# MODELING PC-CORRECTION OF LASER BEAMS IN THE ATMOSPHERE 

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#### Abstract

Compensation for distortions of a light beam in the atmosphere by means of a $P C$-mirror has been examined theoretically and experimentally. Similarity ratios in laboratory modeling of the correction of atmospheric distortions of laser radiation by means of a PC-MBSS mirror have been derived. The effect of Raman scattering on the quality of reconstruction of the corrected beam in the turbulent atmosphere has been studied. It is shown that the efficiency of PC-correction may be significantly limited by errors in the phase conjugation (PC) achieved by a real mirror and by nonlinear phenomena attendant to the propagation of the conjugate wave in the atmosphere.


## 1. Atmospheric distortions substantially degrade

 the operating efficiency of optical communication systems and systems of optical detection and ranging. It has been proposed that the phenomenon of phase conjugation (PC) in' a nonlinear medium be used to compensate for these distortions. ${ }^{1-5}$ Such a form of PC-correction has been most completely developed for short laser pulses (with durations less than $1 \mu \mathrm{~s}$ ). At low intensities of the laser radiation in a clear atmosphere free of any aerosol burden turbulent fluctuations are responsible for the beam wavefront distortions. Additional beam distortions caused by nonlinear optical phenomena are important at high radiation intensities. For short laser pulses of duration less than the time it takes an acoustic wave to propagate across the beam the nonlinear phenomena of thermal relaxation (for the pulsed-periodic regime) and Raman scattering (for a single pulse) have the lowest energy thresholds.Investigations of possible ways of compensating for atmospheric distortions of a laser beam with a PC-mirror under real conditions are met with considerable difficulties. For this reason it is necessary to use laboratory modeling based on the principles of similarity theory in order to examine the main features of such an adaptive correction. In this paper, on the basis of laboratory modeling and numerical simulation we examine the efficiency of correction of short laser pulses in the atmosphere with the help of a PC-mirror.

Various propagation conditions of the conjugate beam in the atmosphere are studied:
a) the beam is not subject to nonlinear distortions in the turbulent medium,
b) the beam undergoes Raman scattering as it propagates in the turbulent medium,
c) the distortions of each individual corrected pulse are caused by heating of the moving medium
resulting from the passage through it of the uncorrected pulses following one sifter the other with high repetition rate.

The application of similarity theory to the above-formulated problem consists in the following. The system of equations and boundary conditions which describes the processes of radiation propagation not only in the atmosphere, but also in the active medium of the PC-mirror, is reduced to dimensionless form. The processes of propagation in the laboratory and under natural conditions are similar if the corresponding parameters in the dimensionless equations and the boundary conditions are identical. ${ }^{6}$
2. A diagram of a scheme of optical communication in the atmosphere which makes use of a PC-mirror is shown in Fig. 1. Single-mode radiation of the sounding laser 1 with complex field amplitude $A_{p r}$ propagates in the atmosphere along the $z$ axis. The origin of the $z$ axis is located in the exit plane of the sounding laser. The distorting medium lies within the limits $0 \leq z \leq L_{1}$. The probing radiation is incident on the receiving aperture of the telescope 2, which is located in the plane $z=L_{1}$. After an $N$-fold telescopic narrowing, the beam passes through the optical amplifier 3 with intensity gain coefficient $K_{g}$ and then enters the active medium of the PC-mirror 4 which lies within the limits $L_{2} \leq z \leq L_{3}$. The modulator 5, designed for temporal modulation of the radiation, introduces information into the beam reflected from the PC-mirror. The modulated amplified beam with complex amplitude $A_{c}$ and phase-conjugate to the probe beam propagates in the atmosphere counter to this beam. In the plane 7, which is conjugate to the plane 6 of the exit aperture of the sounding laser, under certain conditions the beam wavefront is undistorted and $\cdot$ carries the introduced information.


