RECONSTRUCTION OF THE SPATIAL SPECTRAL CHARACTERISTICS OF THE OCEAN-ATMOSPHERE BOUNDARY FROM OPTICAL SOUNDING DATA

A.B. Murynin and E.A. Lupyan

Institute of Space Research of the Academy of Sciences of the USSR, Moscow

Received September 27, 1989

It is shown that isolines of the slopes of the sea surface are fractal curves and can be characterized by a fractional dimension, and the fractional dimension is determined by the exponent of the rises in the surface. It is shown that for a monotonic "slope-brightness" transfer function the isolines of the image can be related to the isolines of the surface. The dependence of the fractal dimension of the isolines on the spectral index p is obtained for a two-dimensional isotropic random field. The inverse function, which makes it possible to evaluate the spectrum of the surface from the fractal dimension of the isolines, is constructed.

Introduction. In many problems of monitoring the state of the sea surface it is necessary to reconstruct the spectra of the heights and slopes of the waves from remote-sensing data. in particular, from the optical images. Although there are many works devoted to this problem (see Refs. 1–4) and the references presented there), efficient methods for solving it have been developed only in the linear approximation, i.e., when the brightness of the image is assumed to be proportional to the slopes of the sea surface. In many cases, however, the transfer function which transforms the field of the heights and slopes of the surface into the brightness field of the image is strongly nonlinear (for example, in regions where a large number of bright spots are observed in images obtained both in sunlight and with point artificial Illumination[5]). In addition, it is impossible to obtain an analytical relation between the spectrum of the optical image and the spectrum of the waves, so that in order to reconstruct the different spectral characteristics of the waves special methods must be developed for solving the problem. In particular, it is possible to search for characteristics of the image that, on the one hand, do not depend on the specific form of the transfer function and, on the other hand, are related with the characteristics of the wave spectrum.

In this paper we shall show that when the transfer function is invertible, such a characteristic for wave fields having a power-law spectrum is the fractal dimension of the isolines of the brightness of their images. (An explanation of the concept of fractal dimension can be found in Refs. 6 and 8).

Formulation of the problem. The process of the formation of an optical image of the sea surface is described well on the basis of the model of mirror reflection and refraction.[4] In this model the brightness of an element of the image $B(x, y)$ is a function of the gradient (slope) of an element of the sea surface α and can be represented In the form

$$B(x, y) = B_0(x, y) + B_1(x', y', \alpha(x', y')),$$

where x', y' are the coordinates of points on the sea surface which in the image have the coordinates x and y, and $B_0(x, y)$ is the smooth trend in the brightness (slowly varying function of the coordinates), which is determined by the conditions of illumination and is independent of the sea surface. There are a number of well-developed methods for taking into account B_0, so that in what follows we shall analyze only the transfer function B_1, describing the modulation of the brightness fields by the slopes of the surface. The form of the function B_1 is quite complicated, since in the general case it depends on a large number of parameters: the height of the sun above the horizon or the source of illumination, the presence of atmospheric haze, the form of the cloud cover, and other optical characteristics of the atmosphere.

In many cases (for example, when surveyed with a quite narrow-field objective), B_1 can be represented in the form $B_1 = B_1(\alpha(x', y'))$. Let B_1 be a monotonic function of α. In practice this situation is often encountered, for example, under conditions of solar illumination in regions not encompassing the central part of the solar path, or under conditions of diffuse elimination when imaging at small angles.

