ESTIMATION OF THE RADIATIVE HEAT INFLUX EFFECT ON THE HEIGHT OF THE GROUND TEMPERATURE INVERSION BOUNDARY

M.A. Konovalova and S.L. Odintsov

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences
Received December 13, 1991

The radiative heat influx effect on the evolution of the height of the ground temperature inversion boundary is estimated. We derived the differential equation that provides the estimate of the variation of this height depending on different profiles of radiative cooling. The results of numerical modeling are discussed.

In the studies of the important problems such as the analysis of propagation channels for electromagnetic and acoustic waves in the boundary atmospheric layer, remote sensing of optical and meteorological parameters, modeling of the processes of atmospheric pollutant transformations, and the like, the certain role is played by the prediction of the evolution of the upper boundary of the ground temperature inversion. The general approach to this problem has been formulated in a number of paper (see, for example, Refs. 1–6). However, in our opinion, some related questions must be refined. The main goal of the present paper is the estimation of the effect of radiative heat influx profile on the evolution of the ground temperature inversion boundary after the sunset.

For convenience we will use hereafter the notions T–inversion and θ–inversion implying the region of the boundary atmospheric layer with increasing absolute (T) and potential (θ) temperatures. Recall that the potential temperature θ is the temperature of an air parcel which it takes after rising or descending dry adiabatically from the initial height corresponding to the pressure p to the height corresponding to a pressure of 1000 mbar. It is related to the absolute temperature of the given air parcel by the formula $\theta = T\,(1000/p)^{2/7}$. The heights of the T– and θ–inversions are denoted by h_T and h_θ. Following Ref. 1, we take the heat–conduction equation in the form

$$\frac{\partial\theta}{\partial t} = -\frac{\partial(\overline{w'\theta'})}{\partial z} + \left(\frac{\partial\theta}{\partial t}\right)_p , \tag{1}$$

as a basis for our study, where z is the vertical coordinate, t is the time, w' and θ' are the random pulsations of the vertical component of the wind velocity and of the potential temperature. The bar above $w'\theta'$ denotes averaging of this product (the vertical turbulent temperature flux) over an ensemble of realizations. The second term in the right side of Eq. (1) is related to the radiative heat influx (sink). Advection, latent heat influxes, and so on are neglected. Both the underlying surface and the atmosphere in the horizontal plane are assumed to be homogeneous. Note in addition that the use of the potential temperature θ rather than the absolute temperature T is caused first of all by a more compact form of heat–conduction equation (1) which is a consequence of the first law of thermodynamics.

Let us assume, following, for example, Refs. 1–6, that the night profile of the potential temperature within the θ–inversion is given by the formula

$$\theta(z,\, t) = \theta_h(t) - [\theta_h(t) - \theta_0(t)]\,[1 - z/h_\theta(t)]^\alpha . \tag{2}$$

Here the subscripts h and 0 imply that the appropriate quantities are taken at the heights $z = h_\theta$ and $z = 0$. The power α varies, as a rule, between 2 and 4 according to the experimental data (see, for example, Ref. 1).

The second term in the right side of Eq. (10) is determined by the vertical gradient of the effective flux of the long–wave radiation. Its value depends primarily on the humidity and the atmospheric temperature. In general it includes integrals, i.e., Eq. (1) is in fact rather complicated integro–differential equation. The peculiarity of the vertical profile of $(\partial\theta/\partial t)_p$ consists in the fact that at the upper boundary of the ground θ–inversion the night radiative cooling is much less than at the underlying surface. Taking this peculiarity into account we can use a simplified representation of the radiative influx (sink) with the main features of its height dependence, for example

$$\left(\frac{\partial\theta}{\partial t}\right)_p = \frac{\partial\theta_h}{\partial t} + \left(c\,\frac{\partial\theta_0}{\partial t} - \frac{\partial\theta_h}{\partial t}\right)\left(1 - \frac{z}{h_\theta}\right)^n , \quad 0 \le z \le h_\theta . \tag{3}$$

Here $c \lesssim 1$ is the parameter which takes into account the turbulent heat flux at the ground. It is usually assumed (see Refs. 1 and 7) that $n = 1$ and $\theta(z > h_\theta) = \text{const}$. However, we will not restrict ourselves to these conditions.