FIG. 1. Diagram of scheme of optical communication through the atmosphere using PC.
3. At present the most developed method of PC is that based on Mandel'shtam-Brullouin stimulated scattering (MBSS) of the beams focused into the active medium. The system of equations which describes the propagation of radiation not only in the distorting medium, but also in the active medium of the PC-MBSS-mirror has the form ${ }^{5}$

$$
\begin{equation*}
\left[2 i k_{1} \frac{\partial}{\partial z}-\Delta_{\perp}-k_{1}^{2} \frac{\tilde{\varepsilon}}{\varepsilon_{1}}+i k_{1} \alpha\right] A_{\mathrm{pr}}=0, \quad 0<z<L_{1} \tag{1}
\end{equation*}
$$

$$
\begin{equation*}
\left[2 i k_{2} \frac{\partial}{\partial z}-\Delta_{\perp}+i k_{2} G\left|A_{\mathrm{st}}\right|^{2}\right] A_{\mathrm{p}}=0, \quad L_{2}<z<L_{3} \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
\left[2 i k_{2} \frac{\partial}{\partial z}+\Delta_{\perp}+i k_{2} G\left|A_{p}\right|^{2}\right] A_{\mathrm{St}}=0, \quad L_{2}<z<L_{3} \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
\left[2 i k_{1} \frac{\partial}{\partial z}+\Delta_{\perp}+k_{1}^{2} \frac{\tilde{\varepsilon}}{\varepsilon_{1}}-i k_{1} \alpha\right] A_{c}=0, \quad 0<z<L_{1} \tag{4}
\end{equation*}
$$

Here $A_{\mathrm{st}}$ and $A_{p}$ are the complex amplitudes of the Stokes wave and the pump wave, respectively, $G$ is the MBSS gain coefficient, $\Delta_{\perp}$ is Laplacian in the plane $z=$ const, $k_{1}$ is the wave number in the distorting medium, $k_{2}$ is the wave number in the active medium, $\alpha$ is the radiation absorption coefficient in the distorting medium, $\varepsilon_{1}$ is the mean dielectric constant of the unperturbed medium, and $\tilde{\varepsilon}$ is the increment to the dielectric constant caused by the perturbations of this medium. At $z=0$ the complex amplitude of the probe wave has the form
$A_{\mathrm{pr}}(x, y, 0)=\sqrt{I_{\mathrm{pr}}} \exp \left(-\frac{x^{2}+y^{2}}{2 a_{0}^{2}}\right)$,
where $I_{p r}$ is the axial intensity and $a_{0}$ is the radius of the beam. We shall assume that on the interval $L_{1}<z<L_{2}$ the radiation is affected only by the amplifier and the lens and that the length of this interval $L_{2}-L_{1}$ is much less than $L_{1}$ or $L_{4}=L_{3}-L_{2}$. We can then write

$$
A_{p}\left(x, y, L_{2}\right)=
$$

$=N \sqrt{K_{g}} A_{\mathrm{pr}}\left(N x, N y, L_{1}\right) \exp \left[\frac{i k_{0}\left(x^{2}+y^{2}\right)}{2 z_{f}}\right]$,
$A_{c}\left(x, y, L_{1}\right)=$
$N^{-1} \sqrt{K_{g}} A_{\mathrm{st}}\left(N^{-1} x, N^{-1} y, L_{2}\right) \exp \left[\frac{i k_{0}\left(x^{2}+y^{2}\right)}{2 z_{f}}\right]$,
where $z_{f}$ is the focal length of the lens, and $k_{0}$ is the wave number in vacuum. For $z=L_{3}$ the boundary condition for Eq. (3) takes the form
$A_{\text {st }}\left(x, y, L_{3}\right)=U(x, y)$,
where $U(x, y)$ is a random field.
In the case in which the conjugate beam propagates in the dielectric field $\tilde{\varepsilon}$ of the moving medium caused by its radiative heating, system of equations (1)-(4) is completed by the equations for $\tilde{\varepsilon}(x, y, z)$ and the radiation-driven temperature of the medium $T(x, y, z)$. If heating of the medium occurs as a result of the passage through it of a train of uncorrected pulses with sufficiently high repetition rate, the additional equations have the form ${ }^{7}$
$\left[2 i k_{1} \frac{\partial}{\partial z}+\Delta_{\perp}+k_{1}^{2} \frac{\tilde{\varepsilon}}{\varepsilon_{1}}-i k_{1} \alpha\right] A_{\mathrm{t}}=0, \quad 0<z<L_{1} ;$
$\tilde{\varepsilon}(x, y, z)=\frac{\mathrm{d} \tilde{\varepsilon}}{\mathrm{d} T} T(x, y, z) ;$
$V \frac{\partial T}{\partial x}=\alpha \frac{\left|A_{\mathrm{t}}\right|^{2}}{\rho C_{\mathrm{p}}}$.
Here $V$ is the velocity of the moving medium, $\rho$ is the density and $C_{p}$ is the specific heat of the medium, and $A_{t}$ is the time-averaged complex amplitude of the uncorrected pulses. For $z=L_{1}$ we shall assume that this time-averaged complex amplitude has the form
$A_{t}\left(x, y, L_{1}\right)=\sqrt{I_{t}} \exp \left(-\frac{x^{2}+y^{2}}{2 a_{0}^{2}}\right)$,
where $I_{t}$ is the average axial intensity of the beam and $a_{0}$ is the beam radius.
4. After transforming to the dimensionless variables

