

## SPECTRAL EXTINCTION COEFFICIENTS AND ASYMMETRY OF SCATTERING OF LIGHT BY AEROSOL WITH WEIGHTED-MEAN PARTICLE-SIZE SPECTRUM

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Received September 27, 1989*

*The optical characteristics of aerosol with a weighted-mean particle-size spectrum  $dN / d(\log r) = A(r_0^{\bar{\nu}} + |r - r_0|^{\bar{\nu}})^{-1}$  are studied.*

*It is shown that for an effective index of refraction of the particles  $m = 1.53 - 0.007i$  the spectral behavior of the extinction coefficients and the shape of the scattering phase function of the particles of such an aerosol correspond to the actually observed optical characteristics of a continental aerosol.*

*The semiempirical dependences between the optical and microstructural characteristics of the aerosol with a weighted-mean particle-size spectrum are obtained.*

Atmospheric aerosol is an extremely variable formation, in both time and space. The particle-size spectrum (the distribution of the number of particles over the sizes) can also change very strongly when the meteorological, synoptic, and other conditions change.

In Refs. 1 and 2 the weighted-mean particle-size spectrum, which can be regarded as an estimate of the mathematical expectation of the form of random functions of this type, was determined by statistical analysis of more than 250 spectra based on the results of measurements of the spectra as random functions.

The obtained spectrum, which is unlike the log-normal or  $\gamma$  distributions, was approximated well by an analytic function of the form

$$\frac{dN}{d \log r} = \frac{A(r, \nu', k)}{r_0^{\bar{\nu}} + |r - r_0|^{\bar{\nu}}},$$

where  $r_0$  is the modal radius of the particles.

In the monograph of V.E. Zuev and G.M. Krekov<sup>3</sup> it was pointed out that the obtained approximation function has the drawback of having many parameters. As indicated in Ref. 4, for optically active particles this function can be significantly simplified. With a definite approximation the coefficient  $A(r, \nu', k)$  may be assumed to be a constant  $A(r, \nu', k) \approx A$ , and the exponent  $\nu = \bar{\nu}$ . In this case the weighted-mean distribution function can be approximated by the simplest two-parameter function of the type

$$\frac{dN}{d \log r} = \frac{A}{r^{\bar{\nu}} + |r - r_0|^{\bar{\nu}}}. \quad (1)$$

For the continental aerosol in the lower troposphere. In accordance with Refs. 1 and 2,  $r_0 = 0.03 \mu\text{m}$  and  $\bar{\nu} = 3$ .

It was interesting to compare the optical properties (the form of the scattering phase function and the spectral behavior of the extinction coefficients) of such an aerosol with data from direct in situ measurements of the optical characteristics of the continental.

The forms (elongations) of the light-scattering phase function  $\gamma(\theta)$  of an aerosol were compared according to the asymmetry factor of the phase function:

$$\Gamma = \frac{\int_0^{\pi/2} \gamma(\theta) \sin \theta \, d\theta}{\int_{\pi/2}^{\pi} \gamma(\theta) \sin \theta \, d\theta}.$$

There are very few published measurements of the asymmetry factors. Moreover, most of the published values refer to the measurements integrated over the spectrum. The asymmetry factors were not determined in all cases of measurements of the phase function. Definite estimates of the actually observed factors  $\Gamma$  can nonetheless be made.

Thus the median value  $\Gamma = 8-10$  was obtained from direct optical measurements of the scattering phase function of an aerosol, performed by G.I. Gorchakov and his coworkers<sup>5</sup> near Zvenigorod. T.P. Toropova et al.<sup>6</sup> established based on the measurements of the scattering phase function in the layer of the atmosphere near the ground that the average value of the asymmetry factor  $\Gamma = 4.5-7.2$  and the maximum values  $\Gamma_{\text{max}} = 8-11.3$ . Analogous results were obtained in integral measurements (close to the results of measurements of  $\Gamma$  in the section of the spectrum  $0.5-0.7 \mu\text{m}$ ) were obtained by O.D. Barteneva, Zidentopf, Foitsik, Tsshaek, et al.

