PARAMETRIZATION OF TURBULENT HEAT INFLUXES IN ESTIMATIONS OF HEIGHT OF NEAR–SURFACE TEMPERATURE INVERSION BOUNDARY

M.A. Konovalova and S.L. Odintsov

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk
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A differential equation is derived for the height of the near–surface temperature inversion boundary with the use of parametrization of the turbulent heat influx in the form of the k–model. Analytical solutions of this equation are obtained for some particular cases.

A reliability of forecasting these or others meteorological elements is to a great extent determined by the accuracy of prescribed input parameters of the prognostic models. A routine control of these parameters allows one to estimate the prognostic reliability of the models and to introduce necessary corrections. Thus, one of the most important parameters affecting the formation of the near–surface temperature inversion is the turbulent heat influx Q_T. Its values near the underlying surface enter into the prognostic models for the height h_θ of the inversion θ (see, for example, Refs. 1–3). Together with conventional and not always sufficiently accurate measurements of Q_T using standard meteorological sensors it is quite acceptable to employ optical or acoustical methods and facilities of diagnostics of the atmosphere. In this case the variants are possible when it is more convenient to estimate not the pulse components ω' and θ' (pulses of the vertical wind velocity and the potential temperature) forming $Q_T = -\partial(\overline{\omega'\theta'})/\partial z$, where z is the vertical coordinate (the bar above the product denotes averaging over the realization ensemble), and the coefficient of turbulent thermal diffusivity $k(z, t)$ (m^2/s) appeared in the parametrization is

$$Q_T = -\frac{\partial}{\partial z}(\overline{\omega'\theta'}) = \frac{\partial}{\partial z}\left(k\frac{\partial\theta}{\partial z}\right), \tag{1}$$

where θ is the average temperature value. In this connection, it is necessary to make relevant changes in the prognostic relations by replacing $\overline{\omega'\theta'}$ by the coefficient k. It is this problem that is examined in this paper.

Let us assume that the temperature field evolution with formation of the near–surface inversion is governed only by the processes of vertical turbulent heat exchange and radiative cooling. With parametrization (1) the heat equation for the atmospheric boundary layer is written as

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(k\frac{\partial\theta}{\partial z}\right) + \frac{\partial\theta_h}{\partial t} + \left(c\frac{\partial\theta_0}{\partial t} - \frac{\partial\theta_h}{\partial t}\right)\left(1 - \frac{z}{h_\theta}\right)^n, \tag{2}$$

where the subscripts h and 0 denote that the temperature corresponds to the heights $z = h_\theta$ and $z = 0$, and the factor c 1 takes into account the turbulent heat exchange at the level of the underlying surface. Within the θ inversion, i.e., in the region of $\partial\theta/\partial z > 0$ the potential temperature profile is taken in the form

$$\theta(z, t) = \theta_h(t) + [\theta_0(t) - \theta_h(t)]\,[1 - z/h_\theta(t)]^\alpha, \tag{3}$$

and above the inversion it is taken in the form

$$\theta(z, t) = \theta_m(t) + (z_m - z)\,\gamma(t), \tag{4}$$

where θ_m is the temperature at a height $z_m > h_\theta$ and γ is the temperature gradient.

On the basis of Eq. (1) and the model of temperature profile (3)–(4) and using the approach from Ref. 1 we obtain the following differential equation for h_θ:

$$(\varphi_0 + \varphi_1 h_\theta)\,h_\theta\frac{\mathrm{d}h_\theta}{\mathrm{d}t} = F_3 h_\theta^3 + F_2 h_\theta^2 + F_1 h_\theta + F_0, \tag{5}$$

where $\varphi_0 = (n + 1)(\theta_0 - \theta_k)$, $\varphi_1 = (2n + 1 - \alpha)\,\gamma$,

$F_3 = (\alpha - n)\dfrac{\mathrm{d}\gamma}{\mathrm{d}t}$, $F_2 = -\dfrac{\partial}{\partial t}[(\alpha - n)\theta_k + (n + 1 - c\alpha - c)\theta_0]$,

$F_1 = (\alpha + 1)(n + 1)\,\alpha\,\gamma\,k_0$, $F_0 = (\alpha + 1)(n + 1)\,\alpha k_0$, and