Making use of Eqs. (1)–(3) and the condition $(\overline{w'\theta'})_h = 0$ we derive, according to the approach of Ref. 1 on the basis of a transition to the differential equation in h_θ, the following relation:

$$(\theta_0 - \theta_h)\,\frac{dh_\theta}{dt} + h_\theta\,\frac{\partial}{\partial t}\left[\frac{\alpha - n}{n + 1}\,\theta_h + \left(1 - c\,\frac{\alpha + 1}{n + 1}\,\theta_0\right)\right] =$$
$$= (\alpha + 1)(\overline{w'\theta'})_0 . \tag{4}$$

To estimate the effect of the unsteady temperature above the inversion on the evolution of its upper boundary, we specify the simplest model of linear decrease of θ with height described by the formula

$$\theta(z \ge h_\theta,\, t) = \theta(z_m,\, t) + \gamma(t)\,(z_m - z) . \tag{5}$$

Here z_m is the reference height (the θ values at this height are denoted by the subscript m), γ is the potential temperature gradient above the θ–inversion ($\gamma \ge 0$). With the use of Eq. (5) for evaluating θ at $z = h_\theta$ Eq. (4) can be written down as

0235-6880/92/07 485-03 $02.00

$$(\varphi_0 + \varphi_1 h_\theta)\frac{\mathrm{d}h_\theta}{\mathrm{d}t} = f_2\, h_\theta^2 + f_1 h_\theta + f_0 \,, \tag{6}$$

where $\varphi_0 = (n + 1)(\theta_0 - \theta_m - \gamma\, z_m)$, $\varphi_1 = (2n + 1 - \alpha)\gamma$,

$f_0 = (n + 1)(\alpha + 1)(\overline{w'\theta'}\,)_0$,

$f_1 = \frac{\partial}{\partial t}[(\alpha - n)(\theta_m + \gamma\, z_m) + (n + 1 - c\alpha - c)\theta_0]$,

$f_2 = (\alpha - n)\,\frac{\mathrm{d}\gamma}{\mathrm{d}t}$.

The subscripts adjacent to φ and f are not related to the height.

In general Eq. (6) has not yet been solved; however, in some particular cases the solution is possible. For example, for $2n + 1 = \alpha$ and γ = const Eq. (6) is reduced to

$$\frac{\mathrm{d}h_\theta}{\mathrm{d}t} = (h_\theta f_1 + f_0)/\varphi_\theta \tag{7}$$

and has the solution

$$h_\theta(t) = e^{-F}\left[h_\theta(t_0) + 2(n + 1)\int\limits_{t_0}^{t}\frac{(\,\overline{w'\theta'}\,)_0}{\theta_0 - \theta_m - \gamma\, z_m}\,e^{F}\,\mathrm{d}t\right] \tag{8}$$

where t_0 is the initial time and

$$F(t) = \int\limits_{t_0}^{t}\frac{1}{\theta_0 - \theta_m - \gamma\, z_m}\left[\frac{\partial\theta_m}{\partial t} + (1 - 2c)\,\frac{\partial\theta_0}{\partial t}\right]\mathrm{d}t \,. \tag{9}$$

In particular, for $n = 1$, $\alpha = 3$, $\gamma = 0$, and $\theta_m = \theta_h$ = const we obtain the results of Ref. 1. Some other combinations of the parameters are available; for them the analytical solution of Eq. (6) can be obtained.

The height h_θ of the θ–inversion boundary may be related to the height h_T of the T–inversion boundary with the use of Eq. (2) and an approximate equality (at the ground pressure being equal to 1000 mbar)

$$\theta(z,\, t) \approx T(z,\, t) + \gamma_a\, z \,, \tag{10}$$

where $\gamma_a = 0.01$ K/m is the adiabatic temperature gradient.

Equality (10) is a consequence of the fact that the temperature T of the adiabatically descending air parcel increases by about 1 degree per 100 meters. Finally we have

$$h_T = h_\theta\left\{1 - \left[\frac{\gamma_a\, h_\theta}{\alpha(\theta_\theta - \theta_0)}\right]^{1/(\alpha - 1)}\right\}. \tag{11}$$

Based on the above–derived equations, we estimated numerically the effect of the radiation heat influx (sink) profile and of the possible unsteady temperature above the inversion on h_θ and h_T. The following set of the parameters were chosen as a reference one: $n = 1$, $\alpha = 3$, $c = 0.95$, $\theta_m = 0$, $\partial_m/\partial t = 0$, $z_m = 1$ km, $h_\theta(t_0) = 20$ m, $\Delta_H = \theta_h - \theta_0(t_0) = 3$ K, $t_0 = 20$ h, LT, i.e., a bit later after the sunset. The temperature θ_0 was described by curve *1* shown in Fig. 1*a*, the heat flux at the ground $Q_0 = c_p\, \rho(\,\overline{w'\theta'}\,)_0$ was described by the function plotted in Fig. 1*a*. Under these conditions the profiles of the absolute (dashed curves) and potential (solid curves) temperatures are shown in Fig. 1*c*. The figures *1*, *2*, and *3* adjacent to the curves indicate 20, 24, and 4 h, LT, respectively. This model of the temperature profile satisfies the conditions used in Ref. 1. The evolution of the heights h_T and h_θ of the boundary under these conditions is shown in Fig. 1*d*, by the curves *1* (solid curves are for h_θ and dashed curves are for h_T).