$$
\begin{align*}
& x^{\prime}=\frac{x}{a_{0}}, \quad y^{\prime}=\frac{y}{a_{0}}, \quad z=\frac{z}{k_{1} a_{0}^{2}}, \quad A_{\mathrm{pr}}^{\prime}=\frac{A_{\mathrm{pr}}}{\sqrt{I_{\mathrm{pr}}}}, \\
& A_{\mathrm{p}}^{\prime}=\frac{A_{\mathrm{p}}}{\sqrt{I_{\mathrm{pr}}}}, \quad A_{\mathrm{st}}^{\prime}=\frac{A_{\mathrm{st}}}{\sqrt{I_{\mathrm{pr}}}}, \quad z^{\prime \prime}=\frac{z}{k_{2} a_{0}^{2}}, \tag{13}
\end{align*}
$$

$A_{\mathrm{t}}^{\prime}=\frac{A_{\mathrm{t}}}{\sqrt{I_{\mathrm{t}}}} ;$
the system of equations and boundary conditions (1)-(12) has the form
$\left[2 i \frac{\partial}{\partial z^{\prime}}-\Delta_{\perp}^{\prime}-T^{\prime}+i \theta\right)_{A_{\mathrm{pr}}^{\prime}}=0, \quad 0<z^{\prime}<L_{1}^{\prime} ;$
$\left[2 i \frac{\partial}{\partial z^{\prime \prime}}-\Delta_{\perp}^{\prime}-G^{\prime}\left|A_{\mathrm{st}}^{\prime}\right|^{2}\right] A_{\mathrm{p}}^{\prime}=0, L_{2}^{\prime}<z^{\prime \prime}<L_{3}^{\prime \prime} ;$
$\left[2 i \frac{\partial}{\partial z^{\prime \prime}}+\Delta_{\perp}^{\prime}+G^{\prime}\left|A_{\mathrm{p}}^{\prime}\right|^{2}\right] A_{\mathrm{st}}^{\prime}=0, L_{2}^{\prime}<z^{\prime \prime}<L_{3}^{\prime \prime} ;$
$\left[2 i \frac{\partial}{\partial z^{\prime}}+\Delta_{\perp}^{\prime}+T^{\prime}-i \theta\right] A_{\mathrm{c}}^{\prime}=0, \quad 0<z^{\prime}<L_{1}^{\prime} ;$
$\left[2 i \frac{\partial}{\partial z^{\prime}}+\Delta_{\perp}^{\prime}+T^{\prime}-i \theta\right] A_{\mathrm{t}}^{\prime}=0, \quad 0<z^{\prime}<L_{1}^{\prime} ;$
$\frac{\partial T^{\prime}}{\partial x^{\prime}}=R\left|A_{t}^{\prime}\right|^{2} ;$
$A_{\mathrm{t}}^{\prime}\left(x^{\prime}, y^{\prime}, 0\right)=\exp \left[-\left(x^{\prime 2}+y^{\prime 2}\right)\right] ;$
$A_{\mathrm{pr}}^{\prime}\left(x^{\prime}, y^{\prime}, L_{1}^{\prime}\right)=\exp \left[-\left(x^{\prime 2}+y^{\prime 2}\right)\right] ;$
$A_{p}^{\prime}\left(x^{\prime}, y^{\prime}, L_{2}^{\prime}\right)=$
$=N \sqrt{K_{\mathrm{g}}} A_{\mathrm{pr}}^{\prime}\left(N x^{\prime}, N y^{\prime}, L_{1}^{\prime}\right) \exp \left[\frac{i \gamma\left(x^{\prime 2}+y^{\prime 2}\right)}{2 z_{f}^{\prime \prime}}\right] ;$
$A_{c}^{\prime}\left(x^{\prime}, y^{\prime}, L_{1}^{\prime}\right)=$
$=N^{-1} \sqrt{K_{g}} A_{\mathrm{st}}^{\prime}\left(N^{-1} x^{\prime}, N^{-1} y^{\prime}, L_{2}^{\prime}\right) \exp \left[\frac{i \gamma\left(x^{\prime 2}+y^{\prime 2}\right)}{2 z_{f}^{\prime \prime}}\right] ;$
$A_{\mathrm{st}}^{\prime}\left(x^{\prime}, y^{\prime}, L_{3}^{\prime \prime}\right)=U^{\prime}\left(x^{\prime}, y^{\prime}\right)$
Here
$U^{\prime}=\frac{U}{\sqrt{I_{\mathrm{pr}}}}, \quad G^{\prime}=G I_{\mathrm{pr}} k_{2} a_{0}^{2}, \quad \theta=\alpha k_{1} a_{0}^{2}$,
$z_{\mathrm{f}}^{\prime \prime}=\frac{z_{\mathrm{f}}}{k_{2} a_{0}^{2}}, \quad \gamma=\frac{k_{0}}{k_{2}}, \quad L_{1}^{\prime}=\frac{L_{1}}{k_{1} a_{0}^{2}}$,
$L_{2}^{\prime}=\frac{L_{2}}{k_{1} a_{0}^{2}}, \quad L_{3}^{\prime \prime}=L_{2}^{\prime}+L_{4}^{\prime \prime}, \quad L_{4}^{\prime \prime}=\frac{L_{4}}{k_{2} a_{0}^{2}} ;$
$R=\frac{\alpha k_{1}^{2} I_{\mathrm{t}} a_{0}^{3}}{\varepsilon_{1} V \rho C_{\mathrm{p}}} \frac{\mathrm{d} \tilde{\varepsilon}}{\mathrm{d} T}, \quad T^{\prime}=k_{1}^{2} a_{0}^{2} \frac{\mathrm{~d} \tilde{\varepsilon}}{\mathrm{~d} T} T$.

