

## OPTICAL CHARACTERISTICS OF THE DUST TRAILS OF METEORS

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Received April 19, 1990*

*An algorithm for describing the processes of coagulation in the trails of bright meteors and for calculating of the optical characteristics of the dust trails formed by these meteors is developed. The model of quasi continuous fractionation of the meteor bodies in the atmosphere is assumed and the dependence of the coefficient of heat transfer on the mass of the body is taken into account. The initial spreading of the trail in the establishment of the thermal equilibrium with the atmosphere is treated as a linear explosion on the axis of the trail, and its consequent spreading is caused by turbulent diffusion. It is shown that the model of thermal coagulation unlike the model of Brownian coagulation is applicable to real meteor trails.*

*According to the numerical calculations there appear every day in the Earth's atmosphere about one hundred optically dense meteor dust trails which can be observed in the day time from space in the spectral range 0.24–0.26  $\mu\text{m}$  or during dusk from the Earth's surface in the spectral range 0.3–1.0  $\mu\text{m}$ .*

### INTRODUCTION

Dust particles with masses more than  $10^{-8}$  g, which enter the atmosphere from outer space are almost completely evaporated in the upper atmosphere. In so doing gaseous trails are formed in which the partial pressure of the vapors of meteoric matter ( $\text{SiO}_2$ ,  $\text{MgO}$ ,  $\text{Fe}_2\text{O}_3$ ,  $\text{CaO}$ ,  $\text{Al}_2\text{O}_3$ , etc.) is several orders of magnitude greater than the pressure of the saturated vapors of this latter at the atmospheric temperature, and, therefore, condensation-coagulation processes leading to the formation of meteor dust trails are initiated. Dusk observations show that very bright meteors (bolids) can produce thick dust trails which are visible for tens of minutes, spreading during this time up to hundreds of meters in the lateral direction and bending under the action of the altitude gradients of the wind velocity.<sup>1</sup>

Owing to the diffuse spreading of the trail, the concentration of the vapors of the meteoric matter and the smallest nuclei of the dust particles arising in the trail falls off rapidly, wherefore, the coagulation can be sufficiently efficient only at the initial stage of the meteor trail existing when the initial concentration of vapor is considerably high. In this connection, an adequate description of the processes of formation of the initial structure of the meteor gas trail and its subsequent diffuse spreading is a matter of great importance.

The dust trails of bolids generate the strongest inhomogeneities in the light scattering characteristics in the middle atmosphere at altitudes 30–100 km. In this respect, only luminous clouds (which appear sometimes within the very thin layer of the mesopause

near the polar latitudes) can compete with the dust trails. It is most probable in this case that the water vapor condensation nuclei in luminous clouds are small dust particles of meteoric origin.

Dust trails of the bolids can be observed not only from the Earth's surface during dusk when they are illuminated by the direct solar rays. At the altitudes higher than 45 km the dust trails were found to be quite visible in the UV range during the daytime from onboard spacecraft at the maximum of the absorption band of ozone (0.24 – 0.26  $\mu\text{m}$ ) against the background of the Earth's daytime atmosphere, which is very dark in this spectral range. Taking into consideration the variety of extra-atmospheric velocities and angles of entrance into the atmosphere of the meteor bodies, the dust trails may be similar to the trails of rockets due to condensation on those segments of their trajectories where the rocket engines are operated.

In this paper the stages of formation and destruction of the meteor gas-dust trails are studied, the optical characteristics of the trails are calculated, and estimates are made of the critical masses of meteor bodies with different velocities and densities and whose trails can be revealed with the help of different observational methods (laser sounding, ground-based twilight and spaceborne observations).

### THE INITIAL STAGE OF FORMATION OF THE METEOR TRAIL

Since the dust trails observed from the Earth's surface usually are due only to bright meteors generated by sufficiently large meteor bodies with starting

masses  $M_0 > 1$  g, we are primarily interested in the trails of large meteor bodies.

