PHASE CORRECTION OF NONLINEAR DISTORTIONS OF A LASER BEAM ON A VERTICAL ATMOSPHERIC PATH

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Received September 30, 1990

Stationary thermal self-action of a coherent beam oh a vertical path in the atmosphere is investigated by the method of numerical modeling. The choice of beam intensity profile, the effect of the altitude profile of the wind direction, and the effectiveness of segmented and flexible correctors are investigated.

The effectiveness of phase correction of high-power radiation propagating on a vertical path in the atmosphere, for which phase distortions are characteristically concentrated near the radiating aperture, have been investigated previously[1–8] both theoretically[1,4–8] and by means of physical modeling.[2,3] The possibilities of modal[1,3,6,8] and segmented correctors[2,6] have been studied. This paper is a continuation of preceding papers,[5–7] but unlike them in this paper factors such as the intensity profile of the initial beam, the dependence of the wind direction on height above the earth's surface, the number of degrees of freedom of modal and zonal correctors, as well as some variants of the simultaneous effect of these factors are investigated more systematically.

Consider a transmitting aperture with area S, emitting a coherent optical beam with wavelength λ in the vertical direction. It is well known that because the medium absorbs the radiation the gas in the beam channel is heated. The heating in the steady-state regime is described by the equation

$$\cos\beta\,\frac{\partial T}{\partial x} + \sin\beta\,\frac{\partial T}{\partial y} = \frac{\alpha}{\rho V_\perp C_p}\,I(x,\ y); \qquad (1)$$

Here .x and y are coordinates in the transverse section of the beam; T is the temperature of the medium; α is the absorption coefficient of the atmosphere at the wavelength λ; ρ is the density and C_p is the heat capacity of air; $V_\perp = V_x^2 + V_y^2$ is the component of the wind velocity that is perpendicular to the direction of propagation of the radiation; and, $\cos\beta = V_x/V_\perp$ and $\sin\beta = V_y/V_\perp$. In this case "wind" includes both the atmospheric wind itself and a correction to it, associated with the slewing while tracking a moving target. All these parameters and the intensity distribution in the cross section of the beam are, in this case, functions of the longitudinal coordinate h (altitude). The heating of the medium changes the index of refraction of the medium n by the amount $\delta n = n_T T$. A section of the path of length δh introduces into the phase of the complex amplitude of the optical field $E(x,\ y)$ a distortion equal to $\delta\varphi = n_T T(x,\ y)\delta h$.

The propagation of radiation is described by the parabolic equation of quasioptics

$$2ik\,\frac{\partial E}{\partial h} = \Delta_\perp E + 2k\delta n E, \qquad (2)$$

where $\Delta_\perp$ is the transverse Laplacian.

In solving Eqs. (1)–(2) numerically we assumed that because the phase distortions decrease rapidly with altitude they are concentrated directly in the plane of the radiating aperture. Then the solution of the problem of propagation of radiation reduces to calculating the phase distortions of the field $E(x,\ y)$ and then solving the problem of free diffraction ($\delta n = 0$). The total distortion introduced into the phase by the atmosphere is determined in this approximation by the formula

$$\varphi(x,\ y) = \frac{2\pi}{\lambda}\int T(x,\ y,\ h)\ n'_T(h)\,dh. \qquad (3)$$

Let $D(\beta) = \cos\beta\partial/\partial x + \sin\beta\partial/\partial y$ and D^{-1} be the operator that is the inverse of D. We shall assume that $\beta(h) = $ const. Then we obtain

$$\varphi(x,\ y) = \frac{2\pi}{\lambda}\,D^{-1}(\beta)I(x,\ y)\int \frac{\alpha n'_T}{\rho V_\perp C_p}\,dh. \qquad (4)$$

In solving the problem of self-action it is convenient to introduce the normalized coordinates $x' = x/a_0$ and $y' = y/a_0$. Then $D'^{-1} = a_0 D^{-1}$. Here a_0 is the size of the beam. The next step is to introduce the characteristic intensity

$$I_0 = \left[\frac{2\pi}{\lambda a_0}\int_0^H \frac{\alpha n'_T}{\rho V_\perp C_p}\,dh\right]^{-1}, \qquad (5)$$

after which the characteristic power is determined automatically: $P_0 = I_0 \cdot a_0^2$. Now $\varphi(x,\ y) = D'^{-1}(\beta) \times$ $\times\ (I(x',\ y')/I_0)$.

