.7\left[P(t_j) \Big/ \left\{\alpha\left[\frac{d^2P_{\rm s}(t)}{d\,t^2}\right]_{t=t_j}\right\}\right]^{1/5}, \qquad (11)$$

where $P(t_j) = P_{\rm s}(t_j) + \bar{P}_{\rm b}$; $P_{\rm s}(t_j)$ is the optical power of a signal on the photocathode at the time t; P_b is the mean optical power of the background on the PMT's photocathode; $a = \eta/(h\nu)$ is the constant coefficient (J^{-1}); η is the quantum efficiency of the PMT's photocathode; h is the Planck constant; and ν is the optical frequency of laser radiation.

Using the method of finite differences we can reduce expression (11) to the form

$$T_j = \frac{P(t_j)}{4\alpha\left[\hat{P}(t_j) - P(t_j)\right]^2}\ , \qquad (12)$$

where the integrated mean estimate of the signal at time t_j is determined by the expression

$$\hat{P}_{\rm s}(t_j) = E_j / T_j - \bar{P}_{\rm b}\ , \qquad (13)$$

where E is the energy accumulated in the integrator during the observation time T_j.

Minimum dispersion of the estimate of a signal $m_2[T_{\rm opt}]$ an optimal interval $T_{\rm opt}$, can be found from expression (10) by transforming the first term

$$m_2[T_{\rm opt}] = \frac{D_U\,\tau_0}{2\,T_{\rm opt}} + \frac{2\,D_U\,\tau_0}{T_{\rm opt}} = 2.5\,\frac{D_U\,\tau_0}{T_{\rm opt}} \qquad (14)$$

or by representing it in terms of the optical power

$$m_2[T_{\rm opt}] = 1.25\,F\,\kappa_1^2\,P_{\rm s}(t_j)\,[1 + 1/q_1]\,/\,(\alpha\,T_{\rm opt})\ , \qquad (14a)$$

where q_1 is the input signal–to–noise ratio

$$q_1 = P_{\rm s}(t_j) / \bar{P}_{\rm b}\ . \qquad (15)$$

By dividing expression (14a) by $U_{\rm s}^2(t_j)$ and taking the square root of the obtained expression we find the total relative error δ_Σ in the estimate of the signal

$$\delta_\Sigma = \left\{1.25\,F\left[1 + 1/q_1\right] \big/ \left[\alpha\,T_{\rm opt}\,P_{\rm s}(t_j)\right]\right\}^{1/2}. \qquad (16)$$

The total relative error contains two components, i.e., the dynamic and the fluctuation ones

$$\delta_\Sigma = \left\{\delta_{\rm TD}^2 + \delta_{\rm TF}^2\right\}^{1/2},$$

where $\delta_{\rm TD}$ and $\delta_{\rm TF}$ are the dynamic and fluctuation components of the total relative error, respectively.

These components of the error can be found from expression (14) in which the first term determines the dynamic error and the second term determines the fluctuation one:

$$\delta_{\rm TD} = \left\{0.25\,F\left[1 + 1/q_1\right] \big/ \left[\alpha\,T_{\rm opt}\,P_{\rm s}(t_j)\right]\right\}^{1/2}. \qquad (17)$$

$$\delta_{\rm TF} = \left\{F\left[1 + 1/q_1\right] \big/ \left[\alpha\,T_{\rm opt}\,P_{\rm s}(t_j)\right]\right\}^{1/2}. \qquad (17a)$$

It is seen from expressions (17) and (17a) that for an optimal time of observation there is a unique proportion between $\delta_{\rm TD}$ and $\delta_{\rm TF}$, i.e.,

$$\delta_{\rm TF} / \delta_{\rm TD} = 2.$$

By introducing the relative error δ_0 of recovering the signal and noise mixture we can reduce expression (12) to the form

$$E_{\rm thr} = \frac{0.56\,F\,(\,|\delta_0| + 2.24)}{\alpha\,\delta_0^2}\ , \qquad (18)$$

where $E_{\rm thr}$ is the threshold energy equal to the energy of the signal and noise mixture in the interval $[t - T_j/2, t_j + T_j/2]$

$$E_{\rm thr} = T_j\,\hat{P}(t_j)\ , \qquad (19)$$

$\hat{P}(t_j)$ is the estimate of the total optical power of a signal and an external background on the photocathode.

The dynamic component of the relative error $\delta_{\rm 0TD}$ of recovering the signal and background mixture is

$$\delta_{\rm 0TD} = \hat{P}(t_j) / P(t_j) - 1 = \delta_0 / 2.24\ . \qquad (20)$$

Depicted in Fig. 1 is the absolute value of the relative error $|\delta_0|$ of recovering the signal and external background mixture as a function of the accumulated energy $E_{\rm thr}$, normalized to the coefficient of the PMT's noise F at the radiation wavelength $\lambda = 0.532\ \mu$m.