For these meteor bodies in the first approximation we can neglect the time of evaporation of the fragment in comparison with the time expended for the complete destruction of the meteor body after starting its quasicontinuous fractionation. Then the evaporation rate of the meteor matter can then be written as<sup>4</sup>

$$\frac{dM}{dt} = - \frac{\Lambda A M^{2/3} \rho v^3}{2 Q_g \delta_0^{2/3}}, \tag{1}$$

where  $M$  and  $v$  are the mass and velocity of the meteor body at the altitude  $h$ ,  $\rho$  is the density of the atmosphere,  $\Lambda$  is the coefficient of heat transfer,  $A$  is the shape coefficient of the meteor body,  $\delta_0$  and  $Q_g$  are the density and specific energy of fractionation of the meteor body.

In the isothermal atmosphere

$$dt = \frac{dl}{v} = \frac{dh}{v \cos z} = \frac{H dp}{\rho v \cos z}, \tag{2}$$

where  $dl$  is the segment of the path along the visible trajectory of the meteor,  $z$  is the zenith angle of the meteor radiant (the angle concluded between the velocity vector of the meteor and the plumb line), and  $H$  is the reduced altitude of the homogeneous atmosphere.

From Eqs. (1) and (2) we find

$$\frac{dM}{dp} = - \frac{\Lambda A H M^{2/3} v^2}{2 Q_g \delta_0^{2/3} \cos z}. \tag{3}$$

Integrating Eq. (3) for constant  $\Lambda$ ,  $A$ , and  $v$ , we obtain

$$M^{1/3} = M_0^{1/3} - \frac{\Lambda A H v_0^2 \rho}{6 Q_g \delta_0^{2/3} \cos z}, \tag{4}$$

where  $v_0$  is the extra-atmospheric velocity of the meteor body.

We find from Eqs. (1), (2), and (4) the mass of the evaporating matter of the meteor per unit length of the meteor path

$$\frac{dM}{dl} = \frac{M_0 \cos z}{H} \cdot \frac{\rho}{\rho_m} \left[ 1 - \frac{1}{3} \frac{\rho}{\rho_m} \right]^2, \tag{5}$$

the maximum value of  $dM/dl$

$$\left[ \frac{dM}{dl} \right]_m = \frac{4 M_0 \cos z}{9 H}, \tag{6}$$

the density of the atmosphere  $\rho_m$  at the altitude  $h_m$  at which the evaporation rate is maximum

$$\rho_m = \frac{2 Q_g M_0^{1/3} \delta_0^{2/3} \cos z}{\Lambda A H v_0^2}, \tag{7}$$

and the density of the atmosphere  $\rho_t$  at the altitude  $h_t$  where the trail terminates  $\rho_t = 3\rho_m$ .

In accordance with Eq. (5) we take

$$\Lambda = 0.03 + 0.97 \exp(-0.25 M_0), \tag{8}$$

where  $M_0$  is measured in grams.

The meteor trails with masses  $M_0 > 1$  g start to spread explosively because of the almost instantaneous release of the large amount of kinetic energy of the evaporated meteoric matter along the axis of the trail. The initial radius of the meteor trail  $R_i$  can be estimated from the condition of equality of the kinetic energy density of the thermal motion of the molecules in the unperturbed atmosphere

$$\frac{v_0^2}{2} \frac{dM}{dl} = \pi R_i^2 C_a k T, \tag{9}$$

where  $C_a$  is the concentration of molecules in the atmosphere,  $T$  is the atmospheric temperature, and  $k$  is Boltzmann's constant.

From Eqs. (5) and (8) we obtain

$$\frac{M v_0^2 \cos z}{2\pi C_a k T H} \frac{\rho}{\rho_m} \left[ 1 - \frac{1}{3} \frac{\rho}{\rho_m} \right]^2. \tag{10}$$

