STATISTICAL MODEL OF THE SIGNAL IN THE SYNTHESIS OF IMAGES OF SMALL OBJECTS BY THE METHOD OF ACTIVE INTERFEROMETRY

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The statistical characteristics and the probability density of the signal recorded when using the method of active interferometry for synthesis of images of small objects observed through a turbulent atmosphere are analyzed. Two types of signals are studied. They differ by the ratio of the diameter of the illuminating apertures and the correlation radius of the phase distortions of the optical field. The dependence of the statistical characteristics on the distance between the illuminating apertures and on the increment of this distance is analyzed for both cases. It is shown that the distribution of the signals is log-normal. The dependence of the distribution on the dispersion of the phase distortions of the optical field and the number of independent spatial and spatial-frequency regions of correlation of the recorded signal is investigated. The cases in which the distribution approaches a log-normal distribution are determined.

It has been suggested that the method of active interferometry be used to solve the problem of "seeing" through a turbulent atmosphere small distant objects from which the reflected signal is too weak for use of the traditional processing methods associated with telescopic reception. The methods consist essentially of illuminating the object by pairs of mutually coherent sources, separated by different distances, and recording the intensity of the reflected signal integrated over the receiving aperture of the optical system.[1–3] The obtained resolution is determined by the spatial frequency of the interference pattern produced by the sources (i.e., by the distance between them) and is theoretically limited only by the signal-to-noise ratio, while the energy potential is sharply increased by using a large energy collector . (or matrix of collectors), which does not need to be fabricated as carefully as a telescope, for the receiving optics.

It is well known that any optical signal which has passed through a turbulent atmosphere, including after reflection from the surface of the object, is of a random character and depends on the fluctuations of the index of refraction of the atmosphere. To obtain information about the object, it is necessary to use methods for processing random signals analogous to those employed when observing astronomical objects by traditional methods.[4] However the form of the starting information for different methods of observation of objects are different, and in order to realize any processing algorithm, a statistical model of the starting signal must be constructed.

The condition for observation of a small distant object through a turbulent atmosphere means that the requirement that the object is isoplanatic is satisfied. This is equivalent to locating the layer of turbulent atmosphere near the receiving-transmitting system. We shall study illumination of the object by two collimated beams of light, which are mutually coherent, through spatially separated apertures of finite size. The field in the plane of the illuminating apertures will have the form

$$E(\rho) = E_0\left[W(\rho)e^{i\varphi(\rho)} + W(\rho - b)e^{i\varphi(\rho)}\right], \tag{1}$$

where ρ is the radius vector in this plane, b is the vector of spatial separation of the centers of the apertures, $\mathbf{E}_0$ is the intensity of the electric field of the illuminating light, $\varphi(\rho)$ are the atmospheric phase distortions, $W(\rho)$ is the pupil function of the transmitting aperture, and

$$W(\rho) = \begin{cases} 1, & |\rho| \le \frac{d}{2}, \\ 0, & |\rho| > \frac{d}{2}, \end{cases}$$

where d is the diameter of the transmitting aperture. As shown in Ref. 3, a complicated interference pattern forms in the image plane of the object. The interference pattern depends not only on the separation b, but also on $\varphi(\rho)$, as a result of which the recorded integral intensity of the reflected signal assumes the form

$$J = \frac{S_{col}}{2\pi R^2}\int_{\Omega_{ob}}\left\{ |E_1(r)|^2 + |E_2(r)|^2 + + 2\,\mathrm{Re}\,E_1(r)E_2^*(r)\right\} dr, \tag{2}$$

where

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$$E_1(r) = \frac{E_0Q(r)}{\lambda R}\exp\left(i\frac{k}{2R}|r|^2\right)\int_{-\infty}^{+\infty} W(\rho)\exp\left(i\varphi(\rho) + i\frac{k}{2R}|\rho|^2 - i\frac{k}{R}r\rho\right)d\rho;$$

$$E_2(r) = \frac{E_0Q(r)}{\lambda R}\exp\left(i\frac{k}{2R}|r|^2\right)\int_{-\infty}^{+\infty} W(\rho - b)\times \exp\left(i\varphi(\rho) + i\frac{k}{2R}|\rho|^2 - i\frac{k}{R}r\rho\right)d\rho, \quad (3)$$

S_{col} is the area of the receiving aperture of the collector, λ is the wavelength of illumination, $k = 2\pi/\lambda$, R is the distance to the object, r is the radius vector in the image plane of the object, $\mathbf{E}_0Q(r)$ is the distribution of the complex amplitude of the light field in the image plane, and Ω_{ob} is the two-dimensional projection of the object on the image plane.