In this case there exists a one-to-one correspondence between the isolines of brightness and the isolines of the field of slopes $\alpha(x', y')$. Indeed, assume that we are studying isolines of the field of slopes, defined by the equation $\alpha(x, y) = h_\alpha$. The relative number of times n that the field of slopes exceeds the threshold h_α is determined uniquely by the magnitude of this

0235-6880/90/03 264–07 $02.00

threshold and by the distribution of the slopes $\omega(\alpha)$ on the sea surface from the relation

$$n = \int_{h_\alpha}^{\infty} \omega(\alpha)\, d\alpha.$$

In the case when B_1 is a monotonic function of α, and will also be uniquely determined by the relation

$$n = \int_{h_\alpha}^{\infty} \omega_b(b) db, \tag{2}$$

where $h_b = B_1(h_\alpha)$, and $\omega_b(b)$ is the distribution of the brightness In the image. The boundaries of the overshoots above threshold, i.e., the isolines of the brightness in the image, will also be determined uniquely. This corresponds to the fact that the isoline of the field of slopes on the surface will be uniquely mapped into the isoline of the field of brightness in the image. In the case when the imaging is done without spatial distortions, the form of a brightness isoline will not depend on B_1, but rather it will be determined only by the form of the isoline in the field of slopes. Thus in the case when the transfer function determining the formation of images is monotonic the form of the isoline of the image brightness is an invariant. For this reason if it is possible to find a characteristic of this form associated with the spectrum of slopes (and hence the height also) of the sea surface, then the spectral characteristics of the waves can be reconstructed from the image, even when the specific form of B_1 is not known.

The spectrum of the waves in many cases is described well by a power law of the form $\Psi(k_x, k_y) \sim k^{-p}$, where $k = \sqrt{k_x^2 + k_y^2}$. An important problem of the diagnostics and study of waves is the determination of the exponent p. For this reason we shall try to find a characteristic of the form of a isoline that is related to p. We note that the random fields having a power—law spectrum in some range of scales exhibits self-similarity.[6,7] Such fields, in many cases, are fractal fields and they can be described by some fractal (fractional) dimension; in addition, this dimension is directly related to p (see Refs. 7 and 9).

The fractal dimension, being one variant of measure, characterizes precisely the form of the surface and therefore also the form of the isolines of the surface. In addition, the fractal dimension of the isolines can be easily evaluated from the images (see Refs. 6–9).

Once the fractal dimension D of the brightness isolines of the images of the sea surface has been determined, we actually determine the fractal dimension D_α of the isolines of the field of slopes of the surface $\alpha(x, y)$. There then arises the problem of investigating the relation between D_α and p. Once we solve this problem we can use the invariant (for a quite large class of transfer functions) characteristic of the field of slopes of the sea surface to analyze the wave spectrum.

As pointed out in Ref. 7, the fractal dimension of an isoline depends on the correlation function of the surface, which in turn determines the spectrum of the surface. In the general case this relation is quite complicated and depends strongly on the level on which the isoline is drawn. The analytical relation between p and D_α cannot be derived analytically in general form for fields of different dimension. (Some relations can be derived. If a is a function of one variable[7,8]). For this reason, numerical experiments must be performed.

Numerical modeling of fields with different spectra and determination of the fractal dimension of their isolines. To determine the dependence of D on p in the two-dimensional case we performed numerical modeling of the fields of slopes for a surface with a given spectrum, separated isolines on them, evaluated their fractal dimension, and constructed the dependence $D = D(p, n)$, where p is the spectral index and n is the relative number of overshoots determining the level at which the isoline is drawn. As we saw above, the parameter n is invariant with respect to the transfer functions studied. For this reason, it is convenient to use precisely these functions to analyze the results and not the value of the threshold h_α, which must be recalculated for the threshold value of the brightness in the image h_b, which depends on the specific form of the transfer function B_1.

In modeling the problem the surface is regarded as a random Gaussian field of rises of the surface $z = \xi(x, y)$, which has a power-law spectrum $\Psi \sim k^{-p}$. The modeling method is describedin detail in Refs. 9 and 10. After the field was synthesized, the relative area of the overshoots n was fixed and then the threshold for constructing abinary representation h was determined. Then a binary representation of the image was constructed with the threshold h and the boundaries of the overshoots were identified using the algorithm presented in Ref. 9.

To calculate the fractal dimension of an isoline the image is covered with squares with sides of length $r = t, 2t, 3t, \ldots$, where t is the image discretization step, and the dependence of the number of elements of the cover N, i.e., the number of squares containing elements of the image whose brightness is equal to 1, on r is constructed. The fractal dimension 0 is defined as the parameter of the linear approximation of the dependence of $\ln N(r)$ on $\ln r$. The value obtained for the fractal dimension D is compared with the index of the power-law spectrum p fixed in the synthesis of the field of slopes.