Subsequent spreading of the trail occurs under the effect of turbulence. Since the sizes of the particles (as well as their masses) in this stage are still at the molecular level, we use Richardson's law to describe this spreading<sup>7</sup>

$$K(l) = \frac{1}{6} \cdot \frac{d}{dt} l_*^2(t) = \alpha \bar{\epsilon}^{-1/3} l_*^{4/3} \tag{11}$$

where  $l_*(t)$  is the effective diameter of the cloud, which is related to the radius of two-dimensional cross section  $R_0$  by the expression

$$R_0 = \frac{l_*}{\sqrt{6}}, \tag{12}$$

$\alpha$  is the constant in Richardson's law, whose value lies within the limits 0.1–0.3 according to studies performed by other authors (we will take  $\alpha = 0.2$ ),  $\bar{\epsilon}$  is the average value of the rate of dissipation of the turbulent energy. From Eqs. (11) and (12) we can derive an equation for  $R_0^2(t)$ :

$$\frac{dR_0^2}{dt} = \alpha \bar{\epsilon}^{-1/3} 6^{2/3} R_0^{4/3} \approx 3\alpha \bar{\epsilon}^{-1/3} (R_0^2)^{2/3}, \tag{13}$$

i.e.,

$$R_0^2(t) = (R_i^{2/3} + \alpha \bar{\epsilon}^{-1/3} t)^3, \tag{14}$$

where the time  $t$  is counted from the moment of termination of the formation the initial radius  $R_i$  of the meteor gas trail.

### COAGULATIONAL EVOLUTION OF THE PARTICLE MASS SPECTRUM

The natural quantization of particle masses at the microscopic level is determined by the average mass of the single molecule of meteoric matter  $m_1 \approx 10^{-22}$  g. Let  $C_n$  be the concentration of  $n$ -dimensionals, i.e., particles with masses  $m_n = n \cdot m_1$ . The spectral evolution of the particle masses can be described by the well-known coagulation equation<sup>8</sup>

$$\frac{dC_n}{dt} = \frac{1}{2} \sum_{j=1}^{n-1} K_{j,n-j} C_j C_{n-j} - C_n \sum_{j=1}^{\infty} K_{j,n} C_j$$

$$C_n(t=0) = C_{0n}, \quad n = 1, 2, 3, \dots, \infty, \quad (15)$$

where  $C_{0n}$  is the initial distribution and  $K_{ij}$  is the coagulation coefficient.

In the meteor trails at first very small particles are generated. Their concentration is quite high (about  $10^{11}$ – $10^{13}$ ) and the inequality  $K_n \ll 1$  holds true for them. Therefore, the Brownian model of coagulation caused by thermal motion is applicable and the coefficient of coagulation can be written as

$$K_{i,j}^T = \pi(r_i + r_j)^2 \left[ \frac{8kT}{\pi} \left[ \frac{1}{m_i} + \frac{1}{m_j} \right] \right]^{1/2}, \quad (16)$$

where  $r_i$  and  $r_j$  are the radii of the coagulated particles, which are assumed to be spherical, i. e.,

$$m_i = \frac{4}{3} \pi r_i^3 \delta, \quad m_j = \frac{4}{3} \pi r_j^3 \delta. \quad (17)$$

Here  $\delta$  is the density of the particle material.

However, as the coagulation develops in the trails of very bright bolids, quite large particles can also appear. To these particles it is necessary to apply the Brownian model of coagulation with coefficient of coagulation ( $K_n \gg 1$ )

$$K_{i,j}^\delta = 4\pi(r_i + r_j)(D_i + D_j), \quad (18)$$

where  $D_i$  and  $D_j$  are the diffusion coefficients of the particles.

We can us find the limiting value of the radius  $r_0$  of the particles for which the coefficients of the Brownian and thermal coagulation are identical, i.e.,  $K_{i,j}^T = K_{i,j}^\delta$ . To simplify the calculation, we take  $r_i = r_j$  and  $m_i = m_j$ . Then

$$K_{i,j}^T = 4\sqrt{2} \pi r_i^2 \left[ \frac{8kT}{\pi m_i} \right]^{1/2} \sim r_i^{1/2}, \quad (19)$$

$$K_{i,j}^\delta = 16\pi r_i D_i = 16\pi \frac{0.274}{C_a r_i} \left[ \frac{kT}{2\pi m_a} \right]^{1/2} \sim r_i^{-1}, \quad (20)$$

where  $m_a$  is the mass of an air molecule and  $C_a$  is the concentration of the air molecules Equation (20) employs the following expression for the coefficient of diffusion:

$$D_i = \frac{0.274}{C_a r_i} \left[ \frac{kT}{2\pi m_a} \right]^{1/2} \quad (21)$$