Analysis of all these data shows that the average value  $\Gamma$  and the rms deviation  $\delta\Gamma$  of the values of the asymmetry factors of the aerosol scattering phase function in the layer of the atmosphere near the ground during the summer and in the visible particle of the spectrum (0.5–0.7  $\mu\text{m}$ ) are equal to  $\Gamma = 6\text{--}7$  and  $\delta\Gamma = 1$ .

The lack of the experimental data still makes it difficult to draw statistically well-founded conclusions about the spectral behavior of the coefficients  $\Gamma(\lambda)$ . From the existing reports (for example, Ref. 6) it follows only that as the wavelength increases the asymmetry of the scattering phase function increases and decreases and its spectral behavior is sign-alternating.

Angstrom's index

$$\omega = \frac{\ln[\beta_{ex}(\lambda_1)/\beta_{ex}(\lambda_2)]}{\ln(\lambda_2/\lambda_1)}$$

was used to characterize the spectral behavior of the volume aerosol light-scattering coefficients  $\beta_{ex}$ .

A large number of measurements of Angstrom's index have now been accumulated (see, for example, Refs. 6 and 7). The spectral scattering coefficients are extremely variable; Angstrom's index can vary over wide limits ( $0 \leq \omega \leq 2.5$ ) depending on the origin (nature) of the aerosol, the type of underlying surface, the meteorological con-

ditions, the synoptic situation, the section of the spectrum, etc.

Analysis of the experimental data shows that for the layer of continental aerosol near the ground the average value  $\omega$  and the rms deviation of Angstrom's index in the visible part of the spectrum are equal to 1.3 and 0.4, respectively.

From the definition of the function (1) as the weighted-mean it follows that the optical properties of an aerosol with the particle size spectrum (1) should be closed to the average statistical properties of the continental aerosol. One problem addressed in this work was to check this result. On a more general level, it was interesting to determine how the optical properties of the aerosol, whose particles are distributed according to Eq. (1), change as the index of refraction of the particle  $m$  and the values of the parameters of the distribution  $r_0$  and  $\nu$  change.

In this connection we performed Mie calculations of the scattering phase function (the factors  $\Gamma$ ) and the volume extinction coefficients of such an aerosol  $\beta_{ex}(\lambda)$  in ten sections of the spectrum (from 0.3 to 1.06  $\mu\text{m}$ ) for 11 values of the refractive index (from  $m = 1.34$  to  $1.70 - 0.005i$ ) and for a modal parameter of the particles  $r_0$  ranging from 0.005 to 0.1  $\mu\text{m}$  and the exponent  $\nu$  ranging from 2.65 to 3.3. Particles with radii ranging from 0.01 to 20  $\mu\text{m}$  were taken into account. Mie calculations were performed for more than 140 scattering phase functions and extinction coefficients.

TABLE I

$m$	1.34 0.01	1.38— 0.0015i	1.43— 0.0015i	1.5— 0.01i	1.53— 0.007i	1.54— 0.01i	1.6— 0.005i	1.625— 0.005i	1.65— 0.005i	1.675— 0.005i	1.70— 0.005i
$\Gamma$	9.615	8.599	7.722	6.873	6.486	6.372	5.698	5.520	5.367	5.225	5.052
$\Gamma(2)$	9.574	8.645	7.727	6.872	6.486	6.410	5.792	5.589	5.401	5.228	5.067
$\delta\Gamma, \%$	-0.43	0.53	0.06	0	0	0.060	1.65	1.24	0.64	0.05	0.29

Table I gives the results of calculations of the asymmetry factors  $\Gamma$  for scattering of light by aerosol with different indices of refraction  $m = n - \kappa i$  for  $\lambda = 0.55 \mu\text{m}$ ,  $r_0 = 0.03 \mu\text{m}$ , and  $\nu = 3$ .

One can see from this table that the asymmetry factor is closest to the mean statistical value  $\Gamma = 6.5$  for the index of refraction  $m = 1.53 - 0.007i$ .

