SPATIAL FILTERING OF LATERAL SHEAR INTERFEROGRAMS IN HOLOGRAPHIC INTERFEROMETRY OF A FOCUSED IMAGE

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Analysis of a lateral shear interferometer based on double-exposure recording of a hologram of a focused image of a matted screen with two successive Fourier transforms is analyzed. It is shown theoretically and experimentally that spatial filtering in the plane of the hologram enables checking a lens or objective over the field, and spatial filtering in the far diffraction zone or in the plane of the image of the pupil of the lens or objective makes it possible to record the interference pattern characterizing the phase distortions introduced in the wave illuminating the matted screen and in the reference wave by the aberrations of the optical systems forming them.

In Refs. 1 and 2 the method of differential interferometry, using diffusely scattered fields for checking both quasispherical and quasiplanar wavefronts, was implemented based on double exposure recording of lensless Fresnel and Fourier holograms. In both cases the objective speckle fields of two exposures in the plane of the photographic plate were superposed.

In this paper the formation of lateral shear interference patterns when performing spatial filtering is analyzed based on double-exposure recording of a hologram of a focused image by superposing the visual speckle fields of the two exposures in the plane of the photographic plate.

According to Fig. 1 the matted screen 1, which lies in the plane (x_1, y_1), is illuminated with a converging quasispherical wave. The radius of curvature of the wavefront is r. With the help of the lens L_1 an image of the matted screen is constructed in the plane (x_3, y_3) of the photographic plate 2, and a hologram of the focused image is recorded during the first exposure using an off-axis reference plane wave 3. It is assumed that prior to the second exposure the matted screen is displaced in the plane (x_1, y_1) along the x-axis by an amount a, while the photographic plate in the plane (x_3, y_3) is displaced in the opposite direction by an amount b.

We shall present in the Fresnel approximation, neglecting the amplitude and phase factors, which are constant in the plane, the distribution of the complex amplitudes of the fields of the two exposures in the plane of the photographic plate in the form

$$u_1(x_3, y_3) \sim \iiiint\limits_{-\infty}^{\infty} t(x_1, y_1)\exp\left[-\frac{ik}{2r}(x_1^2+y_1^2)\right] \times$$
$$\times \exp i\varphi_1(x_1, y_1)\exp\left\{\frac{ik}{2l_1}\left[(x_1-x_2)^2+\right.\right.$$
$$\left.\left.+(y_1-y_2)^2\right]\right\} + \exp\left[-\frac{ik}{2f_1}(x_2^2+y_2^2)\right]p_1(x_2, y_2) \times$$
$$\times \exp i\varphi_2(x_2, y_2)\exp\left\{\frac{ik}{2l_2}\left[(x_2-x_3)^2+\right.\right.$$
$$\left.\left.+(y_2-y_3)^2\right]\right\}dx_1dy_1dx_2dy_2; \quad (1)$$

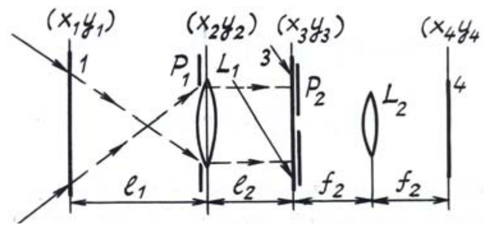

FIG. 1. The optical scheme used for recording and reconstructing a double-exposure hologram, of a focused image: 1) matted screen; 2) photographic plate-hologram; 3) reference beam; 4) recording plane of the interferogram. L_1 and L_2 are lenses; p_1 is an aperture diaphragm; and, p_2 is a filtering diaphragm.

$$u_2(x_3, y_3) \sim \iiiint\limits_{-\infty}^{\infty} t(x_1+a, y_1)\exp\left[-\frac{ik}{2r}(x_1^2+y_1^2)\right] \times$$
$$\times \exp i\varphi_1(x_1, y_1)\exp\left\{\frac{ik}{2l_1}\left[(x_1-x_2)^2+\right.\right.$$
$$\left.\left.+(y_1-y_2)^2\right]\right\}\exp\left[-\frac{ik}{2f_1}(x_2^2+y_2^2)\right]p_1(x_2, y_2) \times$$
$$\times \exp i\varphi_2(x_2, y_2)\exp\left\{\frac{ik}{2l_2}\left[(x_2-x_3-b)^2+\right.\right.$$
$$\left.\left.+(y_2-y_3)^2\right]\right\}dx_1dy_1dx_2dy_2; \quad (2)$$

where k is the wave number; $t(x_1, y_1)$ is the complex transmission amplitude of the matted screen, and is a random function of the coordinates; $\varphi_1(x_1, y_1)$ is a phase function, characterizing the distortions introduced in the wave illuminating the matted screen by wave aberrations of the optical system forming it; f_1 is the focal length of the lens L_1 with the generalized pupil function $p_1(x_2, y_2)\exp i\varphi_2(x_2, y_2)$ which takes into account the wave axial aberrations; l_1 and l_2 are, respectively, the distances between the planes (x_1, y_1), (x_2, y_2) and (x_2, y_2), (x_3, y_3).