Equating  $K_{i,j}^T$  to  $K_{i,j}^\delta$  we obtain from Eqs. (19) and (20)

$$r_0^{2/3} = \frac{0.4\delta^{1/2}}{C_a m_a^{1/3}} \quad (22)$$

Substituting the numerical values  $m_a = 4.8 \cdot 10^{-23}$  g and  $\delta = 2.5$  g/cm<sup>3</sup> (for stone meteor bodies) into Eq. (22). we derive

$$r_0 = \frac{4.4}{(10^{-10} C_a)^{2/3}}, \quad (23)$$

where  $C_a$  is in cm<sup>-3</sup>.

Equation (23) for the altitude interval of interest 30–70 km yields

$$\text{for } h = 30 \text{ km, } r_0 = 0.4 \text{ } \mu\text{m}, \quad (24)$$

$$\text{for } h = 70 \text{ km, } r_0 = 14 \text{ } \mu\text{m}.$$

The limiting value of the particle radius  $r'_0$  at which we transfer over from the model (16) to (18) can be estimated under the following condition: to change the direction of the velocity vector by the angle  $\pi/2$  a particle with mass  $m_j$  must undergo about  $m_j/m_a$  collisions with molecules of air. Regarding the particle as almost immobile in comparison with the molecules of air and neglecting the radius of the molecule in comparison with  $r_j$ , we can find the time interval  $\Delta t$  in which the particle undergoes  $m_j/m_a$  collisions with the molecules of air,

$$\Delta t = \frac{m_j}{\pi r_j^2 \bar{v}_a C_a m_a} \quad (25)$$

where

$$\bar{v}_a = \left[ \frac{8kT}{\pi m_a} \right]^{1/2} \quad (26)$$

is the average thermal velocity of the molecules in the atmosphere. In this time the particle travels a distance

$$\bar{v}_j \Delta t = \left[ \frac{8kT}{\pi m_j} \right]^{1/2} \Delta t. \quad (27)$$

In order that the particle diffuses a distance  $r_i$  the condition  $\bar{v}_j \Delta t < r_i$  must be satisfied. If  $r_i = r_j$  and  $\bar{v}_j \Delta t = r_i$ , we find a limiting value of the radius of the particles for  $\delta = 2.5$  g/cm<sup>3</sup> from Eqs. (25)–(27)

$$r'_0 = \frac{7}{(10^{-10} C_a)^{2/3}}, \quad (28)$$

where  $C_a$  is in  $\text{cm}^{-3}$ , which is close to relation (23).

If  $r_i \gg r'_0$  the Brownian coagulation model with coagulation coefficient (18) is applicable, and if  $r_i \ll r'_0$  the model with coagulation coefficient (16), and for  $r_i \approx r'_0$  interpolation between Eqs. (16) and (18) must be employed.

Only model (16) is applicable for the meteor trails at altitudes  $h > 30$  km, in which nothing but very fine particles with  $r_i \leq 0.1 \mu\text{m}$  are generated. Equations (19), (20), and (24) show that the use of the model (18) will result in a very large overestimation of the efficiency of coagulation in the meteor trails. This is the error that was committed in Ref. 8.

**SYSTEM OF GENERALIZED COAGULATION EQUATIONS**

To make the system (15) practically solvable by numerical methods using modern computers, it is necessary to "generalize" it. Let

$$C_k = \sum_{n=2^{k-1}}^{2^k-1} C_n,$$

i.e.,  $C_k$  is the concentration of the particles whose masses lie in the interval  $2^{k-1}m_1 \leq m \leq 2^k m_1$ . Let  $\tilde{m}_k$  be the characteristic mass and  $\tilde{r}_k$  be the characteristic radius of the particles In this interval, i.e.,