In Eq. (2) only the third (cross) term carries information. This term can be separated from the signal I by different methods, for example, with the help of temporal phase modulation,[1] whence not only the real but also the imaginary part of the product $E_1(r)E_2^*(r)$ is found. As a result the component of the signal carrying information under conditions of a turbulent atmosphere assumes the form

$$q_T(b) = \frac{S_{col}}{\pi R^2}\int_{\Omega_{ob}} E_1(r)E_2^*(r)\,dr = \frac{S_{col}}{\pi\lambda^2R^4}\int_{-\infty}^{+\infty} F(\omega)\iint_{-\infty}^{+\infty} W(\rho_1)W(\rho_2 - b)\times \exp\left[i\varphi(\rho_1) - i\varphi(\rho_2) + i\frac{k}{2R}\left(|\rho_1|^2 - |\rho_2|^2\right)\right]d\rho_1 d\rho_2 \times \int_{\Omega_{ob}} \exp\left[-i\frac{k}{R}r\left(\rho_1 - \rho_2\right) + i\omega r\right]dr\,d\omega, \quad (4)$$

where $F(\omega)$ is the complex Fourier spectrum of the image of the object $I(r) = |\mathbf{E}_0Q(r)|^2$ and ω is the radius vector in the spatial-frequency plane.
Since

$$\int_{\Omega_{ob}} \exp\left[-i\frac{k}{R}r\left(\rho_1 - \rho_2\right) + i\omega r\right]dr \longrightarrow (\lambda R)^2\delta\left[\rho_1 - \rho_2 + \frac{R}{k}\cdot\omega\right],$$

then $q_T(b)$ becomes

$$q_T(b) = \frac{S_{col}}{\pi R^2}\iint_{-\infty}^{+\infty}\tilde{F}(\omega)W(\rho)W\left[\rho - b - \frac{R}{k}\cdot\omega\right]\times \exp\left[i\varphi(\rho) - i\varphi\left[\rho - \frac{R}{k}\cdot\omega\right] + i\omega\rho\right]d\omega\,d\rho, \quad (5)$$

where $\tilde{F}(\omega) = F(\omega)\exp\left(-i\frac{R}{2k}|\omega|^2\right)$.

The expression obtained for $q_T(b)$ characterizes the information-carrying signal for arbitrary ratios of the size of the transmitting aperture d and the spatial correlation radius of the light field ρ_0, determining the "strength" of the fluctuations $\varphi(\rho)$. When the aperture size d is decreased until the condition $d \ll \rho_0$ is satisfied, the function $W(\rho)$ becomes narrower than $\varphi(\rho)$, and then

$$q_T(b) = \frac{S_{col}}{\pi R^2}F\left[\frac{kb}{R}\right]\exp\left[i\frac{k}{2R}|b|^2 + i\varphi(0) - i\varphi(b)\right]. \quad (6)$$

Adaptive noise of different origin (noise in the photodetector, photon noise, errors in determining the cross term (4), etc.) is always present in the recording and measuring systems together with the measured signals. When a complex signal of the type $q_T(b)$ is formed the noise also becomes complex. Thus the total measured signal has the form

$$q_T(b) = \frac{S_{col}}{\pi R^2}\iint_{-\infty}^{+\infty}\tilde{F}(\omega)W(\rho)W\left[\rho - b - \frac{R}{k}\cdot\omega\right]\times \exp\left[i\varphi(\rho) - i\varphi(\rho - \frac{R}{k}\cdot\omega) + i\omega\rho\right]d\omega d\rho + n(b), \quad (7)$$

where $n(b)$ is the complex random (noise) signal.