Thus it is found that the scattering phase function of an aerosol whose particles are distributed according to the distribution (1) and whose refractive index  $m = 1.53 - 0.007i$  has an asymmetry factor (on the average) that agrees adequately with the actually observed scattering phase functions of a continental aerosol. But is the refractive index  $m = 1.53 - 0.007i$  typical for a continental aerosol? According to the data of the radiation commission of IAMAP the main components of the continental aerosol with  $\lambda = 0.55 \mu\text{m}$  have precisely such values of the refractive index

( $m = 1.53 - 0.006i$  for soluble particles and  $m = 1.53 - 0.008i$  for dust).

Since for climatological investigations it is important to have data on the ratio of the fluxes of radiation scattered in the forward and backward hemispheres we tried to construct an analytical approximation of the dependence of the asymmetry factor  $\Gamma$  on the refractive index of the particles and the values of the parameters  $r_0$  and  $\nu$ .

In Ref. 9 a very simple relation was proposed between  $\Gamma$  and the real part of the refractive index  $n$  (for a Jung particle-size distribution)

$$\Gamma \approx \frac{3.6}{n - 1}$$

We obtained the following empirical dependence of the asymmetry factor  $\Gamma$  on  $m$ ,  $r_0$ , and  $\nu$  from calculations of more than 140 scattering phase func-

tions for an aerosol with a weighted-mean particle distribution:

$$\Gamma \approx \frac{a + 0.75(n - \kappa - 1)}{n - \kappa - 1} \quad (2)$$

For the case when  $r_0 = 0.03 \mu\text{m}$  and  $\nu = 3$ ,  $a = 3$ . In the general case

$$a \approx \frac{4}{\bar{\nu}^2(0.15 - |r_0 - 0.03|)} \quad (3)$$

Table I also gives the values of the factors  $\Gamma$  calculated from the formula (2) and the computational errors  $\delta\Gamma$  with the same parameters as for  $\Gamma$ .

One can see from Table I that the error in the approximation of the dependence of  $\Gamma$  on  $m$  by the formula (2) does not exceed 1.65%, and the average error is equal to 0.42%. In the general case, as the wavelength varies from 0.3 to 1.06  $\mu\text{m}$ , the modal radius varies from 0.006 to 0.1  $\mu\text{m}$ , and the distribution parameter  $\nu$  varies from 2.65 to 3.3 the average error  $\delta\Gamma$  and the rms error  $\delta\Gamma$  in the values of  $\Gamma$  determined with the help of the relations (2) and (3) are equal to 1.83 and 13.2%, respectively.

In the expressions (2) and (3) the wavelength dependence of the factor  $\Gamma$  is neglected. The calculations show that the form of the scattering phase function for the particle distribution (1) with  $r_0 \leq 0.03 \mu\text{m}$  changes comparatively little as a function of the wavelength. As one can see from the figure, as the wavelength changes from 0.3 to 1.06  $\mu\text{m}$  the asymmetry factor changes by not more than 15–20%. In the process the coefficient  $\Gamma$  can both increase and decrease, as is

observed under real conditions.<sup>8</sup> For  $r_0 \geq 0.05 \mu\text{m}$ , however, the scattering asymmetry can vary more sharply as a function of the wavelength.

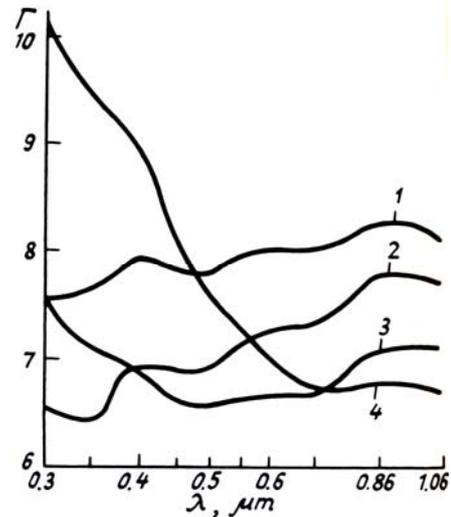


FIG. 1. The spectral behavior of the asymmetry factors of the scattering phase function of an aerosol:  $m = 1.53 - 0.007i$ ,  $\bar{\nu} = 3$ ;  $r_0 = 0.005 \mu\text{m}$  (curve 1),  $r_0 = 0.015 \mu\text{m}$  (curve 2),  $r_0 = 0.03 \mu\text{m}$  (curve 3),  $r_0 = 0.05 \mu\text{m}$  (curve 4).