If $\frac{1}{f_1} = \frac{1}{l_1} + \frac{1}{l_2}$ then the expressions (1) and (2) assume the form

$$u_1(x_3, y_3) \sim \exp\left[\frac{ik}{2l_2}(x_3^2 + y_3^2)\right] \times$$
$$\times \left\{t(-\mu_1 x_3, -\mu_1 y_3)\exp\left[\frac{ik(r - l_1)}{2rl_1}\mu_1^2(x_3^2 + y_3^2)\right] \times\right.$$
$$\left.\times \exp i\varphi_1(-\mu_1 x_3, -\mu_1 y_3) \otimes P_1(x_3, y_3)\right\} ; \tag{3}$$

$$u_2(x_3, y_3) \sim \left\{\exp\frac{ik}{2l_2}\left[(x_3^2 + b)^2 + y_3^2\right]\right\} \times$$
$$\times \left\{t(-\mu_1 x_3, -\mu_1 b + a, -\mu_1 y_3)\exp\left\{\frac{ik}{2rl_1}(r - l_1)\mu_1^2 \times\right.\right.$$
$$\left.\times \left[(x_3^2 + b)^2 + y_3^2\right]\right\}\exp i\varphi_1(-\mu_1 x_3 - \mu_1 b, -\mu_1 y_3) \otimes$$
$$\left.\otimes P_1(x_3, y_3)\right\} ; \tag{4}$$

where $\mu_1 = l_1/l_2$ is the scale transformation factor; $\otimes$ denotes the convolution operation;

$$P_1(x_3, y_3) = \iint\limits_{-\infty}^{\infty} p_1(x_2, y_2)\exp i\varphi_2(x_2, y_2) \times$$
$$\times \exp\left[-\frac{ik}{l_2}(x_2 x_3 + y_2 y_3)dx_2 dy_2\right]$$

denotes the Fourier transform of the generalized pupil function of the lens L_1.

In the expressions (3) and (4) the width of the function $P_1(x_3, y_3)$ determines the size δ of a visual speckle in the plane of the photographic plate. Taking into account only the diffraction limit $\sigma \approx \frac{\lambda l_2}{d}$, [4] where λ is the wavelength of the coherent light source, employed for recording and reconstructing the hologram, and d is the diameter of the pupil of the lens L_1. If within an individual speckle the change in the phase of the spherical wave in the plane (x_3, y_3) with radius of curvature $\frac{rl_1}{\mu_1^2(r - l_1)}$ does not exceed π, then for the region with diameter $D_0 \le \frac{dr}{\mu_1(r - 1)}$ in the plane of the photographic plate we shall remove from the integrand of the convolution integral the factor

$$\exp\left[\frac{ik(r - l_1)}{2rl_1}\mu_1^2(x_3^2 + y_3^2)\right],$$

which characterizes in the expressions (3) and (4) the distribution of the complex amplitude of the field of the spherical wave. In addition, assuming that the conditions $a = \mu_1 b$, $r = \frac{l_1^2}{(l_1 + l_2)}$ are satisfied, we obtain the following distributions of the complex amplitudes of the fields, corresponding to the first and second exposures, for the region in the plane of the photographic plate with diameter $D_0 \le d$:

$$u_1(x_3, y_3) \sim t(-\mu_1 x_3, -\mu_1 y_3) \times$$
$$\times \exp i\varphi_1(-\mu_1 y_3, -\mu_1 y_3) \otimes P_1(x_3, y_3); \tag{5}$$

$$u_2(x_3, y_3) \sim t(-\mu_1 x_3, -\mu_1 y_3) \times$$
$$\times \exp i\varphi_1(-\mu_1 x_3 - a, -\mu_1 y_3) \otimes \exp\left[\frac{ikbx_3}{l_2}\right] \times$$
$$\times P_1(x_3, y_3). \tag{6}$$

Let the double-exposure hologram of a focused image of the matted screen, recorded in this manner, be reconstructed by a copy of the reference wave, the distribution of whose field corresponds, for example, to the distribution for the first exposure. Then the diffraction field in the plane (x_3, y_3) assumes the form

$$u(x_3, y_3) \sim t(-\mu_1 x_3, -\mu_1 y_3)\exp i\varphi_1(-\mu_1 x_3, -\mu_1 y_3) \otimes$$
$$\otimes P_1(x_3, y_3) + \exp i\left[\varphi_3(x_3, y_3) - \varphi_3(x_3 + b, y_3)\right] \times$$
$$\times\left\{t(-\mu_1 x_3, -\mu_1 y_3)\exp i\varphi_1(-\mu_1 x_3 - a, -\mu_1 y_3) \otimes\right.$$
$$\otimes \exp\left[\frac{ikbx_3}{l_2}\right] P_1(x_3, y_3), \tag{7}$$

where $\varphi_3(x_3, y_3)$ the phase function characterizing the distortions of the reference wave owing to wave aberrations of the optical system forming it.