We shall first study the statistical characteristics of the noise. Based on the central limit theorem, the large number of independent sources of noise results in the fact that $n(b)$ can be assumed to be Gaussian and δ-correlated with mean $\bar{n} = \langle n(b)\rangle$ and correlation function

$$\langle n(b_1)n^*(b_2)\rangle = N_0\delta\left(b_1 - b_2\right), \quad (8)$$

where N_0 is the spectral density of the noise power.

We shall begin our analysis of the statistical characteristics of the signal with the simplest case, which is observed when $d \ll \rho_0$. The average value of $q_T(b)$ will have the form

$$\langle q_T(b)\rangle = \frac{S_{col}}{\pi R^2}F\left[\frac{kb}{R}\right]\exp\left[i\frac{k}{2R}|b|^2 - \frac{1}{2}D_\varphi\left(|b|\right)\right], \quad (9)$$

where $D_\varphi(x)$ is the structure function of the phase distortions. The correlation function $q_T(b)$ is determined as follows:

$$Q_T(b_1, b_2) = \langle q_T(b_1)q_T^*(b_2)\rangle =$$

$$= \left[\frac{S_{col}}{\pi R^2}\right]^2 F\left[\frac{kb_1}{R}\right] F^*\left[\frac{kb_2}{R}\right] \exp\left[i\frac{k}{2R}\left(|b_1|^2 - |b_2|^2\right)\right] \times$$

$$\times \exp\left[-\frac{1}{2}D_\varphi\left(|b_1 - b_2|\right)\right], \qquad (10)$$

whence the variance $\sigma^2_{q_T}$ at $b_1 = b_2$ can be found:

$$\sigma^2_{q_T} = \left[\frac{S_{col}}{\pi R^2}\right]^2 \left|F\left[\frac{kb}{R}\right]\right|^2.$$

It is well known that the function $\exp\left(-\frac{1}{2}D_\varphi(x)\right)$ has the characteristic form $\exp(-3.44|x/\rho_0|)^{5/3}$ (Ref. 5), i.e., it is concentrated in the range $|x| \le \rho_0$. Since for a small distant object inequalities of the type

$$\frac{2\pi R}{kl} > \rho_0 \quad \text{or} \quad \frac{R}{kl} \gg \rho_0, \qquad (11)$$

where l is the transverse size of the object and $2\pi R/kl$ is the spatial coherence radius of the object field, are characteristically satisfied, the function $\exp\left(-3.44|b/\rho_0|^{5/3}\right)$ is much narrower than $F\left(\frac{kb}{R}\right)$ and $\langle q_T(b)\rangle$ is different from zero only for distances between the centers of the transmitting apertures $|b| \le \rho_0$ (Fig. 1). Writing $b_2 = b_1 + \Delta b$ the correlation function can be put into the form

$$Q_T(b, \Delta b) = \left[\frac{S_{col}}{\pi R^2}\right]^2 F\left[\frac{kb}{R}\right] F^*\left(k(b + \Delta b)\right) \times$$

$$\times \exp\left[-i\frac{k}{2R}|\Delta b|^2 + i\frac{k}{R}\cdot\Delta b\cdot b - 3.44\left|\frac{\Delta b}{\rho_0}\right|^{5/3}\right]. \qquad (12)$$

From here and the inequalities (11) it is obvious that it is defined in the range of increments Δb: $|\Delta b| \le \rho_0$.

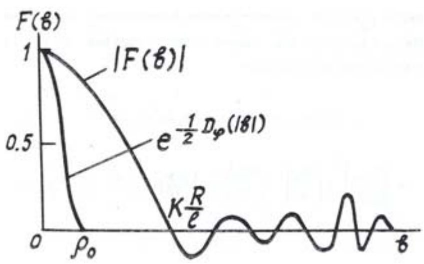

FIG. 1. The ratio of the Fourier spectrum of the object $F(b)$ and the average OTF of the turbulent atmosphere $\exp -\left(\frac{1}{2}D_\varphi|b|\right)$, *forming the signal $q_T(b)$.*

We shall now analyze the probability density $q_T(b)$. Since in a turbulent atmosphere phase fluctuations have a normal distribution,[6] because $\varphi(0)$ and $\varphi(b)$ are independent quantities their difference is also distributed normally, but as a result $q_T(b)$ has a log-normal distribution[6] (Fig. 2):

$$\omega(q_T) = \frac{1}{\sqrt{2\pi}\sigma_\varphi q_T(b)} \exp\left\{-\ln^2[q_T(b)/A]/2\sigma^2_\varphi\right\}, \qquad (13)$$

where σ^2_φ is the variance of the phase fluctuations and $A = \frac{S_{col}}{\pi R^2} F\left(\frac{kb}{R}\right)\exp\left(i\frac{k}{2R}|b_1|^2\right)$. In the expression (13) the condition $\langle\varphi(0) - \varphi(b)\rangle = 0$ was used.