$$\begin{aligned} \tilde{m}_k &= \frac{2^{k-1}m_1 + (2^{k-1} + 1)m_1 + \dots + (2^{k-1} - 1)m_1}{2^{k-1}} \\ &= \left[ 3 \cdot 2^{k-2} - \frac{1}{2} \right] m_1, \end{aligned} \tag{29}$$

and  $\tilde{r}_k$  is related to  $\tilde{m}_k$  by the relation

$$\tilde{m}_k = \frac{4}{3} \pi \tilde{r}_k^3 \delta. \tag{30}$$

The concentration  $C_n$  is assumed to vary smoothly in the  $k$ th interval (i.e., when  $2^{k-1} \leq n \leq 2^k - 1$ ) and the coefficient of coagulation  $K_{i,j}$  varies smoothly as  $i$  runs through the  $l$ th interval, and  $j$  through the  $k$ th interval. Then, employing the relation

$$C_n \approx 2^{-k+1} C_k \tag{31}$$

and introducing the ensemble-averaged coagulation coefficient

$$\tilde{K}_{1,j} = \pi (\tilde{r}_1 + \tilde{r}_j)^2 \left[ \frac{8kT}{\pi} \left[ \frac{1}{\tilde{m}_1} + \frac{1}{\tilde{m}_j} \right] \right]^{1/2}, \tag{32}$$

we obtain from Eq. (15) a system of generalized coagulation equations for  $C_k$  ( $k = 1, 2, 3, \dots$ ):

$$\frac{dC_k}{dt} = \tilde{P}_k - C_k \tilde{L}_k, \quad C_k(t=0) = C_{k0} = \sum_{n=2^{k-1}}^{2^k-1} C_{0n}, \tag{33}$$

where

$$\tilde{P}_k = \frac{1}{2} \sum_{j=1}^k \sum_{l=1}^{\infty} \tilde{K}_{1,j} f_l(k, l, j) C_l C_j, \tag{34}$$

$$\tilde{L}_{1,j} = \sum_{j=1}^{\infty} \tilde{K}_{k,j} C_j. \tag{35}$$

Here

$$f_l(k, l, j) = \begin{cases} 0, & \text{for } j = 1, 2, \dots, k-2, \\ 3 \cdot 2^{1-k} - 2^{1-k} & \text{for } j = k-1, \\ & l = 1, 2, \dots, k-2, \\ 1 - 3 \cdot 2^{1-k-1} + 2^k, & \text{for } j = k; \end{cases} \tag{36}$$

$$f_l(k, k-1, j) = \begin{cases} 3 \cdot 2^{j-k} - 2^{1-k} & \text{for } j = 1, 2, \\ & \dots, k-2, \\ 1, & \text{for } j = k-1, \\ 2^{-2} + 2^{-k}, & \text{for } j = k; \end{cases} \tag{37}$$

$$\begin{aligned} f_l(k, k, j) &= \\ &= \begin{cases} 0, & \text{for } j = k, \\ 1 + 3 \cdot 2^{j-k-1} + 2^{-k} & \text{for } j = 1, 2, \dots, k-1. \end{cases} \end{aligned} \tag{38}$$

Taking Eqs. (36)–(38) into account, we obtain

$$\begin{aligned} \tilde{P}_1 &= 0, \quad \tilde{P}_2 = \frac{1}{2} \left[ \tilde{K}_{1,1} C_1^2 - \tilde{K}_{1,2} C_1 C_2 \right], \\ \tilde{P}_k &= \frac{1}{2} \left[ \tilde{K}_{k-1,k-1} C_{k-1}^2 + \tilde{K}_{k-1,k} C_{k-1} C_k \times \right. \\ &\times \left. \left[ \frac{1}{2} + 2^{1-k} \right] \right] + \sum_{l=1}^{k-2} \left[ \tilde{K}_{1,k-1} C_l C_{k-1} \times \right. \\ &\times \left. \left[ 3 - 2^{1-k} - 2^{1-k} \right] + \tilde{K}_{1,k} C_l C_k \times \right. \\ &\times \left. \left[ 1 - 3 \cdot 2^{1-k-1} + 2^{-k} \right] \right]. \end{aligned} \tag{39}$$