Equation (1) was solved by the marching calculation method, and Eq. (2) was solved by the

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Fourier transform method. The length of the path H was assumed to be equal to 1/3 of the diffraction length $L_d = ka_0^2$, $k = 2\pi/\lambda$. Here and below, for convenience, we shall write all relations in unnormalized coordinates. In so doing, one should remember that the values of a_0 or H can be chosen arbitrarily, provided that the relation $H = 1/3\lambda_0^2\lambda$ is satisfied. The transmitting aperture was assumed to be either a circle of radius $r_0 = 2a_0$ or a square with side $l_0 = r_0\pi^{1/2}$. The intensity profile in the plane of the transmitting aperture was assumed to be one of the following variants of distributions:

$$I(x, y) = \exp\left(-\left(x^2 + y^2\right)/a^2\right) \text{ for the Gaussian,} \quad (6)$$

$$I(x, y) = \exp\left(-\left(x^m + y^m\right)/a^m\right) \text{ for the hyper-Gaussian profile,} \quad (7)$$

$$I(x, y) = \exp\left(-\left(x^2 + y^2\right)^{1/2m}/a^m\right) \text{ for the super Gaussian profile.} \quad (8)$$

The quantity a was, as a rule, set equal to a_0 for a Gaussian profile, R_0 for a super Gaussian profile, and l_0 for a hyper-Gaussian profile. The transmitting aperture was described by the aperture functions

$$M_s = \begin{cases} 1 \text{ when } x^2+y^2<r_0^2 \text{ for the Gaussian and} \\ 0 \text{ when } x^2+y^2>r^2 \text{ for the super-Gaussian profile,} \end{cases} \quad (9)$$

$$M_h = \begin{cases} 1 \text{ when } |x|<l_0 \text{ and } |y|<l_0 \text{ for the hyper-} \\ 0 \text{ when } |x|>l_0 \text{ or } |y|>l_0 \text{ Gaussian profile.} \end{cases} \quad (10)$$

For $m > 8$ the super-Gaussian and hyper-Gaussian intensity profiles fill the apertures M_s and M_h, respectively, practically uniformly, and in addition outside the transmitting aperture the intensity is virtually equal to zero. The isolines of the hyper-Gaussian profile are nearly square. We study below two variants of the hyper-Gaussian profile: the variant defined by the formula (7) and the variant turned by 45 degrees relative to the first variant ("with the diagonal" along the X axis) together with the transmitting aperture.

The numerical solution of the problem of self-action gives the distribution of the normalized intensity of the focused beam in the plane $h = H$. The normalized power P_m/P_0 of the radiation incident in a circle of radius $r = L/ka_0$, equal to the diffraction radius of a focused untruncated Gaussian beam at e^{-1} of maximum intensity on the axis in the focal plane $h = H$, is calculated from the intensity distribution. The computational results are given in the form of curves of the power on target versus the power on the transmitting aperture; these curves are called power optimization curves.

OPTIMIZATION OF THE INTENSITY PROFILE

Before studying the effectiveness of phase correction we shall study the question of the choice of the best intensity profile in the absence of correction. We shall optimize the starting profile in the class of Gaussian, super-Gaussian, and hyper-Gaussian beams. When comparing the results, correctness is obtained when the areas of the transmitting apertures M_s and M_h are equal. We shall confine our attention to the case of constant wind direction $\beta(h) = 0$ (the wind blows along the X axis).