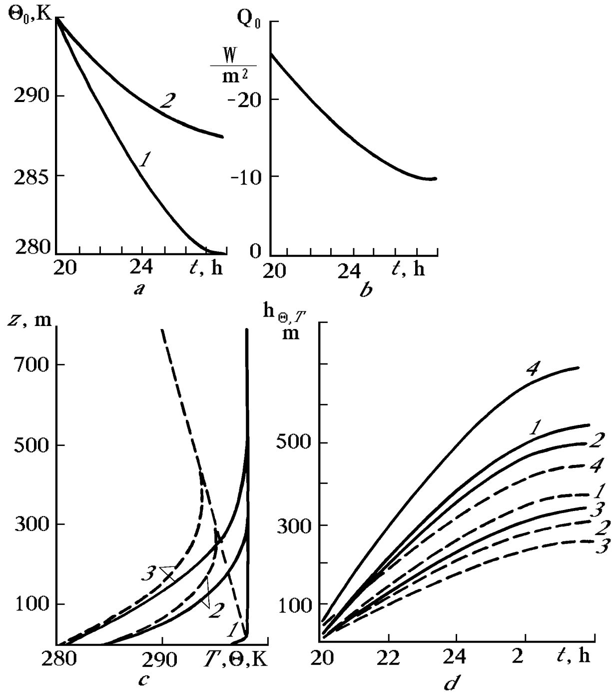

FIG. 1.

In this very figure some other dependences of h_θ and h_T are drawn as an example of sensitivity of the ground inversion height to variation in one or another parameter. Thus, if under condition of conservation of the basic set of the parameters the ground temperature varies according to curve *2* in Fig. 1*a* then the inversion height (h_θ or h_T) will vary with time according to curves *2* in Fig. 1*d*. If in the basic set of the parameters the initial temperature difference between the ground and the boundary of the θ–inversion is $\Delta_H = 5$ K, the quantities h_θ and h_T will vary according to curves *3*. In addition, the evolution of the inversion is strongly affected by its height at the initial time. As an example, curves *4* are shown in Fig. 1*d* obtained at $h_\theta(t_0) = 50$ m.

Considering further the dependence of h_θ on the radiative heat sink profiles we use the above–indicated set of the parameters noting only the changes in it. Let us assume that the potential temperature is constant above the inversion. Then, for $\alpha = 3$ and $n = 1$, 2, and 3 the height h_θ increases throughout the night according to the curves shown in Fig. 2*a* (the figures adjacent to the curves indicate the values of n). The increase of n corresponding to the decrease of radiative cooling within the inversion slows down the rate of increase of its height. It is especially noticeable when going out from $n = 1$ to $n = 2$. However, it should be kept in mind that the effect of nonlinearity of the radiative cooling profile becomes less pronounced when the potential temperature profile tends to the linear one. It is confirmed by Fig. 2*b* in which solid curves are for $\alpha = 2$ and dashed curves for $\alpha = 4$. The figures adjacent to the curves indicate the values of the parameter *n*. We do not give the height h_T because it can be easily calculated from Eq. (11). It is obvious that the nonlinearity of the radiative influx (sink) profiles has an appreciable effect on the inversion height and when predicting it with the use of radiative cooling model (3) special attention should be drawn to the correct choice of the power *n*.