Repeating the procedures enumerated above for different values of the index p and the parameter n makes it possible to construct the dependence $D(p, n)$.

Results of numerical modeling. The dependence of the fractal dimension of isolines on the indices of the power—law spectra of surface rises was studied using the scheme described above. In the modeling process the random fields of slopes were synthesized for surfaces the field of whose rises has a spectrum of the form k^{-p}. The value of the spectral index p varied from 10/3 to 4.5. The range was chosen so as to cover the characteristic situations arising at different stages of wave development.

Examples of the synthesized fields of slopes and their binary images with a threshold $n = 0.5$ and the corresponding isolines are given in Fig. 1.

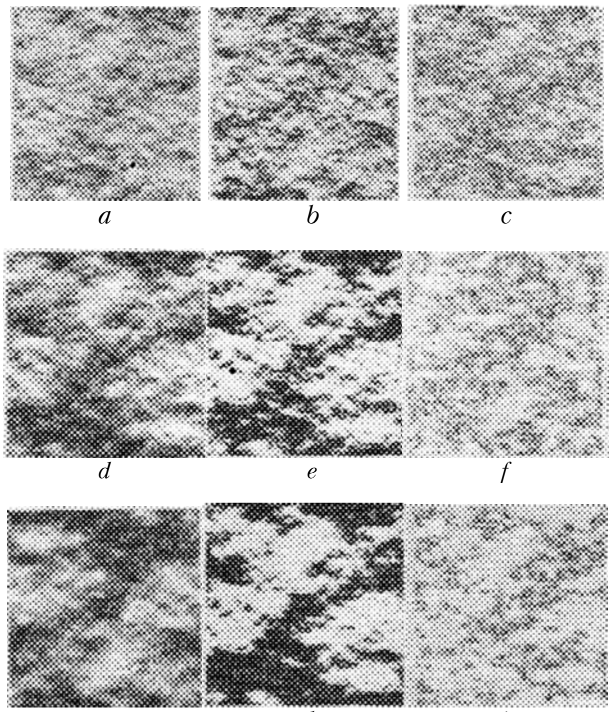

FIG. 1

These examples are given for three different values of p:

1) $p = 10/3$ (Figs. 1*a*, *b*, *c*) — the spectrum with this index is formed owing to nonlinear interaction of waves;[11]

2) $p = 4$ (Figs. 1*d*, *e*, *f*) — the equilibrium Philips spectrum of developed wind-driven waves;[12] and,

3) $p = 4.5$ (Figs. 1*g*, *h*, and *i*) — the spectrum of ripples.[13]

We can see that the structures of the images presented differ strongly, even visually. Thus, for example, for $p = 4$, 5, when the slope field $\alpha(x, y)$ is a quite smooth function, most overshoots of the field above a fixed threshold are clearly delineated two-dimensional regions and their boundaries (isolines of the slopes) are smooth one-dimensional lines. The fractal dimension of the isolines with $p = 4.5$, as will be shown below, is indeed close to unity.

As the index of the power-law spectrum p decreases the irregularity of the isolines increases. For $p = 10/3$, in some sections of the image even the visual difference between the binary brightness field and its isolines is lost. This is in good agreement with the fact that for such a spectral index, as the number of harmonics taken into account in the formation of the field approaches infinity, discontinuities of the field $\alpha(x, y)$ will arise.[7] The dimension of the separate overshoots in this case will approach zero.

We shall show that the isolines are indeed fractal. For this we shall construct the dependence of the number of elements of the cover N on the size of the squares r (radius of the cover). The dependences of $\log(N)$ on r/t are presented in Fig. 2 for different values of p. It is obvious that these dependences can be approximated by straight lines with good accuracy. The slope of the straight lines will determine the fractal dimension of the isolines (see Ref. 9). For comparison, Fig. 2 also shows the dependence $\log(N)$ for a field whose spectrum has a sharp maximum at the wavelength Λ. This dependence has a quite sharp break at $r = \Lambda$, and it cannot be regarded as linear, and correspondingly it is also meaningless to talk about the fractal dimension of an isoline of such a field on the scales studied.