System (33) is nonlinear. Therefore, to solve it, we employ an iterative scheme based on the finite-difference approximation. Transformation from  $C_k(t)$  to  $C_k(t + \Delta t)$  is performed according to the scheme

$$\frac{C_k^{(s+1)} - C_k^{(s)}(t)}{\Delta t} = \tilde{P}_k^{(s)} - C_k^{(s)} \tilde{L}_k^{(s)}, \quad C_k^{(0)} = C_k^{(0)}(t) \tag{40}$$

where  $s$  is the iterative parameter, the tilde ( $\sim$ ) atop  $\tilde{P}_k$  and  $\tilde{L}_k$  indicates that in calculating their values according to Eqs. (34) and (35)  $C_k^{(s)}$  is to be used

$$\begin{aligned} P_k^{(s)} &= \frac{1}{2} \sum_{j=1}^k \sum_{l=1}^k \tilde{K}_{1,j} f_l(k, l, j) C_1^{(s)} C_j^{(s)}, \\ L_k^{(s)} &= \sum_{j=1}^{\infty} \tilde{K}_{k,j} C_j^{(s)}. \end{aligned} \tag{41}$$

We continue the iterative procedure (40) until the increment

$$\max_k \frac{|C_k^{(s+1)} - C_k^{(s)}|}{|C_k^{(s)}|}$$

becomes less than some prescribed value. At this point we are forced to truncate the spectrum at the corresponding value of  $k$ .

It is necessary to take into account that the concentration of all of the particles in the trail decreases. This is due to the spreading of the trail. At each moment of time  $t$  the particles are assumed to be uniformly distributed within a cylinder of radius  $R_0(t)$  and their concentration approaches zero at the distances  $R > R_0(t)$  away from the axis of this cylinder. The value of  $R_0$  is found with the help of Eq. (14). After using procedure (40) to perform the coagulative stage of the transformation from  $C_k(t)$  into  $C_k(t + \Delta t)$ , we obtain

$$C_k(t + \Delta t) := \frac{R_0^2(t)}{R_0^2(t + \Delta t)} C_k(t), \tag{42}$$

and again all of the particles are assumed to be uniformly distributed within the cylinder of the new radius  $R_0(t + \Delta t)$ .

### NUMERICAL REALIZATION) OF THE COAGULATIVE STAGE

Referring to system of equations (33) we transform to the dimensionless time

$$\tilde{t} = t \tilde{K}_{1,1} C_{01} = t K_{1,1} C_{01} \tag{43}$$

and the dimensionless concentrations

$$\tilde{C}_k(\tilde{t}) = C_k(t) / C_{01} = C_k \left[ \frac{\tilde{t}}{\tilde{K}_{1,1} C_{01}} \right] / C_{01}. \tag{44}$$

Using Eqs. (33)–(35) and (43)–(44), we obtain a system of equations for  $C_k$ :

$$\begin{aligned} \frac{d\tilde{C}_k}{d\tilde{t}} &= \frac{1}{2} \sum_{j=1}^k \sum_{l=1}^k f_l(k, l, j) \tilde{K}_{1,j} C_1 \tilde{C}_j - \\ &- \tilde{C}_k \sum_{j=1}^{\infty} \tilde{K}_{k,j} \tilde{C}_j, \end{aligned}$$

$$\tilde{C}_1(t = 0) = 1, \quad \tilde{C}_2(t = 0) = \frac{C_{02}}{C_{01}}, \dots,$$

$$\tilde{C}_k(t = 0) = \frac{C_{0k}}{C_{01}},$$

where

$$\tilde{K}_{1,j} \approx \frac{1}{4\sqrt{2}} \left[ \frac{\tilde{r}_1}{\tilde{r}_1} + \frac{\tilde{r}_j}{\tilde{r}_1} \right] \left[ \frac{\tilde{m}_1}{\tilde{m}_1} + \frac{\tilde{m}_j}{\tilde{m}_j} \right]. \tag{46}$$

Since initially the meteor trail consists of only molecules of evaporated meteoric matter, we set  $C_{02} = C_{03} = \dots = 0$ . Subsequently the solution of system (45) becomes universal, i.e., after solving Eq. (45) once, with the help of the equation

$$C_k(t) = C_{01} \tilde{C}_k(t K_{1,1} C_{01}), \tag{47}$$

which is a consequence of Eq. (44), we find the solution of Eq. (33).