As follows from the expression (7), in the plane of the hologram the diffusely coherent light fields, corresponding to the first and second exposures, are superposed on one another. The identical speckles coincide if their relative slope angle $\alpha = \frac{b}{l_2}$. Therefore, based on Ref. 5 an interference pattern is located in the plane of the hologram. In addition, like in Ref. 5, according to the expression (7), the interference pattern will characterize the phase distortion produced in the wave illuminating the matted screen and the reference wave by the aberrations of the optical systems forming them.

If spatial filtering is performed in the plane of the hologram with the help of an opaque screen with a circular opening centered on the optical axis, then we shall represent the filtering operation as multiplication of the complex amplitude of the light field, described by the expression (7), by the transmission function $p_2(x_3, y_3)$ of the filter.[6] We shall perform the observation in the plane (x_4, y_4), which is the back focal plane of the conversing lens L_2 (Fig. 1). Then we shall represent the diffraction field in the recording plane 4 as a Fourier integral of the field in the filtering plane,

$$u(x_4, y_4) = F\left[p_2(x_3, y_3)u(x_3, y_3)\right], \tag{8}$$

where F is the operation of taking the Fourier transform.

We assume that within the diameter of the filtering opening the condition $\varphi_2(x_3, y_3) - \varphi_3(x_3+b, y_3) + \varphi_1(-\mu_1 x_3-a, -\mu_1 y_3) - \varphi_1(-\mu_1 x_3, -\mu_1 y_3) \leq \pi$ is satisfied. The physical meaning of this condition is that the diameter of the filtering opening does not exceed the width of the interference band for the interference pattern located in the plane of the hologram. Substituting the expression (7) into Eq. (8) we obtain

$$u(x_4, y_4) \sim \left\{F\left[\frac{kx_4}{f_2}, \frac{ky_4}{f_2}\right]p_1(-\mu_2 x_4, -\mu_2 y_4) \times \right.$$
$$\times \exp i\varphi_2(-\mu_2 x_4, -\mu_2 y_4) + F\left[\frac{kx_4}{f_2}, \frac{ky_4}{f_2}\right] \times$$
$$\left.\times\, p_1(-\mu_2 x_4 + b, -\mu_2 y_4)\exp i\varphi_2(-\mu_2 x_4 + b, -\mu_2 y_4)\right\} \otimes$$
$$\otimes\, P_2(x_4, y_4), \tag{9}$$

where f_2 is the focal length of the lens L_2; $\mu_2 = l_2/f_2$ is the scale factor of the transformation; and,

$$F\left[\frac{kx_4}{f_2}, \frac{ky_4}{f_2}\right] = \iint_{-\infty}^{\infty} t(-\mu_1 x_3, -\mu_1 y_3)\exp i\varphi_1 \times$$
$$\times (-\mu_1 x_3, -\mu_1 y_3)\exp\left[-\frac{ik}{f_2}(x_3 x_4 + y_3 y_4)\right]dx_3 dy_3,$$

$$P_2(x_4, y_4) =$$
$$= \iint_{-\infty}^{\infty} p_2(x_3, y_3)\exp\left[-\frac{ik}{f_2}(x_3 x_4 + y_3 y_4)\right]dx_3 dy_3,$$

are the Fourier transforms of the corresponding functions.

It follows from the expression (9) the diffraction field in the recording plane 4 in Fig. 1 is the superposition of two correlated speckle fields, which coincide within the region of overlapping of the images of the pupil of the lens L_1. Therefore the interference pattern characterizing the wave aberrations of the lens L_1 lies in the plane (x_4, y_4). Indeed, if the period of the function $\exp i\varphi_2(-\mu_2 x_4, -\mu_2 y_4) + \exp i\varphi_2(-\mu_2 x_4+b, -\mu_2 y_4)$ is at least an order of magnitude[7] greater than the size of a speckle, determined by the width of the function $P_2(x_4, y_4)$ then in the expression (9) we remove this function from the integrand of the convolution integral. The distribution of them illumination in the recording plane is then determined by the expression

$$I(x_4, y_4) \sim \left\{1 + \cos\left[\varphi_2(-\mu_2 x_4, -\mu_2 y_4) - \right.\right.$$
$$\left.\left. - \varphi_2(-\mu_2 x_4 + b, -\mu_2 y_4)\right]\right\} \times$$
$$\times \left|F\left[\frac{kx_4}{f_2}, \frac{ky_4}{f_2}\right] \otimes P_2(x_4, y_4)\right|^2, \tag{10}$$

which describes the speckle structure modulated by the interference fringes. The interference pattern has the form of a lateral shear interferogram in infinitely wide fringes; the interferogram characterizes the axial wave aberrations of the lens L_1.