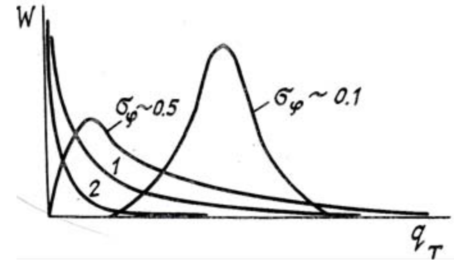

FIG. 2. Distribution of the signal q_T for different variances σ^2_φ.

We shall now study the general case: the signal $q_T(b)$ for $d \gtreqless \rho_0$. Its average value will have the form

$$\langle q_T(b)\rangle = \frac{S_{col}}{\pi R^2}\int_{-\infty}^{+\infty} \tilde{F}(\omega)\exp\left[-\frac{1}{2}D_\varphi\left|\frac{R}{k}\cdot\omega\right|\right] \times$$

$$\times H_0\left[b + \frac{R}{k}\cdot\omega\right]d\omega, \qquad (14)$$

where

$$H_0\left[b + \frac{R}{k}\cdot\omega\right] = \int_{-\infty}^{+\infty} W(\rho)W\left[\rho - b - \frac{R}{k}\cdot\omega\right]\exp\left[i\omega\rho\right]d\rho \qquad (15)$$

is the mutual aberration-free optical transfer function (OTF) of the transmitting apertures. Using the inequalities (11) and the definition of $D_\varphi(x)$, we write

$$\langle q_T(b)\rangle \approx \frac{S_{col}}{\pi R^2}\tilde{F}(0)\int_{-\infty}^{+\infty} \exp\left[-3.44\left|\frac{R\omega}{k\rho_0}\right|^{5/3}\right] \times$$

$$\times H_0\left[b + \frac{R}{k}\cdot\omega\right]d\omega, \qquad (16)$$

whence it is obvious that the average value is different from zero only for distances $|b| < d + \rho_0$. When the condition $d \gg \rho_0$ is satisfied, i.e., when the diameters of the transmitting apertures are large, the function $\exp[-1/2]\ D_\varphi$ becomes much narrower, as a result of which we can write, expanding H_0 in a Taylor series at the point 6 and retaining the first two terms,

$$\langle q_T(b)\rangle \approx \frac{S_{col}}{\pi R^2}\tilde{F}(0)\left[H_0(b)M + \nabla H_0(b)\theta\right], \qquad (17)$$

where

$$M = \int\limits_{\infty}^{\infty} \exp\left[-3.44\left|\frac{R\omega}{k\rho_0}\right|^{5/3}\right] d\omega,$$

$$\theta = \int\limits_{\infty}^{\infty} \left|\frac{R\omega}{k}\right| \exp\left[-3.44\left|\frac{R\omega}{k\rho_0}\right|^{5/3}\right] d\omega.$$

To determine the correlation function $Q_T(b_1,\ b_2)$ we write, according to Ref. 3,

$$Q_T(b_1,\ b_2) = \langle q_T(b_1) q_T^{*}(b_2)\rangle \approx \left[\frac{S_{col}}{\pi R^2}\right]^2 MP \times$$

$$\times \int\limits_{\infty}^{\infty} |F(\omega)|^2 H_0\left[b_1 + \frac{R}{k}\cdot\omega\right] H_0^{*}\left[b_2 + \frac{R}{k}\cdot\omega\right] d\omega, \qquad (18)$$

where

$$P = \int\limits_{\infty}^{\infty} \exp\left[-3.44\left|\frac{\rho}{\rho_0}\right|^{5/3}\right] d\rho.$$