### THE PARAMETERS OF THE METEORIC BODIES AND OF THE SMALL DUST PARTICLES IN THE TRAIL

In accordance with Ref. 5 let us consider the three most characteristic types of meteoric bodies: ferrous, ordinary, and carbonaceous chondrites CI, in which  $\delta_0 = 7.7 \text{ g/cm}^3$  and  $Q_g = 1.3 \cdot 10^{10} \text{ erg/g}$ ,  $\delta_0 = 3.5 \text{ g/cm}^3$  and  $Q_g = 1 \cdot 10^{10} \text{ erg/g}$ , and  $\delta_0 = 2 \text{ g/cm}^3$  and  $Q_g = 0.4 \cdot 10^{10} \text{ erg/g}$ , respectively.

The density of the small dust particles in the trails of ferrous meteoric bodies is assumed to be  $\delta = 7.8 \text{ g/cm}^3$ , and the refractive index is assumed to be  $m = 1.28 - 1.37i$  (Ref. 3). As any stone meteorite consists mainly of  $\text{SiO}_2$ , we take  $m = 1.5$  for all types of stone meteorites.

The volume scattering coefficient of radiation with wavelength  $\lambda$  in the trail can be written in the form

$$\sigma_a = \sum_{k=1}^{\infty} K(\tilde{r}_k, m, \lambda) \pi \tilde{r}_k^2 C_k, \tag{48}$$

where  $K(\tilde{r}_k, m, \lambda)$  is the scattering efficiency of the dust particles.

Since only very small dust particles are generated in the meteor trails for which

$$X_k = \frac{2\pi \tilde{r}_k}{\lambda} < 1, \tag{49}$$

one can use the Rayleigh approximation<sup>3</sup> to calculate their scattering efficiency

$$K(\tilde{r}_k, m, \lambda) = \frac{8}{3} X_k^4 R_1 \left[ \left( \frac{m^2 - 1}{m^2 + 1} \right)^2 \right]. \tag{50}$$

Numerical values of  $K(\tilde{r}_k, m, \lambda)$  for quartz and ferrous particles are given in Ref. 3.

TABLE I. Mean radii  $\tilde{r}_k$ , of quartz particles with different  $k$  ( $k$ -dimensionals).

$k$	1	2	3	4	5	6	7	8	9	10
$\tilde{r}_k, \mu\text{m}$	0.00023	0.00031	0.00040	0.00050	0.00065	0.00080	0.0010	0.0013	0.0017	0.0020
$k$	11	12	13	14	15	16	17	18	19	20
$\tilde{r}_k, \mu\text{m}$	0.0026	0.0031	0.0040	0.0050	0.0066	0.0084	0.010	0.013	0.017	0.020
$k$	21	22	23	24	25	26	27	28	29	30
$\tilde{r}_k, \mu\text{m}$	0.027	0.033	0.042	0.053	0.067	0.084	0.10	0.13	0.17	0.20

TABLE II. Temporal behavior of  $R_0$  and  $s$  in meteor trails generated by a meteoric body with  $M_0 = 100$  g,  $\delta_0 = 3.5$  g/cm<sup>3</sup>, and  $\cos z = 0.6$  at an altitude  $h_m = 59$  km for initial velocities of 15 and 30 km/sec.