As follows from the expressions (5) and (6), information about the phase distortions introduced into the light field by the lens L_1 falls within an individual speckle in the space of the image of the matted screen, which is formed in the plane of the photographic plate by performing two successive Fourier transforms. In contrast to Ref. 5, in a small region of the image on the optical axis the amplitude-phase distribution of the field within each individual speckle results from the diffraction of a plane wave propagating along the optical axis. Then an interference pattern characterizing the axial wave aberrations of the lens L_1 is formed when the spatial filtering presented above is performed. However for a small region of the image of the matted screen centered at a point o determined by the coordinates x_{30} off the optical axis the amplitude-phase distribution of the field within each individual speckle in this region is the result of diffraction of an off-axis plane wave, propagating at an angle $\beta = x_{30}/l_2$ to the optical axis. Then when the spatial filtering is performed an interference pattern, characterizing the combination of axial and off-axis wave aberrations of the lens L_1, is formed. This makes it obvious that spatial filtering must also be performed in the plane of the hologram for aberrational wavefronts, formed in the channels illuminating the matted glass and the reference wave. In the case when there are aberrations, however, changing the position of the center of the filtering diaphragm (for example, from a maximum of an interference fringe for an interference pattern lying in the plane of the hologram to the minimum of the interference fringe) results only in a phase change by n between the correlated speckle fields in the (x_4, y_4) plane, and this is insignificant for differential interferometry.

The range over which the lens L_1 can be checked over the field is limited by the resolution ν_0 of the holographic recording medium and its dimensions. To observe the interference pattern within the entire aperture of the lens L_1, it follows from the condition of collinear similarity[8] that the diameter D of the illuminated region of the matted screen must satisfy the condition $D \geq d/\mu_1$, if $\mu_1 \leq 1$, and $D \geq d$, if $\mu_1 \geq 1$. Then in the direction ν_x the highest value of the spatial frequency ν_m in the spectrum of the hologram is determined by the quantity $1/\lambda \cdot \sin[\theta + \mathrm{arctg} D/2l_1]$, where θ is the angle of incidence of the reference wave. To separate in space the fields reconstructed by the hologram in the zeroth and (± 1)-st orders of diffraction the condition $\theta \geq 3 \cdot \mathrm{arctg} D/2l_1$ must be satisfied. Then the range over which the lens can be checked over the field with known dimensions of the recording medium is determined from the condition $\nu_m \leq \nu_0$. The range of monitoring across the field is therefore larger than in Ref. 5, since the width of the spectrum of spatial frequencies of the hologram is determined only by the angular dimensions of the illuminated region of the matted screen and by the angle of incidence of the reference wave.

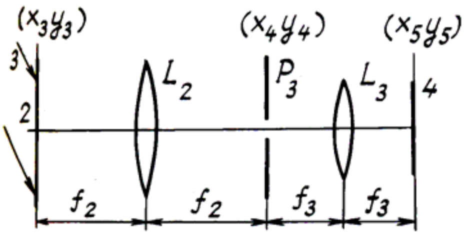

FIG. 2. Spatial filtering of the light field in the far diffraction zone using a collimating system of lenses L_2 and L_3.

To record the interference pattern located in the plane of the image of the matted screen, we shall study the behavior of the spatial filtering of the light field, reconstructed by a double-exposure hologram, on the optical axis in the Fourier plane (Fig. 2). Assuming that the diameter of the lens L_2 is greater than the dimensions of the hologram, we shall represent the diffraction field in the plane (x_4, y_4) as a Fourier integral of the field in the plane of the hologram, described by the expression (7). Then the distribution of the complex amplitude of the light field assumes the form

$$u(x_4, y_4) \sim \Big\{F\left[\frac{kx_4}{f_2}, \frac{ky_4}{f_2}\right] p_1(-\mu_2 x_4, -\mu_2 y_4) \times$$

$$\times \exp i\varphi_2(-\mu_2 x_4, -\mu_2 y_4) + \Phi_2(x_2, y_4) \otimes \Big\{F\left[\frac{kx_4}{f_2}, \frac{ky_4}{f_2}\right] \otimes$$

$$\otimes \Phi_1(x_4, y_4)\Big\} p_1(-\mu_2 x_4 + b, -\mu_2 y_4) \times$$

$$\exp i\varphi_2(-\mu_2 x_4 + b, -\mu_2 y_4)\Big\} \otimes P_2(x_4, y_4), \tag{11}$$

where

$$\Phi_1(x_4, y_4) = \iint_{-\infty}^{\infty} \exp i\left[\varphi_1(-\mu_1 x_3 + a, -\mu_1 y_3) -\right.$$

$$\left. - \varphi_1(-\mu_1 x_3, -\mu_1 y_3)\right] \exp\left[-\frac{ik}{f_2}(x_3 x_4 + y_3 y_4)\right] dx_3 dy_3,$$

$$\Phi_2(x_4, y_4) = \iint_{-\infty}^{\infty} \exp i\left[\varphi_3(-x_3 y_3) - \varphi_3(x_3 + b, y_3)\right] \times$$

$$\times \exp\left[-\frac{ik}{f_2}(x_3 x_4 + y_3 y_4)\right] dx_3 dy_3,$$

are the Fourier transforms of the corresponding functions.