$\theta_k = \theta_m + \gamma z_m$. In Eq. (5) $k_0(t) = k(z = 0, t)$ is the coefficient of turbulent thermal diffusivity at the level of the underlying surface. The following comment concerning k_0 should be done. In a number of studies it was assumed that $k(z) \to 0$ as $z \to 0$. However, if the turbulent temperature flux is not equal to zero at $z = 0$, i.e., if $\overline{\omega'\theta'} \neq 0$, then the parametrization $\overline{\omega'\theta'} = -k\dfrac{\partial\theta}{\partial z}$ implies that for the finite value of $\partial\theta/\partial z$ the coefficient k should be nonzero though its values may be some orders of magnitude lower than unity, as was noted, for example, in Ref. 4.

Analytical solution of Eq. (5) exists only in a few cases. In particular, if the neutral distribution of the potential temperature is conserved above the inversion, i.e., $\gamma = 0$, then the solution of Eq. (5) is

$$h_\theta(t) = \mathrm{e}^W\left[h_\theta^2(t_0) + 2\alpha(\alpha + 1)\int\limits_{t_0}^{t} k_0\mathrm{e}^{-2W}\,\mathrm{d}t'\right]^{1/2}, \tag{6}$$

$$W = \int\limits_{t_0}^{t}\frac{F_2}{\varphi_0}\,\mathrm{d}t',$$

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In the case of $\alpha = 2n + 1$, and $\gamma = 0$, $\theta_m = \text{const}$ we have the equation

$$h_\theta(t) = \Delta^{2c-1}\left\{\left[\frac{h_\theta(t_0)}{\Delta^{2c-1}(t_0)}\right]^2 + 2\alpha(\alpha+1)\int\limits_{t_0}^{t}\frac{k_0}{\Delta^{2(2c-1)}}\mathrm{d}t'\right\}^{1/2}, \qquad (7)$$

where $\Delta = \theta_m - \theta_0(t)$, t_0 is the initial time.

Two variants of the night evolution of k_0 (from 8 p.m. to 4 a.m.) are shown in Fig. 1 by solid lines.

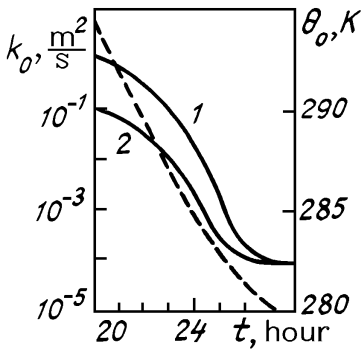

FIG. 1.

Figure 2 presents the time dependence of the inversion height h_θ for these two variants of the k_0 behavior. Figures in Fig. 2 denote the relevant variant of the k_0 behavior. Solid lines correspond to the case of $n = 1$ and dashed lines correspond to $n = 2$. In both cases we assumed $\theta_m = \text{const} = 298$ K and $\gamma = 0$. The θ_0 variation was taken to be a function shown by dashed line in Fig. 1. At the initial moment $\theta_0(t_0) = 20$ m and $\Delta(t_0) = 3$ K. The parameter α is assumed to be 3. In addition, in Fig. 2 the variation of h_θ is plotted by dashed–dotted line taking into account the cooling above the inversion with the rate $\partial\theta_m/\partial t = -0.5$ K/hour for the case $n = 1$, $\alpha = 3$.

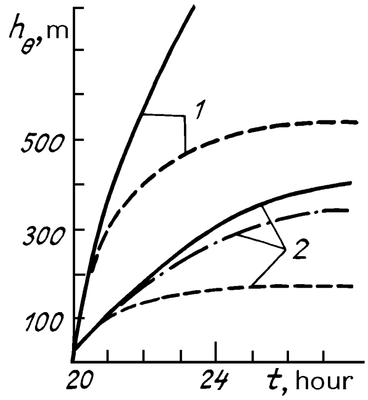

FIG. 2.

The above variations of $h_\theta(t)$ does not require special comments. Note only the essential dependence of h_θ on the radiative cooling profile characterized by the n parameter that was already mentioned in Ref. 3.

Thus, the present results and the data published earlier in Ref. 3 allow one to choose an appropriate variant for prediction of the height of the near–surface temperature inversion boundary within the framework of predetermined models.

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