Analysis of Eqs. (15)-(20) together with boundary conditions (21)-(25) indicates that for laboratory modeling of the above-described processes with $N^{l a b}=1$ and $K_{g}^{l a b}=1$ the following relations must be satisfied:

$$
\begin{align*}
& a_{0}^{1 \mathrm{ab}}=\frac{a_{0}^{\mathrm{atm}}}{N^{\mathrm{atm}} ; \quad \theta^{1 \mathrm{ab}}=\theta^{\mathrm{atm}} ; \quad L_{1}^{\prime \mathrm{ab}}=L_{1}^{\prime \mathrm{atm}} ;} \\
& L_{4}^{\prime \prime \mathrm{ab}}=L_{4}^{\prime \prime \mathrm{atm}} ; \quad z_{f}^{\prime \prime \mathrm{ab}}=z_{f}^{\prime \prime \text { atm }} ; \\
& I_{\mathrm{pr}}^{1 \mathrm{ab}}=\frac{I_{\mathrm{pr}}^{\mathrm{atm}} G^{\mathrm{atm}} R_{2}^{\mathrm{atm}} K_{\mathrm{g}}^{\mathrm{atm}}\left(N^{\mathrm{atm}}\right)^{2}}{G^{1 \mathrm{ab}} k_{2}^{1 \mathrm{ab}}} . \tag{28}
\end{align*}
$$

Here the superscript "lab" refers to the laboratory conditions and the superscript "atm" to the atmospheric conditions. In addition to conditions (28) the condition

$$
\begin{equation*}
R^{1 \mathrm{ab}}=R^{\mathrm{atm}} \tag{29}
\end{equation*}
$$