If functions $\Phi_1(x_4, y_4)$ and $\Phi_2(x_4, y_4)$ are much narrower than the function $P_2(x_4, y_4)$, which determines the size of a speckle in the plane (x_4, y_4), then the expression (11) describes the distribution of two speckle fields which coincide within the region of overlapping of the images of the pupil of the lens L_1. Therefore, as pointed out above, the interference pattern characterizing the wave aberrations of the lens L_1 lies in the far diffraction zone. If an opaque screen p_3 (Fig. 2) with a circular opening, centered on the optical axis, is placed in the focal plane of the lens L_2 and the condition $\varphi_2(-\mu_2 x_4 + b, -\mu_2 y_4) - \varphi_2(-\mu_2 x_4, -\mu_2 y_4) \leq \pi$ is satisfied inside this opening (i.e., the width of the interference fringe for the interference pattern in the plane (x_4, y_4) does not exceed the diameter of the filtering opening), then the light field immediately behind this screen is described by the expression

$$u(x_4, y_4) \sim p_1(x_4, y_4)\Big\{F\left[\frac{kx_4}{f_2}, \frac{ky_4}{f_2}\right] \otimes P_2(x_4, y_4) +$$

$$+ F\left[\frac{kx_4}{f_2}, \frac{ky_4}{f_2}\right] \otimes \Phi_1(x_4, y_4) \otimes \Phi_2(x_4, y_4) \otimes P_2(x_4, y_4)\Big\} \tag{12}$$

We shall represent the diffraction field in the focal plane of the lens L_3 as a Fourier integral of the field in the plane in which the spatial filtering is performed. Then the complex amplitude of the correlation speckle fields, corresponding to the first and second exposures, in the recording plane 4 assumes the form

$$u(x_5, y_5) \sim \Big\{t(\mu_1\mu_3 x_5, \mu_1\mu_3 y_5) \times$$

$$\times \exp i\varphi_1(\mu_1\mu_3 x_5, \mu_1\mu_3 y_5) p_2(-\mu_3 x_5, -\mu_3 y_5) +$$

$$+ t(\mu_1\mu_3 x_5, \mu_1\mu_3 y_5) \exp i\varphi_1(\mu_1\mu_3 x_5 - a, \mu_1\mu_3 y_5) \times$$

$$\times \exp i\left[\varphi_3(-\mu_3 x_5, -\mu_3 y_5) - \varphi_3(-\mu_3 x_5 + b, -\mu_3 y_5)\right] \times$$

$$\times\, p_2(-\mu_3x_5,\ -\mu_3y_5)\Big\}\otimes P_3(x_5,\ y_5), \qquad (13)$$

where $\mu_3 = f_2/f_3$ is the scale factor of the transformation;

$$P_3(x_5,\ y_5)=\iint\limits_{-\infty}^{\infty} p_3(x_4,\ y_4)\exp\left[-\frac{ik}{f_2}(x_4x_5+y_4y_5)\right]dx_4dy_4$$

is the Fourier transform of the transmission function of the filter.

As follows from the expression (13), in the plane $(x_5,\ y_5)$ the speckle fields of the two exposures are superposed on one another and the identical speckles coincide. Therefore the interference pattern, characterizing the phase distortions produced in the wave illuminating the matted glass and in the reference wave by the aberrations in the optical systems forming them, lies in this plane. Thus if the period of the function $\exp i\varphi_1(\mu_1\mu_3x_5,\ \mu_1\mu_3y_5) + \exp i[\varphi_1(\mu_1\mu_3x_5 - a,\ \mu_1\mu_3y_5) + \varphi_3(-\mu_3x_5,\ -\mu_3y_5) - \varphi_3(-\mu_3x_5 + b,\ -\mu_3y_5)]$ is at least one order of magnitude greater than the size of a speckle in the recording plane 4, determined by the width of the function $P_0(x_5,\ y_5)$, then the expression (13) this function can be removed from the integrand of the convolution integral. Then the distribution of illumination in the plane $(x_5,\ y_5)$ will be determined by the expression