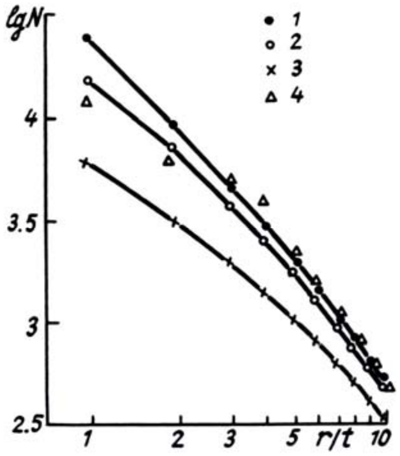

FIG. 2. The number of elements of the cover N versus the radius of the cover r (p = 10/3 (1), 4 (2), 4.5 (3); a non-power-law spectrum with a local maximum (4)).

Figure 2 also shows that the straight lines corresponding to different values of the index of the power—law spectrum have different slopes. This means that the fractal dimension of isolines of such fields is different. The dependences of the fractal dimension on n and p are given in Fig. 3.

Figure 3*a* shows the dependence of the fractal dimension of the isolines D on n for different spectra. The sharpest changes occur for small values of p. This is explained qualitatively by the fact that for slowly decaying spectra a large number of independent harmonics participate in the formation of the field at practically all levels n. The structure of the isolines with $n \sim 0.5$ is determined by uncorrelated harmonics, so that these isolines will be highly irregular lines with a large fractal dimension. For small n the structure of the isolines depends on the character of the overshoots of the field, which are also determined by a large number of harmonics, but harmonics which are now correlated.[14] As we have already mentioned, the dimension of these overshoots can approach zero. The dimension of the isolines will also, correspondingly, approach zero.

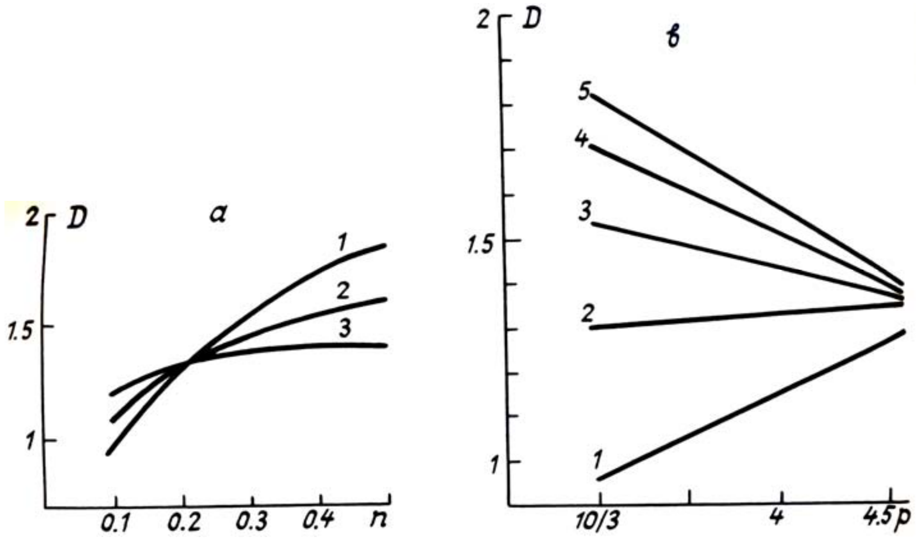

FIG. 3. The dependence of the fractal dimension D on the relative number of overshoots n (p = 10/3 (1), 4 (2), 4.5 (3)) (a) and the index of the spectrum of rises p (n = 0.1 (1), 0.2 (2), 0.3 (3), 0.4 (4), and 0.5 (5)) (b).