Writing, as we did in the preceding case, $b_1 = b$, $b_2 = b + \Delta b$, we can see that $Q_T(b,\ \Delta b)$ is defined in the region $|\Delta b| \le d$ and has a maximum at $\Delta b = 0$, when $Q(b,\ 0)$ is the variance $\sigma^2_{q_T}$:

$$\sigma^2_{q_T} = \left[\frac{S_{col}}{\pi R^2}\right]^2 MP \int\limits_{\infty}^{\infty} |F(\omega)|^2 \left|H_0\left[b + \frac{R}{k}\cdot\omega\right]\right|^2 d\omega. \qquad (19)$$

For large b, such that $|b| \gg d$, in Eq. (18) the high-frequency part of $|F(\omega)|^2$, which oscillates quite rapidly, is integrated and in addition the position of the minima and maxima alternate at distances much shorter them d. For this reason the behavior of the function Q in this region as $|b|$ increases will be: a) monotonically decreasing (as $1/x^2$) if $|\Delta b| \le d$ or b) oscillatory and decreasing (as $\sim (\sin x/x)^2$ if $|\Delta b| \ll d$. For $|b| < d$ the low and middle frequency parts of $|F(\omega)|^2$, in which the fluctuations of the Fourier spectrum are not so characteristic because of the condition that the object be small., are integrated. As a result, the smaller $|b|$, the smaller the oscillations as a function of b and the weaker the dependence of $Q(b, \Delta b)$ on Δb are, i.e., in this region, like in the case studied above, $Q(b, \Delta b) \approx |F(\omega)|^2$.

Before analyzing the probability distribution of the signal $q_T(b)$, we shall study the characteristic that determines its random nature — the instantaneous OTF of the system "transmitting aperture-atmosphere":

$$H_\varphi\left[b + \frac{R}{k}\cdot\omega\right] = \int\limits_{\infty}^{\infty} W(\rho)W\left[\rho - b - \frac{R}{k}\cdot\omega\right] \times$$

$$\times \exp\left[i\omega\rho + i\varphi(\rho) - i\varphi(\rho - \frac{R}{k}\cdot\omega)\right] d\rho. \qquad (20)$$

It was mentioned above that the phase fluctuations of the optical field in a turbulent atmosphere as well as the differences of these fluctuations have a normal distribution, as a result of which the entire integrand in Eq. (20) has a log-normal distribution. The entire OTF H_φ can be expressed approximately by the sum

$$H_\varphi(x) \approx \sum_{j=0}^{H(x)} \tilde{W}_j \exp(i\Delta\varphi_j), \qquad (21)$$

where

$$N(x) \approx \mathrm{IP}\left\{\frac{1}{\pi\rho_0^2}\int\limits_{\infty}^{\infty} W(\rho)W(\rho - x)d\rho\right\},$$

IP indicates the integral part of the expression enclosed in braces, and $x = b + \frac{R}{k}\omega$.

As shown in Ref. 7, the sum of log-normally distributed quantities is also a log-normal quantity for finite N, so that for $(d/\rho_0)^2 \ge 1$ H_φ a log-normal distribution at all frequencies $x = b + \frac{R}{k}\omega$. We shall now study the signal $q_T(b)$ which is a convolution of the spectrum $\tilde{F}(\omega)$ and the instantaneous OTF H_φ at the frequency kb/R:

$$q_T(b) = \frac{S_{col}}{\pi R^2}\int\limits_{-\infty}^{+\infty} \tilde{F}(\omega)H_\varphi\left[b + \frac{R}{k}\cdot\omega\right]d\omega, \qquad (22)$$