$$I(x_5,\ y_5)\sim\Big\{1+\cos\big[\varphi_1(\mu_1\mu_3x_5,\ \mu_1\mu_3y_5)-$$
$$-\varphi_1(\mu_1\mu_3x_5-a,\ \mu_1\mu_3y_5)-\varphi_3(-\mu_3x_5,\ -\mu_3y_5)+$$
$$+\varphi_3(-\mu_3x_5+b,\ -\mu_3y_5)\big]\Big\}\Big|t(\mu_1\mu_3x_5,\ \mu_1\mu_3y_5)\times$$
$$\times\, p_2(\mu_3x_5,\ \mu_3y_5)\otimes P_3(x_5,\ y_5)\Big|^2, \qquad (14)$$

which describes the speckle structure modulated by the interference fringes. The interference pattern has the form of a lateral shear interferogram in fringes of infinite width; this interferogram characterizes the phase distortions produced in the wave illuminating the matted glass and in the reference wave by aberrations in the optical systems forming them. In addition, the spatial extent of the interference pattern, based on the expression (14), is limited by the diameter of the pupil of the lens L_1. In order to observe it within the entire image of the matted screen we shall study the spatial filtering on the optical axis, as shown in Fig. 3.

Then we shall write the diffraction fields in the plane of the hologram, which correspond to the first and second exposure, in the form

$$u(x_3,\ y_3)\sim\exp\left[\frac{ik}{2l_2}(x_3^2+y_3^2)\right]\Big\{t(-\mu_1x_3,\ -\mu_1y_3)\times$$
$$\times\exp\left[-\frac{ik}{f_2}(x_3^2+y_3^2)\right]\exp i\varphi_1(-\mu_1x_3,\ -\mu_1y_3)\otimes$$
$$\otimes P_1(x_3,\ y_3)\Big\}; \qquad (15)$$

$$u_2(x_3,\ y_3)\sim\exp\left\{\frac{ik}{2l_2}\left[(x_3+b)^2+y_3^2\right]\right\}\times$$
$$\times\exp i\left[\varphi_3(x_3,\ y_3)-\varphi_3(x_3+b,\ y_3)\right]\Big\{t(-\mu_1x_3,\ -\mu_1y_3)\times$$
$$\times\exp\left\{-\frac{ik}{2l_2}\left[(x_3+b)^2+y_3^2\right]\right\}\times$$
$$\times\exp i\varphi_1(-\mu_1x_3-a,\ -\mu_1y_3)\otimes P_1(x_3,\ y_3)\Big\}. \qquad (16)$$

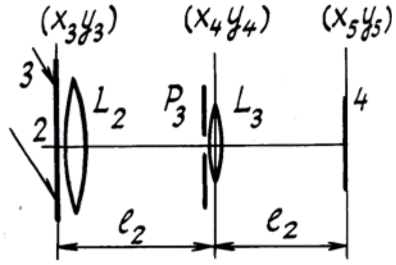

FIG. 3. Spatial filtering of the light field in the image plane of the pupil of the lens L_2.

If the diameter of the lens L_2, lying in the plane of the hologram, is greater than the size of the image of the matted screen and the lens L_2 forms an image of the pupil of the lens L_1 in the plane $(x_4,\ y_4)$, then, assuming unit magnification below in order to simplify the formulas, the diffraction field in the image plane is given by the expression

$$u(x_4,\ y_4)\sim\exp\left[\frac{ik}{2l_2}\left[(x_4^2+y_4^2)\right]\right]\times$$
$$\times\left\{F_1\left[\frac{kx_4}{l_2},\ \frac{ky_4}{l_2}\right]p_1(-x_4,\ -y_4)\exp i\varphi_2(-x_4,\ -y_4)+\right.$$
$$+\Phi_2'(x_4,\ y_4)\otimes\left\{F_1\left[\frac{kx_4}{l_2},\ \frac{ky_4}{l_2}\right]\otimes\Phi_1'(x_4,\ y_4)\right\}\times$$
$$\left.\times\, p_1(-x_4+b,\ -y_4)\exp i\varphi_2(-x_4+b,\ -y_4)\right\}, \qquad (17)$$

where

$$\left\{F_1\left[\frac{kx_4}{l_2},\ \frac{ky_4}{l_2}\right]=\iint\limits_{-\infty}^{\infty} t(-\mu_1x_3-a,\ -\mu_1y_3)\times\right.$$
$$\times\exp\left[\frac{ik}{2l_2}\left[(x_4^2+y_4^2)\right]\exp i\varphi_1(-\mu_1x_3,\ -\mu y_3)\times\right.$$
$$\times\exp\left[\frac{ik}{2l_2}\left[(x_3x_4+y_3y_4)\right]dx_3dy_3,\right.$$

$$\Phi_1'(x_4,\ y_4)=\iint\limits_{-\infty}^{\infty}\exp i\left[\varphi_1(-\mu_1x_3-a,\ -\mu y_3)-\right.$$

$$- \varphi_1(-\mu_1 x_3, -\mu_1 y_3)\Big]\exp\Big[\frac{ik}{2l_2}\Big[(x_3 x_4 + y_3 y_4)\Big]dx_3 dy_3,$$

$$\Phi_2'(x_4, y_4) = \iint\limits_{-\infty}^{\infty} \exp i\Big[\varphi_3(x_3, y_3) - \varphi_3(x_3 + b, y_3)\Big] \times$$

$$\times \exp\Big[-\frac{ik}{l_2}\Big[(x_3 x_4 + y_3 y_4)\Big]dx_3 dy_3$$

are the Fourier transforms of the corresponding functions.