If the energy of the received signal and noise mixture is constant in each observation, the error of the estimate is also constant. Thus, if a device of a postdetector processing keeps the accumulated energy constant, then the tuning of observation lime is done automatically in accordance with the input signal power variations in time.

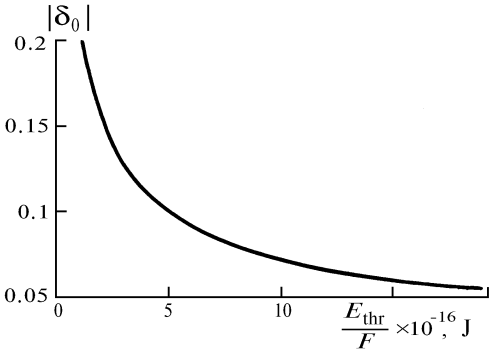

FIG. 1. The absolute value of the relative error δ_0 of recovering the sum of optical power of a signal and background $P(t)$ as a function of the relation between the threshold energy E_{thr} and the PMT's noise coefficient F.

Let us now find the dependence of E_{thr} on the relative error of recovering the signal δ_Σ for different values of the external background. Transform expression (16) by squaring both its sides and multiplying the numerator and the denominator of the resulting expression by $[\overline{P}_s(t_j) + \overline{P}_b]$. Taking into account expressions (19) and (20) and substituting $\delta_{TD} = \delta_\Sigma / 2.24$ we finally obtain

$$E_{thr} = \frac{1.25\, F\,(q_1 + 1)\left[q_1\,(1 + |\delta_\Sigma| / 2.24) + 1\right]^2}{\alpha\, q_1^2\, \delta_\Sigma^2} . \qquad (21)$$

Figure 2 presents the absolute value of the relative error $|\delta_\Sigma|$ of signal recovering as a function of the threshold energy E_{thr} at $\lambda = 0.532$ μm for four values of the input signal–to–noise ratio q_1 (curves *1*, 2, *3*, and *4* at q_1 = 1, 2, 5, and 10, respectively). (A functional block-diagram of a device which performs the described algorithm of processing is depicted in Fig. 3).

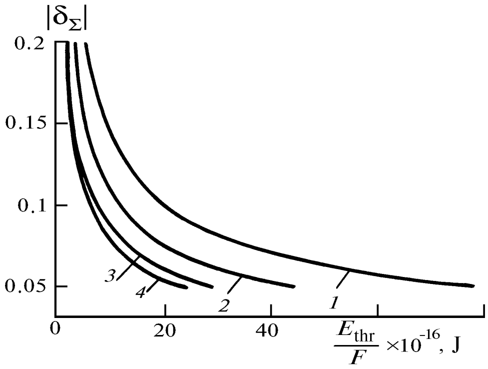

FIG. 2. The absolute value of the relative error δ_Σ of recovering the optical power of a signal (P_s) as a function of the relation between the threshold energy E_{thr} and the PMT's noise coefficient F: 1) q_1 = 1, 2) q_1 = 3, 3) q_1 = 5, and 4) q_1 = 10.

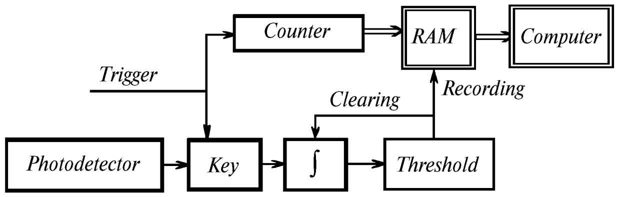

FIG. 3. A functional block–diagram of the processing of a signal.

To reduce the error of recovering the signal caused by a count increment of the counter it is possible to use a variable threshold. In this case the inequality is satisfied

$$E_{thr} / \left[P_s(t_j) + 1/q_1\right] \gg \Delta T ,$$

where ΔT is the count increment of the counter.

Thus the variable threshold provides for a minimum error in recovering the signal in its wide input dynamic range.

THE EFFICIENCY PARAMETER

To estimate the efficiency of the method of adaptive processing of a signal we introduce the efficiency parameter (V_s) which interrelates three basic parameters, i.e., range resolution (r_j), signal power $P_s(t_j)$, and the relative variance of the estimate of a signal (δ_Σ^2)

$$V_s = r_j\, \delta_\Sigma^2\, P_s(t_j) . \qquad (22)$$

As is seen from Eq. (22), the smaller the efficiency parameter V_s, the more efficient is the method of processing. Moreover, it is impossible to improve one of the variables without worsening the remaining ones since the efficiency parameter is a constant value. Let us determine the efficiency parameter for the adaptive method of signal processing. Using the expression for optimal interval of observations we can find the spatial resolution

$$r_j = \frac{C\, F\,(q_1 + 1)}{1.6\, \alpha\, \delta_\Sigma^2\, q_1\, P_s(t_j)} ,$$

and then the efficiency parameter

$$V_s = r_j\, \delta_\Sigma^2\, P_s(t_j) = \frac{C\, F\,(q_1 + 1)}{1.6\, \alpha\, q_1} . \qquad (23)$$

By substituting the values $C = 3 \times 10^8$ m and $\alpha = 2.67 \cdot 10^{17}$ J^{-1} at $\lambda = 0.53$ μm into Eq. (23) we obtain

$$V_s = \frac{0.7\, F\,(q_1 + 1)}{q_1} , \qquad (23a)$$

where the dimensionality of V_s is [m · NW].