Figure 1 shows power optimization curves for the following profiles of the beam intensity: 1) unbounded Gaussian $a = a_0$; 2) Gaussian $a = a_0$ (here and below the beam is limited by the corresponding aperture functions (9) and (10)); 3) "double width" Gaussian $a = 2a_0 = r_0$; 4) super-Gaussian $m = 8$, $a = r_0$; 5) hyper-Gaussian $m = 8$, $a = l_0$; and, 6) hyper-Gaussian, turned by 45 degrees. One can see from the figure that from the standpoint of obtaining the highest power on target more complete coverage of the transmitting aperture is more advantageous (super–Gaussian and hyper-Gaussian profiles, as well as the "double width" Gaussian beam).

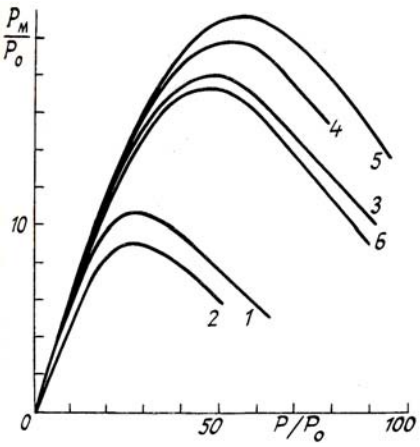

FIG. 1

CHANGE IN WIND DIRECTION WITH INCREASING ALTITUDE

It was assumed above that the wind direction does not depend on the height above the ground $\beta(h)$ = const. In the real atmosphere, however, the wind direction changes gradually from the direction near the ground to the direction in the free atmosphere. The direction of heat flow out of the beam channel could also be affected by the slewing of the beam while tracking a moving target, if the direction of slewing is not the same as the direction of the atmospheric wind.

In the general case, the atmospheric path is described by two profiles — the profile of wind direction $\beta(h)$ and the profile of the normalized nonlinearity parameter:

$$R(h) = \left[\frac{\alpha(h)n'_r(h)}{\rho(h)V_\perp(h)}\right]\left[\frac{\alpha(0)n'_T(0)}{\rho(0)V_\perp(0)}\right]^{-1}. \quad (11)$$

We introduce the concept of effective refraction length of the path, defined as

$$H_R = \int_0^H R(h)dh. \quad (12)$$

Then the total distortion of the phase on the path is given by

$$\varphi(x, y) = \frac{1}{H_R}\int_0^H R(h)D^{-1}(\beta(h)I(x, y)dh. \quad (13)$$

Because in calculating the self-action the phase distortions are assumed to be concentrated in the plane of the transmitting aperture, it is convenient to eliminate the dependence on the coordinate h by introducing a function $h(\beta)$ that is the inverse of $\beta(h)$ and $R(\beta) = R(h(\beta))$. Then the formula (13) will assume the form

$$\varphi(x, y) = \frac{1}{H_R}\int_0^{2\pi} R(\beta)h'_\beta(\beta)D^{-1}(\beta)I(x, y)d\beta. \quad (14)$$

Now, to calculate the phase distortions the function $f(\beta) = R(\beta)h'_\beta(\beta)/H_R$, which characterizes the "angular"' distribution of the distortions, is prescribed instead of the profiles $R(h)$ and $\beta(h)$. The specific form of $f(\beta)$ can be very different for different locations and meteorological conditions. It depends on the direction and velocity of slewing. Here we confined our attention to a uniform distribution

$$f(\beta) = \begin{cases} \text{const } \beta \in [-\beta_0, \beta_0], \\ 0 \quad \beta \notin [-\beta_0, \beta_0]. \end{cases} \quad (15)$$

The constant is determined from the condition $\int f(\beta)d\beta = 1$, which follows from the definition (11). Further, we shall characterize the distribution $f(\beta)$ by the "standard deviation"

$$\sigma_V = \left[\int \beta^2 f(\beta)d\beta\right]^{1/2}, \quad (16)$$

which is applicable to distributions other than the uniform distribution (15).