must also be satisfied.
5. In the problem of compensation for turbulent distortions of low-intensity radiation by a PC-MBSS-mirror the system of equations describing the involved processes together with the boundary conditions is given by Eqs. (1)-(8). As before, $\tilde{\varepsilon}$ in Eqs. (1) and (4) denotes the fluctuations of the dielectric constant. In what follows we shall set $\alpha=0$ in Eqs. (1) and (4) for a turbulent medium. From Eq. (1), which describes the propagation of the probe beam in the turbulent medium, it is easy to derive an equation for the mutual coherence function $\Gamma\left(x_{1}, y_{1}, x_{2}, y_{2}, z\right)$ of this beam. This equation has the form
$2 i k_{1} \frac{\partial \Gamma}{\partial z}+\left(\Delta_{1}-\Delta_{2}\right) \Gamma+$
$+\frac{i \pi k_{1}^{3}}{2} H\left[x_{1}-x_{2}, y_{1}-y_{2}\right] \Gamma=0$,
where
$H(\rho)=2 \int \Phi_{\varepsilon}[\kappa][1-\cos \kappa \rho] \mathrm{d}^{2} \kappa ;$
where $\Phi_{\varepsilon}$ is the spectrum of the fluctuations $\tilde{\varepsilon}$, which is assumed to take the form

$$
\begin{equation*}
\Phi_{\varepsilon}(\kappa)=0.033 c_{\varepsilon}^{2}\left(\kappa^{2}+\kappa_{0}^{2}\right)^{-11 / 6} \exp \left(-\kappa^{2} / \kappa_{W}^{2}\right) \tag{32}
\end{equation*}
$$

where $C_{\varepsilon}^{2}$ is the structure function of the dielectric constant fluctuations, $\kappa_{0}=\frac{2 \pi}{L_{0}}$, $L_{0}$ is the outer tur-
bulence scale, $\kappa_{m}=\frac{5.92}{l_{0}}, l_{0}$ is the inner turbulence scale. An exact analytic solution of Eq. (27) was derived in Ref. 8 assuming a quadratic approximation of the function $H(\rho)$. Analysis of this solution indicates that the expression for the dimensionless function $\Gamma\left(x_{1}^{\prime}, y_{1}^{\prime}, x_{2}^{\prime}, y_{2}^{\prime}, L_{1}^{\prime}\right) / I_{p r}$ which takes boundary condition (5) into account depends only on the parameters $L_{1}^{\prime}$ and

$$
\begin{equation*}
\beta_{0}=0.365 C_{\varepsilon_{1}}^{2} k_{1}^{7 / 6} L_{1}^{11 / 6} \tag{33}
\end{equation*}
$$

That is, for $L_{1}^{\prime l a b}=L_{1}^{\prime a t m}$ and $\beta_{0}^{\text {lab }}=\beta_{0}^{a t m}$ the following equality holds:

$$
\begin{aligned}
& \left\langle A_{\mathrm{pr}}^{\prime l \mathrm{ab}}\left(x_{1}^{\prime}, y_{1}^{\prime}, L_{1}^{\prime}\right) \cdot A_{\mathrm{pr}}^{\prime \mathrm{ab}}\left(x_{2}^{\prime}, y_{2}^{\prime}, L_{1}^{\prime}\right)\right\rangle= \\
= & \left\langle A_{\mathrm{pr}}^{\prime \text { atm }}\left(x_{1}^{\prime}, y_{1}^{\prime}, \quad L_{1}^{\prime}\right) \cdot A_{\mathrm{pr}}^{\prime a t m}\left[x_{2}^{\prime}, y_{2}^{\prime}, \quad L_{1}^{\prime}\right]\right\rangle
\end{aligned}
$$

The similarity relations for the average characteristics of the processes are derived by the Monte Carlo method. A series of realizations of the random field $A_{p r}^{\prime}\left(x_{1}^{\prime}, y_{1}^{\prime}, L_{1}^{\prime}\right)$ which possess the given coherence function and appropriate statistics is simulated.

These realizations serve as the initial conditions for solving the system of equations (which describes the propagation of the radiation in the active medium of the mirror and the propagation of the corrected beam in the atmosphere) written in terms of the dimensionless variables (13). This system together with its associated boundary conditions comprises Eqs. (16)-(18) and conditions (23)-(25), in which $T^{\prime}=\frac{\tilde{\varepsilon} k_{1}^{2} a_{0}^{2}}{\varepsilon_{1}}$ and the remaining notation is the same as in Eq. (26). When $N^{\text {lab }}=1$ and $K_{g}{ }^{1 a b}=1$ and conditions (28) are fulfilled, the equality

$$
A_{c}^{\prime a \mathrm{ab}}\left(x^{\prime}, y^{\prime}, 0\right)=C A_{c}^{\prime a t \mathrm{~m}}\left(x^{\prime}, y^{\prime}, 0\right)
$$

where $C$ is some constant, will be satisfied for every realization. Since this equality holds for individual realizations, the average characteristics of the corrected beams agree to within a constant factor. Thus, conditions (28) and the equality $\beta_{0}^{l a b}=\beta_{0}^{a t m}$ must be satisfied in order to adequately model the processes of correction using the MBSS-mirror under laboratory conditions, where is defined by Eq. (33).