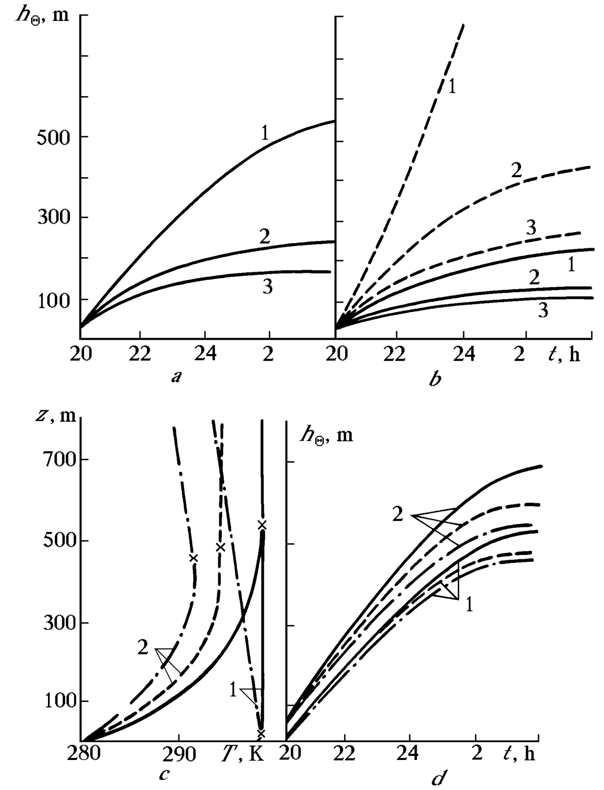

FIG. 2.

We have treated the case of the constant potential temperature above the θ–inversion without the radiative and turbulent heat exchange above h_θ. However, in reality the processes of night cooling of the atmosphere can also take place above the inversion. To estimate their effect on the evolution of the θ–inversion height, let us first consider the case in which above the θ–inversion the neutral behavior of the potential temperature is preserved, i.e., $\partial\theta/\partial z = 0$ for $z > h_\theta$, but the uniform cooling takes place, i.e., $\partial\theta_m/\partial z = \text{const} < 0$. In Fig. 2*c* the profiles of $\theta(z)$ are shown for $t_0 = 20$ h, LT (curve *1*) and $t_0 = 4$ h, LT of the next day (curve *2*). Solid curves correspond to the case of the constant temperature above h_θ, dashed curves to the uniform cooling with the rate $\partial\theta_m/\partial t = -0.5$ K/h (the curves coincide at the time t_0). The heights with $\partial\theta/\partial z = 0$, i.e., the boundaries of the ground θ–inversion at the given time are indicated by crosses. We can compare different h_θ under these conditions using the family of curves *1* shown in Fig. 2*d* (notation of curves are the same as in Fig. 2*c*). The family of curves *2* corresponds to the case of $h_\theta(t_0) = 50$ m for other parameters being the same as in the basic set. As can be seen from Fig. 2*d*, the cooling processes above the inversion can lead to the noticeable variations in its height.

Super–adiabatic temperature distribution above the θ–inversion also affects the variation of h_θ, though in a smaller extent. As an example, the model profiles of $\theta(z)$ are shown in Fig. 2*c* by dot–dash curve at the times $t_0 = 20$ h, LT (curve *1*) and $t_0 = 4$ h, LT (curve *2*) with the following changes in the basic set of the parameters: $\partial\theta_m/\partial t = -0.5$ K/h and $\partial\gamma/\partial t = 0$, i.e., under condition of uniform cooling above h_θ and constant gradient of the potential temperature $\gamma = 0.61$ K/100 m. The evolution of the height h_θ in this case is shown by dot–dash line in the family of curves *1* in Fig. 2*d*. The family of curves *2* at $h_\theta(t_0) = 50$ m also comprises dot–dash curve corresponding to the examined case.

Finally it should be noted that the height distribution of the radiative heat influxes (sinks) may have an appreciable effect on the behavior of the ground radiative inversion. The variations in h_θ in this case are comparable and sometimes exceed the variations attendant to the changes in the other parameters and functions. In this paper we had no goal to perform the exhaustive modeling of the evolution of the ground inversion height and only wanted to underline the necessity of taking into account the possible nonlinearity of the height distribution of radiative cooling of the atmosphere in modeling. More detailed analysis of complicated time dependences of such quantities as the underlying surface temperature, turbulent heat flux, temperature gradient above inversion, and the like, can be made, for example, when solving Eq. (6).

REFERENCES

1. T. Yamada, J. Appl. Meteorol. **18**, No. 4, 256–531 (1979).
2. J.C. Andre and L. Mahrt, J. Atmos. Sci. **39**, No. 4, 864–878 (1982).
3. J.M. Godowitch, J.K. S. Ching, and J.F. Clarke, J. Climate Appl. Meteorol. **24**, No. 8, 791–804 (1985).
4. S.S. Zilitinkevitch and A.S. Monin, Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana **10**, No. 6, 587–599 (1974).
5. M.E. Berlyand, *Prediction and Control of Atmospheric Pollution* (Gidrometeoizdat, Leningrad, 1985).
6. S.P. Beschastnov, Meteorolog. Gidrolog., No. 8, 33–39 (1987).
7. F.T.M. Nieuwstadt, J. Appl. Meteorol. **19**, No. 12, 1445–1447 (1980).