In the case of large values of v the dependence of the fractal dimension on n is very weak. In this case the behavior of the field is determined by the long-wavelength components in a quite large range of values n. As n decreases the dimension nonetheless starts to decrease and. irrespective of the spectral index, as n approaches 0 D approaches zero. The difference of D from unity for $n \approx 0.5$ in the case of smooth fields, for example, for $p = 4.5$, is evidently connected with the fact that we are regarding the fractal dimension as some intermediate asymptotic property in a limited range of scales.[6]

Figure 3*b* shows the dependence of D on p for different values of n. We can see that their behavior depends strongly on the level n. From the viewpoint of the problem of separating sections of the surface with different indices p a binary representation with $n = 0.5$ is optimal. First, for this value of n, as one can see from Fig. 3*b*, the dependence $D(p)$ is steepest. Second, the largest number of image elements belongs to the isoline under study for $n = 0.5$, and this decreases (compared with other values of n) the error in the determination of the dimension of the isoline D and therefore also the spectral index p associated with it.

For practical applications it is very important that the dependence of D on p in the range studied is linear (Fig. 3*b*). This makes it easy to solve the problem of finding p for a known fractal dimension of the isoline D. The dependence of D on p can be represented in the form

$$D(p) = \beta_0(n) + \beta_1(n)(p - 3). \tag{3}$$

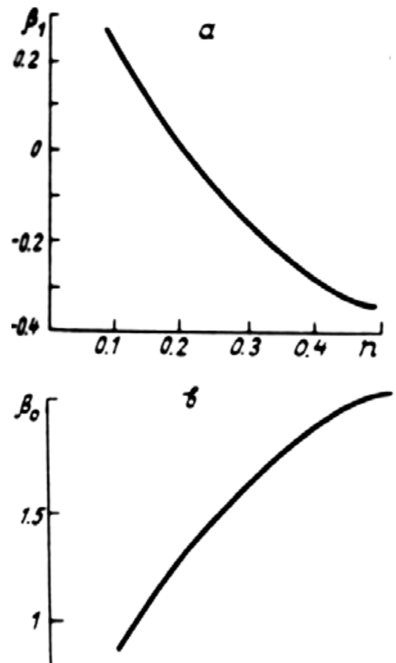

FIG. 4. The weighting functions $\beta_1(n)$ (a) and $\beta_0(n)$ (b).

The weighting functions $\beta_1(n)$ and $\beta_0(n)$ were found based on numerical modeling. They are presented in Fig. 4. They can be easily used to solve the problem formulated, i.e., for finding the spectral index from the experimentally determined fractal dimension.

Applications. The results presented above make it possible to implement the proposed method for analyzing optical images of the sea surface. The analysis is performed by eliminating the trend in the brightness, analyzing the histogram of the brightness and calculating the threshold h_b corresponding to a given value of the relative area of the overshoots n, separating the boundary, and calculating the fractal dimension D and the index p from the formula (3).

The method of estimating the wave spectrum, based on analysis of the fractal dimension of the brightness isolines, has a number of advantages over other methods for solving this problem. The main advantage is the invariance with respect to the conditions of illumination. The method is applicable under any conditions giving a monotonic relation between the brightness of the elements of the sea surface and their slopes in some direction.

Such monotonicity is observed in most situations with natural illumination of the sea surface. The possibility of estimating the spectra of the surface from the bright-spot componentof the brightness field is most significant. This makes it possible to use both the bright spots in the region of the solar track and the images formed by systems of artificial pulsed illumination, intended for operating in the dark.[5] The main limitation of this method is that it is oriented toward analyzing power-law spectra. True, with its help it is possible to determine that the spectrum under study is not a power-law spectrum, but the structure of the non-power-law spectrum cannot be studied.

Investigations of the correlation between the different harmonics of surface waves with the help of analysis of the dependence of the fractal dimension of an isoline on its level could become an important development of the method. This is especially important, because there now exist many problems in which the interaction between different harmonics is important.

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