The computational results are presented in Fig. 2. The graphs are enumerated as follows: hyper-Gaussian beam ($a = r_0$, $m = 8$): 1) $\sigma_V = 0.5$, 2) $\sigma_V = 1.0$, and 3) $\sigma_V = 1.5$; Gaussian beam ($a = a_0$):4) $\sigma_V = 0.5$, 5) $\sigma_V = 1.0$ and 6) $\sigma_V = 1.5$. One can see that as the standard deviation σ_V increases the power on target increases substantially (by a factor of 2–3), while the advantage of having a filled aperture (hyper-Gaussian beam) is preserved.

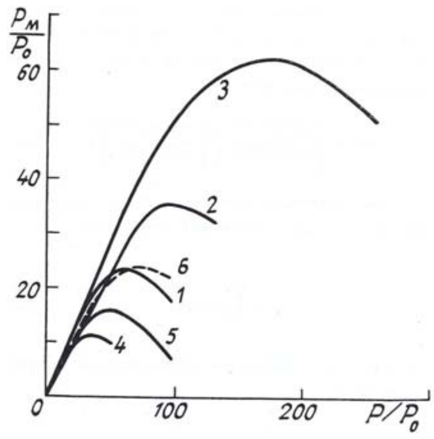

FIG. 2

EFFECTIVENESS OF PHASE CORRECTION

We shall study the effectiveness of two types of phase correctors: a segmented with a hexagonal configuration of the segments and a "flexible" corrector, which compensates the first few classical aberrations. The wind direction is assumed to be constant $\beta(h) = 0$.

Two variants of a segmented corrector were studied: 1) compensation of the average phase within each segment and 2) compensation of the average phase and tilt within each segment. The number of compensator transmitting aperture was set equal to 7.19 and 37.

Modeling of the flexible corrector consisted of calculating the best approximation of the phase distortions within the transmitting aperture with Zernike polynomials by the method of least squares, followed by subtraction of these aberrations from the phase of the radiation. The aberration polynomials are enumerated as follows: tilt (1, 2), defocusing (3), astigmatism (4, 5), coma (6, 7), and spherical aberration (8).

Figure 3 shows the power optimization curves for a Gaussian beam with an effective size $a = a_0$ on the aperture M_s with correction of phase distortions using a segmented corrector with the following parameters (the number in parentheses is the number of degrees of freedom of the corrector):

1–7 segments, correction of the average phase and tilt (21),

2–19, segments, correction of the average phase (19),

3–19 segments,correction of the average phase and tilt (57),

4–37 segments,correction of the average phase (37).

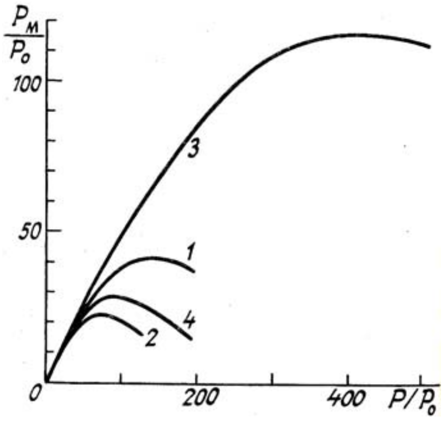

FIG. 3

The total number of degrees of freedom of the corrector, presented in parentheses, is equal to the product of the number of degrees of freedom of one segment by the number of segments. The use of a segmented corrector makes it possible to increase the maximum power on target by a factor of 3 or 4, while for a corrector with 19 elements and three degrees of freedom for each segment the maximum power on target increases by more than an order of magnitude. Thus if the effectiveness of the 37-element corrector were not unexpectedly low, one could say the effectiveness of the corrector depends directly on the total number of degrees of freedom.