$t, s$	$R_0, m$		$s = \sigma_m / \sigma_a$		$k(C_{\text{max}})$		$k(s_{\text{max}})$		$k = k_{\text{max}}$		
	15	30	15	30	15	30	15	30	15	30	
$3 \cdot 10^{-5}$	—	1.14	—	0.005	1	—	1	—	8	—	
$2 \cdot 10^{-4}$	—	1.14	—	0.011	1	—	2	—	10	—	
0.004	0.0075	1.14	4.7	0.38	0.005	3	1	7	5	15	11
0.01	—	1.14	—	1.6	—	5	—	9	—	17	—
0.04	0.045	1.14	4.7	14	0.05	8	2	12	5	20	14
0.1	0.12	1.15	4.7	60	0.24	10	4	13	8	21	16
0.4	—	1.20	—	470	—	13	—	16	—	23	—
0.9	1.0	1.77	5.0	1300	5.9	14	9	18	12	26	20
2.3	2.5	1.5	5.3	3300	18	16	10	19	14	26	20
4.3	—	1.8	—	4300	—	16	—	20	—	26	—
11	9.2	3.0	7.0	3000	51	17	12	21	16	27	22
21	28	5.3	12	1300	39	18	14	21	17	27	24
31	38	8.0	16	650	28	18	14	22	17	27	24

RESULT OF NUMERICAL CALCULATIONS

We have obtained numerical solutions of the problem for different values of the parameters of the meteoric bodies  $M_0$ ,  $v_0$ , and  $\delta_0$ . Table I shows the average values of the radii  $\tilde{r}_k$  of quartz particles with different values of  $k$  ( $k$ -dimensionals). To illustrate the dynamics of the coagulation in the bolid trails, Table II shows the temporal change of some of the characteristics: the radius of the trail  $R_0$ , the coefficient of relative turbidity of the trail  $s = \sigma_m / \sigma_a$  (where  $\sigma_a$  is the volume coefficient of Rayleigh scattering of light due to air molecules at the given altitude), the values of  $k = k(C_{\text{max}})$  for the particles whose concentration is maximum, the values of  $k = k(s_{\text{max}})$  for those particles which give the main contribution to  $s$ , the maximum value  $k = k_{\text{max}}$  for which the particle concentration has

still not fallen to zero. Table II presents the results calculated for the trails of two bolid.

One can see from Table II that the efficiency of coagulation falls off as the bolid velocity  $v_0$  increases (as a result of the growth in this case of the altitude and initial radius of the trail  $R_1$  are increased), and as the mass  $M_0$  and density  $\delta_0$  of the meteoric body decrease. For such relatively small values of  $M_0$  ( $M_0 \approx 100$  g) the coagulation in the trail practically terminates before any noticeable spreading of the trail can take place as a result of turbulent diffusion (after a time of the order of 1 sec). Only very small dust particles with radii  $\tilde{r}_k \ll 1 \mu\text{m}$  have time to form in the trail during this time. However, coagulation processes in the trails of slow dense meteoric bodies with large masses ( $M_0 > 1$  kg) go on efficiently even at the stage of diffuse spreading of the trail. Particles of radii

$\tilde{r}_k \geq 0.1 \mu\text{m}$ , which scatter the light most efficiently, can be generated in such trails but those particles occur very rarely among the primordial particles of space dust, i.e., micrometeorites.<sup>2</sup>

The dust trails of bolids with  $R_{0s} > H$  can be observed from the Earth's surface at dusk when their visible surface brightness can be more than twice the background brightness of the sky in the spectral range  $0.3\text{--}1 \mu\text{m}$  and also in daytime from a satellite in the spectral region  $0.24\text{--}0.26 \mu\text{m}$ . If  $v_0 = 11 \text{ km/s}$  and  $\cos z = 0.6$ , such trails are generated by ferrous meteoric bodies with masses  $M_0 > 10 \text{ g}$ , dense stone meteoric bodies with masses  $M_0 > 30 \text{ g}$ , and carbonaceous chondrites CI with  $M_0 > 100 \text{ g}$ .

The dust trails of slow dense meteoric bodies with masses  $M_0 > 10 \text{ kg}$  can be observed for tens of minutes, the diameter of trail during this time increases up to hundreds of meters. Generally, such trails are several tens of kilometers in length, but for almost tangential penetration of the very large meteoric bodies into the Earth's atmosphere the lengths of their dust trails can reach hundreds of kilometers.

The frequencies of formation of the observed dust trails of bolids are quite high because about 100 me-

teoric bodies with masses  $M_0 > 1 \text{ kg}$  penetrate into the Earth's atmosphere every twenty-four hours.

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