ON THE FLUX METHODS IN THEORY OF RADIATIVE TRANSFER

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Received July 26, 1991

Some aspects of the use of flux methods for solving the problems of radiative transfer in scattering media are discussed. The accuracy of the multiple reflection method and its practical implementations are considered.

From Editorial Board

The discussion about the essence of the multiple reflection method was initiated to a certain extent by the Editorial Board of this Journal, and it is self–evident that the reader himself can judge the arguments of the parties.

It is likely that it makes sense to recall the essence of the problem. In the opinion of A. Borovoi, manipulations with the one–dimensional radiative transfer equation can be associated only with the two–flux approximation. The authors of the method of multiple reflections insist on larger contribution treating their proposal as an heuristic method appealing to physical reasoning.

The positive results of this discussion are undoubtedly confirmed by new estimates of the accuracy of this method (true, item 3 of Sec. 3 of this paper looks somewhat strange: if some doubts are cast upon the reliability of the data of numerical calculations performed by the opponent, then why not to recalculate these data?). It has been found that some nontrivial contingencies may arise when this method is implemented for the solution of specific problems.

1. INTRODUCTION

At present numerical methods of calculation of radiation fluxes in bounded scattering volumes are fundamental, but they need rather long machine time.[1] For solving such problems in real time, analytic methods are developed, for instance, the FA method[2] and modernized Eddington δ–method.[3] The basic problem in the development of such methods is the correct formulation of fundamental equations and the corresponding formulation of boundary conditions. The solution of these equations with necessary approximations is a subject of mathematical physics and does not need to address often to physical processes of radiative transfer.

In the development of the multiple reflection (MR) method the most important problem is the construction of such a model of radiative transfer in spatially bounded scattering media (SBSM) which allows us to solve the problem of radiative transfer using simple and exact solutions for the one–dimensional medium. In this case the one–dimensional medium represents a part of the model of radiative transfer in the SBSM rather than a description of some real physical object. On further construction of the model a combination of three one–dimensional media oriented in perpendicular directions provides the solution of the problem of radiative transfer in the SBSM.

The basis of such a combination is the six–flux representation of scattering phase function,[4] which can be used to write down the transfer equation in the form[5]

$$\mu_i \frac{dI_i}{d\tau} = I_i - \Lambda \sum_{j=1}^{6} \delta_{ij} I_i \, . \quad (1)$$

Thus, the problem is reduced to the solution of Eq. (1) rather than the two–flux system of equations as Borovoi[6] argues. However, the solution procedure proposed in Ref. 5 has insufficient accuracy for the calculations of radiation fluxes in scattering media with absorption, whereas the asymptotes of the MR method allow us to reach an error of no more than 1.5–2% (see Ref. 7).

It should also be added that the six–flux representation of scattering phase function permits one to obtain a one–to–one correspondence between the parameters of a medium and coefficients of Eq. (1). The derived solution can be conveniently used for real–time calculations of integral radiation fluxes coming from a bounded volume of a scattering medium. Such calculations were performed in modeling the radiation balance of atmosphere. Great interest in this problem (calculation of integral fluxes) in the special literature[8,9] is verification of this thesis. The MR method is also applicable in the case of broken clouds.[10]

2. THE TWO–FLUX APPROXIMATION

When considering the flux methods, the analogy to the two–flux method is drawn because they use the system of equations

$$\begin{aligned} \frac{dI_1}{d\mathbf{I}} &= -\alpha' I_1 + \beta' I_2 \, , \\ -\frac{dI_2}{d\mathbf{I}} &= -\alpha' I_2 + \beta' I_1 \, , \end{aligned} \quad (2)$$

which is the particular case of Eq. (1), where $\mathbf{I}$ is the chosen direction and α' and β' are the coefficients determined by the parameters of the medium.[11] In this case, as was pointed out in Ref. 6, the key problem of the two–flux approximation is the determination of the coefficients α' and β' rather than the solution of the system of equations (2). Sufficiently large number of papers is devoted to this problem (see Refs. 11 and 12), in which the relation of the coefficients α' and β' to the parameters of the medium is found by a semiempirical method while the MR method permits one to establish a one–to–one correspondence between the parameters of the medium and radiation. Moreover, the MR method permits one to take into account the transverse dimensions of the medium what is most important in the experimental study. Hence, despite this method is similar in appearance to the two–flux method, the model of radiative transfer in the MR method is constructed in such a manner that it would be better to say about the six–flux approximation, i.e., the balance of

0235-6880/93/05 367-03 $02.00

radiation fluxes is considered in three pendicular directions. But the MR method method also differs from the six–flux method because it permits one to calculate the radiative transfer in a bounded volume.

3. THE ACCURACY OF THE MR METHOD

When analyzing the accuracy of the MR method by way of comparison[6] of the MR and the Monte Carlo methods (MC), consideration must be given to the following points:

1. The error cannot be calculated formally for signals of different levels. For instance, if the reflection is 99.99% and the transmission is 0.01%, than the error in determining the transmission can be very large because the signal variations caused by statistical error of the standard method (in this case it is the MC method) exceeds the signal level.
2. The reliability of the data presented in Table I of Ref. 6 must be tested. For instance, when the longitudinal optical thickness is 100 and 200, the transmission determined by the standard method is about 1% (see Ref. 2), what is physically impossible. Nevertheless, the conclusion has been drawn that the error of the MR method is 100%. Therefore, the conclusions about the accuracy of the method cannot be drawn based on such comparisons.
3. We think that the larger is the optical thickness, the smaller must be the discrepancy. But it can be seen from Table II of Ref. 6 that the error varies irregularly. Such an irregularity can be explained only by random factors (such, for instance, as reading of graphic information from the curves drawn on a logarithmic scale).
4. The problem of the adequacy for the construction of the model of photons leaving the medium is more difficult in the MC and MR method (in Ref. 2 this problem was left unexplained). The errors can be also introduced as a result of different geometry.
5. It should be noted that the MR method yields good results at large optical thicknesses what is confirmed by comparison of the asymptotic results obtained by the MR method with the data of the exact solution (in asymptote) reported in Ref. 7.