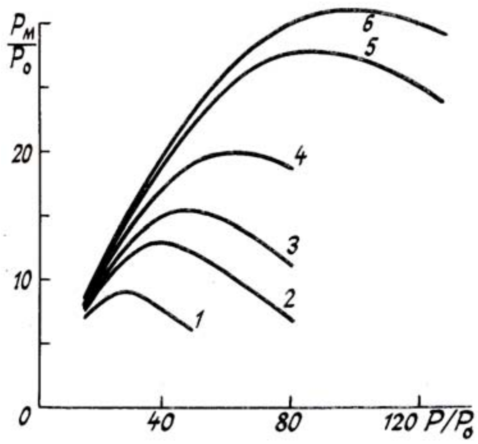

FIG. 4

Figure 4 shows power optimization curves of the same beam with the use of a modal corrector: 1) without correction; 2) tilt correction; 3) tilt and defocusing correction; 4) tilt, defocusing, and astigmatism corrections; 5) corrections of the first seven aberrations; and, 6) correction of eight aberrations.

The computational results are presented for the case $\beta(h)$ = const. One can see that the effectiveness of correction increases monotonically as the number of corrected aberrations increases. When all eight classical aberrations (from tilt to spherical aberration) are corrected, the maximum power on target is tripled.

Next, the effect, of the change in wind direction on the path as the height above the ground increases on the effectiveness of correction of the lowest order modes was investigated for beams with different intensity profiles. Table I gives the maximum values of the power in the receiving aperture, which were obtained by optimizing the power of the source. The first figure is the maximum power without correction, the second figure is the maximum power with tilt correction, and the third figure is the maximum power with defocusing correction. The index $m = 8$ for super-Gaussian and hyper-Gaussian profiles. The size a is equal to a_0, $2a_0$, and l_0 for the Gaussian, super-Gaussian, and hyper-Gaussian profiles, respectively.

TABLE I.

Type of beam	$\sigma_v = 0$	$\sigma_v = 0.75$	$\sigma_v = 1.5$
Gaussian	9:13:15	13:21:32	24:25:60
Super-Gaussian	20:50:60	28:76:265	66:77:337
Hyper-Gaussian	21:125:34	27:68:34	62:72:35
Hyper-Gaussian turned by 45 degrees	17:30:24	27:70:35	65:77:36

It follows from Table I that in the absence of phase correction the super-Gaussian and both variants of the hyper-Gaussian intensity profiles are approximately equivalent and give a 2 to 2.5 times higher intensity than Gaussian filling of the aperture. The hyper-Gaussian profile is sharply distinguished in the case of tilt correction, but only if the wind direction is constant on the entire path, which is most likely to happen on a horizontal path under conditions of beam slewing than in the case of vertical propagation. Defocusing correction make it possible to achieve higher effectiveness for a super-Gaussian intensity profile when the amplitude of variation of wind direction is large. At the same time, in the case of a hyper-Gaussian profile defocusing correction unexpectedly resulted in a worse result. This last fact can be explained as follows. First, the least-squares method used to calculate the aberration coefficients is optimal only from the standpoint that it gives the smallest rms residual distortion of phase on the transmitting aperture, but it does not give the maximum possible power on target, especially if the number of corrected aberrations is small. Second, this deficiency could be connected with the square shape of the transmitting aperture of the hyper-Gaussian beam. Such an aperture is poorly matched with the corresponding aberration polynomial, which is proportional to $\rho^2 = x^2 + y^2$ and which has "circular" symmetry.

On the whole the results presented in Table I indicate that variation of wind direction with altitude changes radically the ratio of the contributions of the lowest order aberrations and thereby strongly affects the effectiveness of the modal corrector. As the amplitude of the change in wind direction increases, the contribution of aberrations which do not exhibit "circular" symmetry (i.e., they are not invariant under a rotation of the coordinate system] decreases and conversely the contribution of aberrations which do have circular symmetry (defocusing and spherical aberration) increases.

It should be noted that the approximation, introduced above, of a phase screen placed in the plane of the transmitting aperture obviously overestimates the ' effectiveness of correction. However this factor is determining only if the number of degrees of freedom of the corrector is large. In the case when the number of degrees of freedom is small, the maximum effectiveness of correction is determined by the accuracy with which the corrector reproduces prescribed predistortions of the phase.

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