Because the size of the image of the matted screen is limited in space, the light field behind the hologram will have a distribution that is characteristic of a speckle field,[9] in addition, as follows from the expression (17), the speckle fields corresponding to the first and second exposures are superposed on one another with identical speckles coinciding within the region of the overlapping of the images of the pupil of the lens L_1. Therefore the correlated speckle fields, filtered by the aperture diaphragm p_3 of the lens L_3 (Fig. 3), whose diameter does not exceed the width of an interference fringe for the interference pattern lying in the plane (x_4, y_4) and characterizing axial aberrations of the lens L_1, directly behind the aperture diaphragm are determined by the expression

$$u(x_4, y_4) \sim p_3(x_3, y_3)\exp\Big[\frac{ik}{l_2}(x_3^2 + y_4^2)\Big] \times$$

$$\times \Big\{F_1\Big[\frac{kx_4}{l_2}, \frac{ky_4}{l_2}\Big] + F_1\Big[\frac{kx_4}{l_2}, \frac{ky_4}{l_2}\Big] \otimes \Phi_1'(x_4, y_4) \otimes$$

$$\otimes \Phi_2'(x_4, y_4)\Big\}. \quad (18)$$

If it is assumed that the lens L_3 forms an image of the plane (x_3, y_3) in the plane (x_5, y_5) (to simplify the formulas we shall assume unit magnification), then the complex amplitude of the diffraction field in the recording plane 4 (Fig. 3) will, assume the form

$$u(x_5, y_5) \sim \exp\Big[\frac{ik}{2l_2}(x_5^2 + y_5^2)\Big]\Big\{t(\mu_1 x_5, \mu_1 y_5) \times$$

$$\exp\Big[-\frac{ik}{l_2}(x_5^2 + y_5^2)\Big]\exp i\varphi_1(\mu_1 x_5, \mu_1 y_5) \otimes P_3(x_5, y_5) +$$

$$+ t(\mu_1 x_5, \mu_1 y_5)\exp\Big[-\frac{ik}{2l}(x_5^2 + y_5^2)\Big] \times$$

$$\times \exp i\varphi_1(\mu_1 x_5 - a, \mu_1 y_5)\exp i\Big[\varphi_3(-x_5, -y_5) -$$

$$- \varphi_3(-x_5 + b, -y_5)\Big] \otimes P_3(x_5, y_5)\Big\}, \quad (19)$$

where

$$P_3(x_5, y_5) = \iint\limits_{-\infty}^{\infty} p_3(x_3, y_3) \times$$

$$\times \exp\Big[-\frac{ik}{l_2}(x_4 x_5 + y_4 y_5)\Big]dx_4 dy_4$$

is the Fourier transform of the transmission function of the aperture diaphragm p_3.

It follows from the expression (19) that in the plane (x_5, y_5) the speckle fields of two exposures are superposed on one another and identical speckles are incident within the region of overlapping of the images of the matted screen. Therefore the interference pattern characterizing the phase distortions of the wave illuminating the matted screen and of the reference wave lies in this plane. Therefore if the period of the function $\exp i\varphi_1(\mu_1 x_5, \mu_1 y_5) + \exp i[\varphi_1(\mu_1 x_5 - a, \mu_1 y_5) + \varphi_3(-x_5, -y_5) - \varphi_3(-x_5 + b, -y_5)$ is at least an order of magnitude greater than the size of a speckle in the recording plane 4, determined by the width of the function $P_3(x_5, y_5)$, then in the expression (19) we remove this function from the integrand of the convolution integral. Then the distribution of the illumination in the plane (x_5, y_5) is determined by the expression

$$I(x_5, y_5) \sim \Big\{1 + \cos\Big[\varphi_1(\mu_1 x_5, \mu_1 y_5) -$$

$$- \varphi_1(\mu_1 x_5 - a, \mu_1 y_5) - \varphi_3(-x_5, -y_5) +$$

$$+ \varphi_3(-x_5 + b, -y_5)\Big]\Big\}\Big|t(\mu_1 x_5, \mu_1 y_5) \times$$

$$\times \exp\Big[-\frac{ik}{2l}(x_5^2 + y_5^2)\Big] \otimes P_3(x_5, y_5)\Big|^2, \quad (20)$$

which describes the speckle structure, modulated by the interference fringes. The interference pattern has the form of a lateral shear interferogram in fringes of infinite width. This pattern characterizes the phase distortion produced in the wave illuminating the matted glass and in the reference wave within the entire image of the matted screen by aberrations in th channels forming these waves.