The relative variance for the Kramer–Rao boundary (KRB) is

$$\delta^2_{\mathrm{KRB}} = \frac{D_{\mathrm{KRB}}}{P^2_{\mathrm{s}}(t_j)} = \frac{(q_1 + 1)}{\alpha\, P_{\mathrm{s}}(t_j)\, r_j\, q_1} = \frac{C\,(q_1 + 1)}{2\alpha\, P_{\mathrm{s}}(t_j)\, r_j\, q_1}$$

In accordance with Eq. (22) we find the uncertainty function for KRB

$$V_{\mathrm{KRB}} = r_j\, \delta^2_{\mathrm{KRB}}\, P_{\mathrm{s}}(t_j) = \frac{C\,(q_1 + 1)}{2\,\alpha\, q_1}\,. \qquad (24)$$

The PMT noise coefficient (F) does not enter into expression (24) since it has been obtained for Poisson statistics of photoelectrons and for an ideal PMT with $F = 1$. Therefore to compare different methods of processing with the potential one the PMT noise coefficient should be taken 1.

For different methods to be compared we introduce the efficiency coefficient K_{eff}.

$$K_{\mathrm{eff}} = \frac{V_{\mathrm{KRB}}}{V_{\mathrm{s}}}\,. \qquad (25)$$

It characterizes the difference between the method under study and the potential one $K_{\mathrm{eff}} \le 1$. For an adaptive processing of a signal K_{eff} is found from Eqs. (23a) and (24). Its value is 0.8.

SIMULATION OF THE POSTDETECTOR PROCESSING OF A SIGNAL

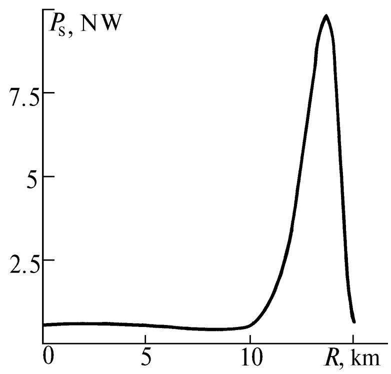

FIG. 4. A reference model of a signal; P_{s} – the optimal power of the signal, NW *and R – the distance between a* 15–km *atmospheric layer and the Earth's surface,* km.

To make numerical simulations we used a typical model of a lidar return signal assuming the lidar to be onboard a satellite and intended for acquiring profiles of atmospheric aerosol. A 15–km atmospheric layer was investigated, the meteorological visual range of the atmosphere S_M was taken to be 2 km.

Figure 4 depicts a signal used in simulation and Fig. 5 presents a relative error in recovering this signal with minimum input signal–to–noise ratio $q_1 = 1$.

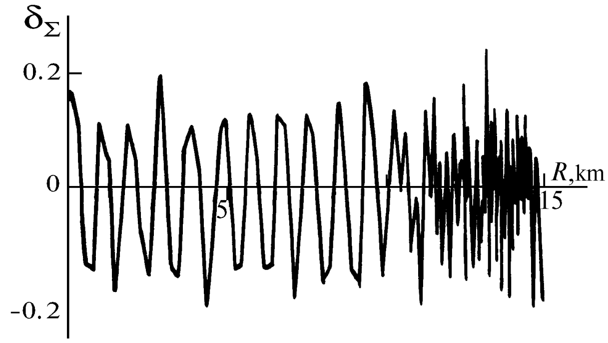

FIG. 5. A relative error δ_Σ of recovering the initial signal; $q_{1\min} = 1$, R is the distance between a 15–km *atmospheric layer and the Earth's surface,* km and $E_{\mathrm{thr}} = 15.6 \cdot 10^{-16}$ J.

CONCLUSIONS

1. An adaptive postdetector processing of a signal enables recovering of weak signals at the expense of increasing time of observation and worsening the spatial resolution, while a strong signal is recovered from noise at a shorter time and with higher spatial resolution. In both cases the estimate of a signal is the best one in the sense of the criterion of minimum standard deviation of the signal.

2. The described method of processing a signal makes it possible to detect a signal in a wide dynamic range what is its important advantage against a device for processing a signal with an analog–to–digital converter.

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