Since $\tilde{F}(\omega)$ is the spectrum of a two-dimensional' finite function (the object has sharp boundaries), determined in the region that is poorly resolved by the traditional observational systems (the condition that the size of the object is small), the domain of $\tilde{F}(\omega)$ is much larger than the domain of $H_\varphi\left(b + \frac{R}{k}\omega\right)$ which exists only near $H_0\left(b + \frac{R}{k}\omega\right)$, i.e., $\frac{k}{R}(b - d) \le |\omega| \le \frac{k}{R}(b + d)$. For this reason the region of integration in Eq. (22) is also bounded by the same limits (see Fig. 3). According to the sampling theorem, in order to determine the spectrum $\tilde{F}(\omega)$ of a finite function $I(r)$, for example, in the direction α, the vector b must be changed in steps of size

$$\Delta b_\alpha \approx \left[\frac{1}{2} \div \frac{1}{3}\right]\frac{2\pi R}{kl_\alpha}, \tag{23}$$

where l_α is the size of the object in the direction α. The condition that the object be small is characterized by the fact that when traditional observation methods are employed it gives poor resolution of the object (even if the atmosphere is not turbulent). This means that when telescopes of sufficiently good quality, having input pupils with acceptable diameters and not of large capacity from the technological viewpoint, are used, the image of the object will consist only of several resolution elements. To illuminate the object, in this case, there is no sense in using collimators with a large aperture diameter, since one of the advantages of the method of active interferometry is that expensive telescopes are not used in it. In this connection, the condition that the object be small can be characterized by the relation

$$\frac{2\pi R}{kl} \overset{>}{\underset{<}{\approx}} d, \tag{24}$$

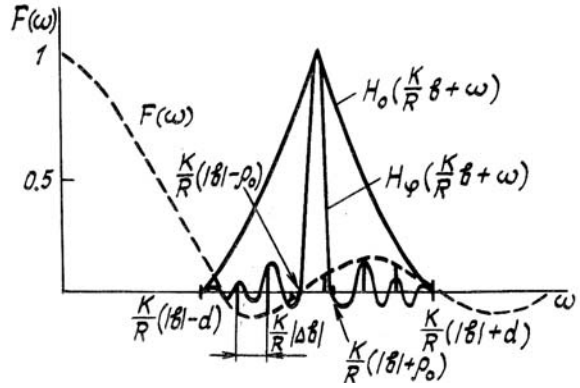

FIG. 3. The relation between the Fourier spectrum of the object $F(\omega)$ instantaneous OTF $H_\varphi(\omega)$ and the average OTF $H_0(\omega)$ forming the signal $q_T(b)$ in a turbulent atmosphere.

whence, using the relation (23), it follows that

$$|\Delta b| \overset{>}{\underset{<}{\approx}} d\left[\frac{1}{2} \div \frac{1}{3}\right], \tag{25}$$

where $|\Delta b| = \Delta b_\alpha$ in the direction α. Thus only several (M) independent frequencies of the spectrum $\tilde{F}(\omega)$, separated from one another by a distance $|\Delta b|$, contribute to the integral in Eq. (22):

$$q_T(b) \approx \frac{S_{col}}{\pi R^2}\sum_{m=0}^{M}\tilde{F}_m H_{\varphi m}(b), \tag{26}$$

where

$$M \approx \left[\frac{2d}{|\Delta b|}\right]^2.$$

With the help of Eq. (21) $q_T(b)$ can be expressed in the form

$$q_T(b) = \frac{S_{col}}{\pi R^2}\sum_{m=0}^{M}\sum_{j=0}^{N}\tilde{W}_j\tilde{F}_m\exp(i\Delta\varphi_{jm}), \tag{27}$$

which characterizes the summation of $M\times N(b)$ independent log-normally distributed quantities. For finite $N = (d/\rho_0)^2 \geq 1$ multiplication by M, under the condition (25), will not change much the distribution of the sum in $q_T(b)$, since according to Mitchell's work[8] the log-normal law is very stable with respect to summation and approaches the normal law according to the central limit theorem extremely slowly. However, if $N = (d/\rho_0)^2 \gg 1$ the summation over the $N\times M$ realizations may be appreciable. To estimate the number of realizations $L = NM$ with which the distribution $q_T(b)$ can be regarded as being normal, we shall expand the probability distribution of the sum of L independent random quantities in Hermite polynomials[7]

$$f_L(z) = \frac{1}{2\pi}e^{-\frac{z^2}{2}}\left[1 + \frac{\gamma_1}{64L^{1/2}}h_3(z) + \frac{\gamma_2}{24L}h_4(z) + \ldots + 0\left(L^{-3/2}\right)\right], \tag{28}$$