It is not difficult to show that when the double-exposure hologram, recorded by the method presented above, of the focused image ii reconstructed in (+1)–st order of diffraction, spatial filtering in the far diffraction zone using the collimating system of lenses L_2 and L_3 (Fig. 2) results in recording of an interference pattern whose spatial extent is limited by the diameter of the pupil of the lens L_1. To record the interference pattern within the entire image of the matted screen the lens L_3, with aperture diaphragm p_3 (Fig. 3), placed at a distance l_2 from the hologram, is sufficient by itself, because in this case there is present in the plane of the hologram a phase factor of the converging spherical wave with radius of curvature l_2.

It should also be noted that when the interference pattern characterizing the phase distortions of the wave illuminating the matted screen and of the reference wave is recorded, carrying out spatial filtering off the optical axis will result in distortion of the pattern owing to the off-axis wave aberrations of the lens L_1.

In the experiment the double-exposure holograms of the focused image of the matted screen were recorded on photographic plates of the type Mikrat-VRL, using an He-Ne laser, at the wavelength 0.63 μm. The matted screen and the photographic plate were secured on optical measuring tables from the OSK-2 optical bench. The focal lengths of the lenses L_1, 15–30 mm in diameter, were equal to 140–200 mm. As an example Fig. 4a shows a lateral shear interferogram, obtained when performing spatial filtering in the plane of the hologram on the optical axis by reconstructing a hologram with an unseparated laser beam with a diameter of ≈1 mm. The interference pattern characterizes the spherical aberration of the lens L_1 with f_1 = 140 mm and d = 20 mm with prefocal defocusing. The double-composition hologram was recorded with unit magnification for an Illuminated region on the matted screen 40 mm in size. Prior to the second exposure the matted screen and the photographic plate were shifted by 0.8 ± 0.002 mm.

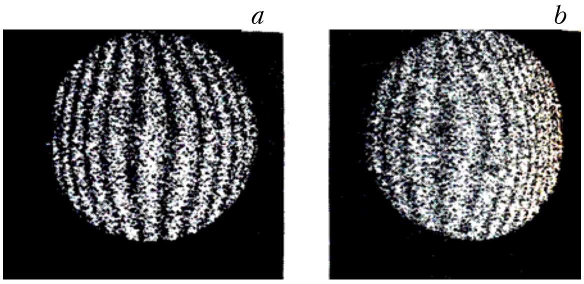

FIG. 4. Lateral shear interferogram characterizing the wave aberrations of the lens L_1 being checked. The interferogram was recorded when performing spatial filtering in the plane of the hologram: a) on the optical axis; b) off the optical axis at the edge of the image of the matted screen.

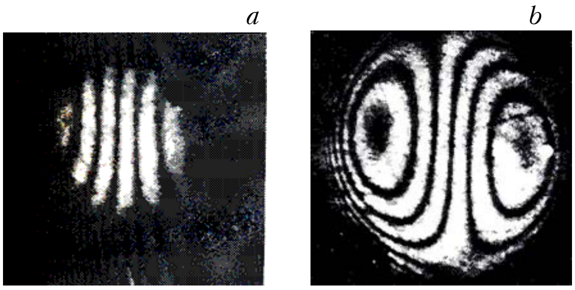

FIG. 5. Lateral shear interferogram characterizing the aberrations of the wave illuminating the matted screen and the reference wave, during spatial filtering: a) in the far diffraction zone; b) in the plane of the image of the pupil of the lens L_1.

Figure 4b shows a lateral shear interferogram obtained by performing spatial filtering in the plane of the hologram at a point corresponding to the edge of the image of the matted screen ($x_{30} \simeq$ 20 mm, y_{30} = 0). The interference pattern in this case characterizes the combination of axial and off-axis wave aberrations[10] of the lens L_1.

Figure 5a shows the form of the recorded interference pattern, located in the plane of the hologram, when performing spatial filtering on the optical axis as shown in Fig. 2. Its spatial extent corresponds to the diameter of the lens L_1, and it characterizes the wave aberrations of the optical systems forming the wave illuminating the matted screen and the reference wave. When the spatial filtering on the optical axis is performed in the manner shown in Fig. 3, however, the interference pattern shown in Fig. 5b, within the entire image of the matted screen is recorded.

In conclusion it should be noted that the double-exposure method, studied in this work, for recording a hologram of a focused image of a matted screen makes it possible to check wave aberrations of lenses and objects over the field of the image by performing a spatial filtering in the plane of the hologram. This method gives a wider range than the well-known method of Ref. 5 for the same spatial resolution of the medium in which the hologram is recorded.

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