FIG. 2. Block diagram of the experimental setup.
6. A diagram of the laboratory setup is shown in Fig. 2. A single-mode probe beam consisting of the biharmonic of a neodymium laser 1 (with wavelength
$\lambda=0.53 \mu \mathrm{~m}$, energy up to 0.2 J , pulse duration at half-maximum 40 ns , beam diameter 5 mm , and nearly diffraction-limited beam divergence 0.26 mrad ) passes through the cell 2 filled with the distorting medium. It is focused by the lens 3 into the cell 4 filled with carbon tetrachloride $\left(\mathrm{CCl}_{4}\right)$, in which MBSS is excited. The radiation reflected from the PC-MBSS-mirror then passes again through the distorting medium 2. It is then analyzed. The total beam energy $W_{1}$ after reflection from the mirror was measured with a calorimeter 5. The energy of the radiation reflected from the MBSS-mirror and passed through the distorting medium $W_{2}$ is measured at the nearly diffraction-limited angle $\theta=0.3 \mathrm{mrad}$ with the calorimeter 6 outfitted with a diaphragm 7. The energy ratio $W_{2} / W_{1}$ is a measure of the accuracy of compensation for distortions of the radiation.


FIG. 3. Calculated (solid curve) and experimental (dashed curve) values of the correction parameter $W_{2} / W_{1}$ as a function of the parameters $R(a)$ and $\beta_{0}(b)$.

In our modeling of the compensation for thermal distortions ${ }^{5}$ the parameters of the distorting medium and radiation had the following values: $L_{1}^{l a b}=30$ $\mathrm{cm}, L_{4}^{l a b}=20 \mathrm{~cm}, z_{f}^{l a b}=12 \mathrm{~cm}, \rho^{l a b}=0.8 \mathrm{~g} / \mathrm{cm}^{2}$, $C_{p}=2.4 \mathrm{~J} / \mathrm{g} \cdot \operatorname{deg}, \varepsilon_{1}^{l a b}=1.85, \varepsilon_{2}^{l a b}=2.1(\mathrm{di}-$ electric constant of the active medium), $d \tilde{\varepsilon}^{l a b} / d T=$ $1.1 \cdot 10^{-3} \mathrm{deg}^{-1} V^{\text {lab }}=0.33 \mathrm{~cm} / \mathrm{s}, \alpha=0.04 \mathrm{~cm}^{-1}$, $G_{l a b} \approx 0.0058 \mathrm{~cm} / \mathrm{MW}, 0<I_{t}^{l a b}<1 \mathrm{~W} / \mathrm{cm}^{2}$, and $a_{0}=0.25 \mathrm{~cm}$.

For the radiation of a chemical DF-laser ( $\lambda^{a t m}=3.8 \mu \mathrm{~m}$ ), whose distortions are compensated by means of a PC-MBSS-mirror ${ }^{9}$ based on compressed (22 atm) $\mathrm{SF}_{6}\left(G^{a t m} \approx 3.5 \cdot 10^{-2} \mathrm{~cm} / \mathrm{MW}\right)$, the above
parameters correspond to the following parameters of the atmosphere and beam: $L_{1}^{a t m}=7 \cdot 10^{5} \mathrm{~cm}$,
$V^{a t m}=10^{3} \mathrm{~cm} / \mathrm{s}, a^{a t m}=100 \mathrm{~cm}$, $\alpha_{a t m}=2.26 \cdot 10^{-6} \mathrm{~cm}^{-1}, I_{p r}^{a t m}=0.4 \mathrm{~W} / \mathrm{cm}^{2}$, $K_{g}^{a t m}=10^{3}, 0<I_{t}^{a t m}<12 \mathrm{~W} / \mathrm{cm}^{2}$, and $N^{a t m}=400$.