where $h_n(z)$ are the Hermite polynomials; γ_1 and γ_2 are the skewness and excess; and, $z = (x - L\langle x\rangle)/\sigma_x L^{1/2}$, where x is a random quantity, $\langle x\rangle$ is the average value of x, and σ_x^2 is the variance of x, respectively. Since the term $\dfrac{\gamma_1 h_3(z)}{64L^{1/2}}$ makes the largest contribution to $f_L(z)$ from the entire sum (after the one), the quantity L_{cr} can be estimated from the condition

$$\frac{\gamma_1 h_3(z)}{64\sqrt{L_{cr}}} \ll 1 \quad \text{or} \quad L_{cr} \geq 0.029\,\gamma_1^2 h_3^2(z). \tag{29}$$

For a log-normal distribution the skewness[7] can be determined as

$$\gamma_1 = (\exp\sigma_{\Delta\varphi}^2 + 2)\sqrt{\exp\sigma_{\Delta\varphi}^2 - 1}\,. \tag{30}$$

Since when the phase differences $\Delta\varphi = \varphi(\rho) - \varphi(\rho - R\omega/k)$ are formed phases which are not correlated with one another are actually calculated (and the transmitting apertures are separated by the vector b, for which the condition $b > \rho_0$ is satisfied), we have

$\sigma^2_{\Delta\varphi} \equiv \sigma^2_\varphi$. Since the Hermite polynomial $h_3(z) = z - 3z$, being multiplied by $\exp(-z^2/2)$, makes an appreciable contribution for small and middle values of z (for large z the product approaches zero as $\exp(-z^2/2)$), to calculate L_{cr} we shall set $h_3(z) \sim 1$. Table I gives the estimates of the values for different variances of the phase σ^2_φ and the corresponding limiting values $(d/\rho_0)^2_{cr}$ for different values of $M = (2d/|\Delta b|)^2$.

TABLE I.

σ_φ	L_{cr} \ M	4	9	25	100
0.8π	$1.5\cdot10^5$	$4\cdot10^4$	$1.7\cdot10^4$	$6\cdot10^3$	$1.5\cdot10^3$
π	$1.25\cdot10^9$	$3.2\cdot10^8$	$1.4\cdot10^8$	$4.75\ 10^7$	$1.25\cdot10^7$
1.5π	$3\cdot10^{23}$	$7.5\cdot10^{22}$	$3.3\cdot10^{22}$	$1.2\cdot10^{22}$	$3\cdot10^{21}$

One can see from Table I that the variance of the phase fluctuations σ^2_φ has the strongest effect on L_{cr} and on the limiting ratio of the area of the transmitting aperture d^2 and the area of the cell of spatial coherence of the phase distortion ρ_0^2. The smaller the variance, the more rapidly $q(b)$ becomes normally distributed. For weak fluctuations ($\sigma_\varphi \sim 0.1\pi - 0.2\pi$) the distribution of $q(b)$ can be regarded as normal, even for relatively small ratios $(d/\rho_0)^2$; this agrees with the results of Ref. 8.

In a turbulent atmosphere, however, the variance of the phase usually reaches large values $\sigma_\varphi \sim (10\pi)^2$,[6] as a result of which for any acceptable ratios $(d/\rho_0)^2 \gg 1$ the signal $q_T(b)$ has a log-normal distribution. Thus the random signal $q_T(b)$ consists of two components: an information-carrying signal $q_T(b)$ and noise $n(b)$. The noise $n(b)$ is normally distributed and δ-correlated with spectral power density N_0. The signal $q_T(b)$ for $(d/\rho_0)^2 \ll 1$ is distributed log-normally with zero mean for real values $||b| > \rho_0$ and correlation function (10) and variance

$$\sigma_g^2 = \left(S_{col}/\pi R^2\right)^2 \left|F(kb/R)\right|^2.$$

For $(d/\rho_0) \gtreqless 1$ the signal $q_T(b)$ is distributed log-normally for the values $\sigma_\varphi \geq 0.2\pi$, while for $(d/\rho_0)^2 \gg 1$ the limit σ_φ at which the distribution becomes normal increases approximately up to 0.5–0.7π. In a turbulent atmosphere with average and strong fluctuations Of the phase the distribution is log-normal for virtually all values $(d/\rho_0)^2 \gg 1$.

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