It is well known that for a sufficiently wide spectrum of particle sizes, distributed according to Jung's law. Angstrom's index  $\omega$  is related with the distribution parameter  $\nu^*$  by the relation

$$\omega = \nu^* - 2.$$

TABLE II

$r_0, \mu\text{m}$	$\lambda, \mu\text{m}$	0.3	0.347	0.40	0.45	0.50	0.55	0.60	0.694	0.860	$\bar{\omega}, \delta\omega$
		-0.347	-0.4	-0.45	-0.50	-0.55	-0.60	-0.694	-0.860	-1.06	
0.005	$\omega$	1.051	0.998	1.219	0.973	1.005	1.019	1.047	0.929	0.982	1.025
	$\delta\omega, \%$	-1.8	3.4	-15.4	6.0	2.6	1.2	1.0	11.1	5.0	1.5
0.015	$\omega$	1.313	1.233	1.412	1.163	1.180	1.175	1.178	1.045	1.069	1.196
	$\delta\omega, \%$	-11.7	-5.9	-17.8	-0.3	-1.7	-1.3	-1.5	11.0	8.5	-2.3
0.03	$\omega$	1.404	1.378	1.570	1.342	1.379	1.366	1.351	1.214	1.205	1.350
	$\delta\omega, \%$	-4.7	-2.9	-14.8	-2.7	-2.9	-2.0	-9.2	10.3	11.1	-2.0

For the average value of the parameter  $\nu^* = 3$  the average value of Angstrom's parameter  $\omega$  must be equal to 1. In reality, as we have already pointed out, the average value  $\omega = 1.3$ . It was thus interesting to determine the value of the index  $\omega$  for particles distributed according to the distribution (1). Within the framework of this work we calculated more than 140 values of the index  $\omega$  in

quite wide ranges of the parameters  $\lambda$ ,  $m$ ,  $r_0$ , and  $\nu$ . This enabled us to approximate the dependence of the index  $\omega$  on  $m$ ,  $r_0$  and  $\nu$ :

$$\omega \approx \bar{\nu} \left[ 1 + \frac{n - \kappa - 1}{2} (1.15\bar{\nu} - 2) \left( \frac{r_0}{r_0 + 0.025} \right)^2 \right] - 2. \quad (4)$$

Table II gives the results of the Mie calculations of the indices  $\omega$  for  $m = 1.53 - 0.007i$  and  $v = 3$  for three values of the modal particle sizes  $r_0 = 0.005, 0.015, \text{ and } 0.03 \mu\text{m}$ . The table also gives the errors  $\delta\omega$  in the calculation of the values of  $\omega$  from the formula (4).

As follows from this table, the average error  $\delta\omega$  and the rms error  $\delta\omega$  in the values of the index  $\omega$  calculated from the formula (4) are equal to  $-0.95\%$  and  $8.48\%$ , respectively. For  $r_0 = 0.03 \mu\text{m}$  the average value of Angstrom's index is  $\omega = 1.35$ , which is very close to the mean statistical value  $\omega = 1.3$ .

### CONCLUSIONS

1. The semiempirical dependences of the optical characteristics of aerosol on its microstructure obtained in this work (the relations (2)–(4)) could find application in the interpretation of the results of statistical investigations of the optical properties of atmospheric aerosol under different climatic conditions.

2. The close agreement of the values obtained for the asymmetry factor  $\Gamma$  and Angstrom's index  $\omega$  with the mean statistical values of these characteristics apparently confirms the fact that in developing statistically well-founded optical models of the atmosphere the distribution (1) can be used as a weighted-mean particle-size distribution of the continental aerosol.

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