Experimental (curve 1) and theoretical (curve 2) dependences of the accuracy of correction $W_{2} / W_{1}$ vs the dimensionless parameter $R$ for $L_{1}^{\prime}=3.8 \cdot 10^{-3}$ are plotted in Fig 3a. Curve 2 of this figure was obtained by numerical solution of system of equations (15)-(20) with boundary conditions (21)-(25). It can be seen from the figure that the experimental data agree with the theoretical results within the measurement error. The correction accuracy decreases as the thermal distortions defined by $R$ increase.

In the modeling of the compensation for the turbulent distortions in Ref. 4 the parameters of the distorting and active media and the radiation were the following: $L_{1}^{l a b}=34 \mathrm{~cm}, L_{4}^{l a b}=20 \mathrm{~cm}, z_{f}^{l a b}=12 \mathrm{~cm}$, $G_{l a b} \approx 0.0058 \mathrm{~cm} / \mathrm{MW}, a_{0}^{l a b}=0.25 \mathrm{~cm}$, and $0<C_{\varepsilon}^{2 l a b} \leq 10^{-7} \mathrm{~cm}^{-2 / 3}$. For a beam of the biharmonic of a neodymium laser, with radius $a_{0}^{a t m}=0.25 \mathrm{~cm}$, whose distortions are compensated by means of a $\mathrm{CCl}_{4}$-based PC-MBSS-mirror, these parameters correspond to the following parameters of the atmosphere and beam: $L_{1}^{a t m}=3 \mathrm{~km}, \quad L_{4}^{a t m}=L_{4}^{l a b}$, $z_{f}^{a t m}=z_{f}^{l a b}, \quad 0<0<C_{\mathrm{s}}^{2 a t m}<7.6 \cdot 10^{-15} \mathrm{~cm}^{-2 / 3}$, $I_{p r}^{a t m}=38 \mathrm{~W} / \mathrm{cm}^{2}, K_{g}^{a t m}=1$, and $N^{a t m}=100$.

Figure 3b shows the experimental (curve 1) and theoretical (curve 2) dependences of the average correction accuracy $W_{2} / W_{1}$ on the dimensionless parameter $\beta_{0}$ for $L_{1}^{\prime}=3.8 \cdot 10^{-3}$. Curve 2 in this figure was obtained by multiple computer solution of system (15)-(18) with conditions (22)-(25) for different realizations of the random field of the normalized dielectric constant. It can be seen from the figure that within the measurement accuracy the experimental data agree with the theoretical results. The
accuracy of correction using the MBSS-mirror decreases as turbulent distortions of the beam increase.
7. In Fig. 3 the decrease in the correction accuracy with increase of the beam distortions is caused by errors of the MBSS-based PC of the focused beams. The quality of compensation for the distortions also deteriorates as a result of the effect of nonlinear phenomena on the conjugate beam, in particular, Raman scattering. The system of equations which describes the Raman scattering of the conjugate beam in the atmosphere has the form
$\left[2 i k_{1} \frac{\partial}{\partial z}+\Delta_{\perp}+k_{1}^{2} \frac{\tilde{\varepsilon}}{\varepsilon_{1}}\right] A_{\mathrm{c}}=i k_{1} \delta \frac{\omega_{1}}{\omega_{\mathrm{st}}}\left|A_{\mathrm{st}}\right|^{2} A_{\mathrm{c}}$,
$\left[2 i k_{\mathrm{st}} \frac{\partial}{\partial z}+\Delta_{\perp}+k_{\mathrm{st}}^{2} \tilde{\varepsilon}_{1}\right] A_{\mathrm{st}}=-i k_{\mathrm{st}} g\left|A_{\mathrm{c}}\right|^{2} A_{\mathrm{st}}$,
where $A_{\mathrm{St}}$ and $k_{\mathrm{St}}$ are the complex amplitude and wave number of the Stokes wave, $\omega_{1}$ and $\omega_{\text {st }}$ are the frequencies of the conjugate and Stokes radiation, respectively, and $g$ is the gain coefficient for Raman scattering. Let us now consider the ideal operation of the PC-mirror. The boundary condition for $A_{c}$ at $z=L_{1}$ becomes

$$
\begin{equation*}
A_{c}\left(x, y, L_{1}\right)=k_{g}^{1 / 2} A_{p r}^{*}\left(x, y, L_{1}\right) . \tag{36}
\end{equation*}
$$