Based on these facts we can disprove the opinion of Borovoi[6] about the inconsistency of the MR method.

4. PRACTICAL IMPLEMENTATION OF THE MR METHOD

This method is most promising for real–time calculation of radiation balance and interpretation of experimental data, especially of model measurements, when spatial boundedness of the medium must be considered. In addition, the MR method yields the lateral intensity distribution $dI/d\tau_x$ (see Ref. 13). The analogy in appearance with the two–flux approximation leads to the possibility of asserting that the MR method is inapplicable for the description of the object in the form of a pile of plates, bars, etc., because in this case other physical phenomena[6] must be also taken into account. But the MR method is inapplicable for the description of such objects, and multiple reflections are used only for vivid derivation of the transfer equation. In addition, this method allows one to solve efficiently other problems, for example, to calculate the radiation fluxes in the scattering medium bounded by reflecting surfaces.[14]

When elaborating the MR method, the use of the model medium in the form of a parallelepiped is determined by the standard choice of boundary conditions and mathematical apparatus for the solution of the problem; moreover, going to the medium of arbitrary geometry presents no problems. The use of the integral radiation characteristics in flux methods[11] is although not a serious problem in practical implementation of this method. Moreover, the MR method provides a possibility to determine seven components of radiation balance in the volume of scattering medium (in the particular case of conservative symmetrical medium there are only three such components). For comprehensive study of the angular distribution of scattered radiation brightness in scattering volume one can use n–flux methods,[15] however, two– or six–flux approximations suffice for the most part of practical problems.

Experimental determination of integral characteristics of radiation scattered by volume of a medium is an ordinary problem and the subject matter of many works in photometry.[16]

The measurement of the radiation flux Φ is based on the trivial relation $\Phi = \int\limits_{4\pi} I d\omega$. Practically this problem is solved with the help of a photometric sphere (for small volumes of the medium) or by scanning the scattering object. In so doing a receiver can scan the volume boundary, or a scanning aperture can vary.

A comparison of theoretical and experimental results allows us to make a conclusion that the flux methods describe fairly well the real processes of radiative transfer.[11]

As a concrete example of practical implementation of the MR method, we give the method of determining a quantum survival probability Λ in highly turbid media.

To determine Λ, the coefficient R_∞ must be measured. In doing so the dependence of R_∞ on the transverse optical dimensions must be taken into account. As our computations for $\Lambda = 0.9$ show, R_∞ is independent of the transverse optical dimensions already at τ_y, $\tau_z \geq 10$. The decrease of quantum survival probability results in smaller optical dimensions. Consequently, as Λ changes, the transverse optical dimensions must be carefully monitored for weakly absorbing media.

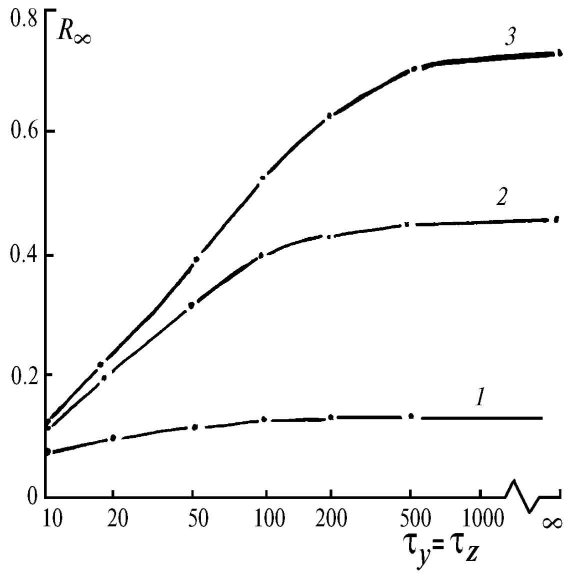

FIG. 1.

Elaborated method of determining Λ was experimentally checked for model suspensions of polystyrene latex with the particle size $d = 0.08$ μm. Collimated monochromatic radiation flux ($\lambda = 0.63$ μm) was used to obtain Rayleigh scattering described by integral parameters of the scattering phase function $\eta \approx \beta = 0.2$ and $\mu = 0.15$ (see Ref. 7). Measurement was carried out by scanning a face of a cell with a light quide connected to a photomultiplier FÉU–79. The accuracy of measuring Λ was determined by the error in measuring the light flux reflected by a layer of investigated medium in the case of isotropic and Rayleigh scattering while for unisotropic scattering – by the accuracy of determining the parameters η, β, and μ.

The other practical implementation of the MR method is the case of diffuse reflection of radiation from spatially bounded disperse media. The method allows one to calculate the reflection coefficients for different samples. These coefficients strongly depend on the transverse optical dimensions of a medium.

In conclusion it should be noted that the MR method is the basis for critical analysis of many experimental and theoretical works in the field of radiative transfer in scattering media available to us. This method initiates the development of new calculational methods taking into account the spatial boundedness of the medium and devices harnessing this method.

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