The system of equations for the probe beam and the conjugate beam (1), (34), (35) with boundary conditions (5) and (36) was numerically solved. The spectrum of the fluctuations $\tilde{\varepsilon}$ was taken to be of the form (32). The calculations were performed for the following values of the parameters: $\lambda=1.06 \mu \mathrm{~m}$, $L_{1}=4.5 \mathrm{~km}, \quad C_{\varepsilon}^{2}=4 \cdot 10^{-15} \mathrm{~cm}^{-2 / 3}, l_{0}=1 \mathrm{~cm}$, $a_{0}=10 \mathrm{~cm}$, and $g=2.5 \cdot 10^{-12} \mathrm{~cm} / \mathrm{W}$. The parameter $g$ was chosen for Raman scattering in the rotational transitions of the nitrogen molecule. It was assumed that $k_{1}=k_{\mathrm{St}}$. The results of the calculations are given in Figs. 4 and 5.


FIG. 4. Computed contour lines of normalized beam intensity for the probe beam at $z=0(a)$ and $z=L_{1}(b)$ and the conjugate beam in the plane $z=0$ for $\tilde{g}=20(a)$ and 80 (c).

Figure 4 depicts the contour lines of the normalized intensity. The low-intensity probe beam in the plane $z=0$ is presented in Fig. 4a, Fig. 4b shows this beam after it has passed through the turbulent medium at $z=L_{1}$ for a single realization of the random dielectric field. In these figures the intensity is scaled to the maximum intensity of the probe beam. For low intensities of the conjugate beam in the plane $z=0$ it has a form identical to that of the initial probe beam in this plane (Fig. 4a). If the intensity of the conjugate beam exceeds a certain threshold value, then this beam is not reconstructed in the plane $z=0$ (Fig. 4c). Here the intensity is scaled to the maximum intensity of the conjugate beam in the plane $z=0$ without Raman scattering.

Figure 5 shows the dependence of the correction accuracy $W_{2} / W_{1}$ on the parameter $\tilde{g}=g I_{p r} K_{g} L_{1}$, which characterizes the gain increment of the Stokes signal on a path of length $L_{1}$ in a radiation field with the intensity $K_{g} I_{p r}$. It is seen from Fig. 5 that when intensity of the corrected beam exceeds a certain threshold value ( $\tilde{g}>50$ ) the quality of correction decreases with further increase of the intensity of this beam.


FIG. 5. Computed correction accuracy as a function of the parameter $\tilde{g}$.
8. To summarize, the results of laboratory and theoretical investigations indicate that the efficiency of PC-correction of atmospheric distortions of light beams can be substantially limited by errors in phase conjugation by a real mirror and by the nonlinear phenomena that occur in the propagation of the corrected waves in the atmosphere.

## REFERENCES

1. B.Ya. Zel'dovich, N.F. Pilipetskii, and V.V. Shkunov, Phase Conjugation (Nauka, Moscow, 1985), 247 pp .
2. V.P. Lukin, Atmospheric Adaptive Optics (Nauka, Novosibirsk, 1986), 248 pp.
3. O.I. Vasil'ev, S.S. Lebedev, and L.P. Semenov, Kvantovaya Elektron, 14, No. 11, 2347-2348 (1987). 4. O.I. Vasil'ev and S.S. Lebedev, Kvantovaye Elektron. 17, No. 3, 336-338 (1990).
4. O.I. Vasil'ev and S.S. Lebedev, Opt. Atm. 3, No. 2, 150-156 (1990).
5. V.P. Kandidov, D.P. Krindach, V.V. Popov, and S.S. Chesnokov, in: Optical Wave Propagation and Adaptive Optics, Institute of Atmospheric Optics, Siberian Branch of the Academy of Sciences of the USSR, Tomsk (1988), pp. 9-12.
6. J.W. Strohbehn, ed., Laser Beam Propagation in the Atmosphere, Topics in Applied Physics, Vol. 25 (Springer, Berlin, 1978).
7. A.S. Gurvich, A.I. Kon, V.L. Mironov, and S.S. Khmelevtsov, Laser Radiation in the Turbulent Atmosphere (Nauka, Moscow, 1976), 278 pp.
8. S.D. Velikanov, Yu.V. Dolgopolov, V.V. Egorov, et al., Izv. Akad. Nauk SSSR, Ser. Fiz. 52, No. 3, 553-556 (1988).
