Specific features of continuous wavelet transform of pulse signals

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The possibilities of continuous wavelet transform (CWT) in applications to the analysis of model-based pulse signals are shown. It is found that the wavelet analysis can reveal the local signal features and investigate a change of the spectral composition of the pulse signal. The wavelet transforms of the pulse signals in the cases of equilibrium and distortion of activity of regulating systems have been studied. It is shown that, when the activity of regulating systems is distorted, the shape of the pulse signal and, hence, the structure of the obtained wavelet transform change. A new method of identification of informative points of pulse signal is suggested based on the continuous wavelet transform.

Introduction

Pulse signal of radial artery carries information on many physiological processes in the organism. Extraction of this information requires a detailed analysis of the components of pulse signal. For instance, the phase (time) analysis of heart activity, based on calculations of durations of the phases of cardiac cycle and on analysis of their time interrelations, requires a correct determination of informative (characteristic) points, being the extremes of the pulse signal. The pulse wave is a non-stationary, quasi-periodic process whose frequency composition and main parameters are time-dependent and can vary within the observation time interval. Therefore, methods of signal spectral analysis, such as Fourier transform can be insufficient in the study of signal frequency components.

Recently, for analysis of nonstationary signals, a new method, namely, wavelet analysis, is actively used. Its ability to obtain the frequency dependence of the amplitude opens new possibilities in thorough analysis of frequency and time structure of nonstationary signals such as pulse wave. This is especially important for localization of characteristic regions of pulse wave aimed at subsequent identification of the characteristic points on the small-amplitude wave segments, the accuracy of determination of which is critical for correct diagnosis.

This paper is aimed at estimating the possibility and expedience of the wavelet analysis technique application to the problem of automation of the process of identification of functional deviations of the organism through the identification of characteristic points of pulse signal.

Continuous wavelet transform

Wavelet transform of one-dimensional signal consists in its expansion into the basis, constructed from a function (wavelet), possessing certain properties, by means of the scaled changes and shifts. Each function of this basis characterizes both a certain space (time) frequency and its localization in physical space (time).

Wavelets should possess the following properties: (1) localization in time and frequency spaces, (2) possession of zero mean, and (3) finiteness.[1–3] It is possible to cover the whole space through a shift of differently contracted variants of a single basis wavelet function:

$$\psi_{a,b}(t) = \psi\left(\frac{t}{a} - \frac{b}{a}\right) = \psi\left(\frac{t-b}{a}\right), \ a,b \in R,$$

where a is the scaling factor responsible for the wavelet width; and b is the shift parameter, determining the wavelet position on the axis.

Each wavelet has characteristic features in time and frequency spaces; therefore, using different wavelets, it is sometimes possible to identify and highlight more fully some or other properties of the analyzed signal and increase the accuracy of the calculations.

The continuous wavelet transform is carried out through convolution of the analyzed function $f(t)$ with two-parameter wavelet function $\psi_{a,b}$, calculated from the formula[1–3]:

$$W(a,b) = \langle f, \psi_{a,b} \rangle = \int_{-\infty}^{\infty} a^{-1/2}\, \overline{\psi\left(a^{-1}(t-b)\right)}\, f(t)\mathrm{d}t,$$

where the overbar denotes complex conjugate; and $f(t) \in L^2(R)$.

The result of the wavelet transform of one-dimensional series is a two-dimensional array of amplitudes of wavelet transform, i.e., the values of the coefficients $W(a, b)$; containing a combined information on the analyzing wavelet and analyzed signal. The spectrum $W(a, b)$ of one-dimensional signal represents a surface in three-dimensional space, that can be visualized in different ways. Generally, these surfaces are represented in terms of the pattern of wavelet coefficients (wavelet spectrum), being the values of the wavelet coefficients in the scale-time plane, whose values determine the color of the corresponding region of the wavelet pattern.

Application of continuous wavelet analysis to model pulse signals

Consider a sinusoid with a frequency of 1 Hz, removable discontinuity point at $t = 5$ s, and a jump at the point $t = 8$ s (Fig. 1*a*); the wavelet spectrum of its coefficients is presented in Fig. 1*b*.

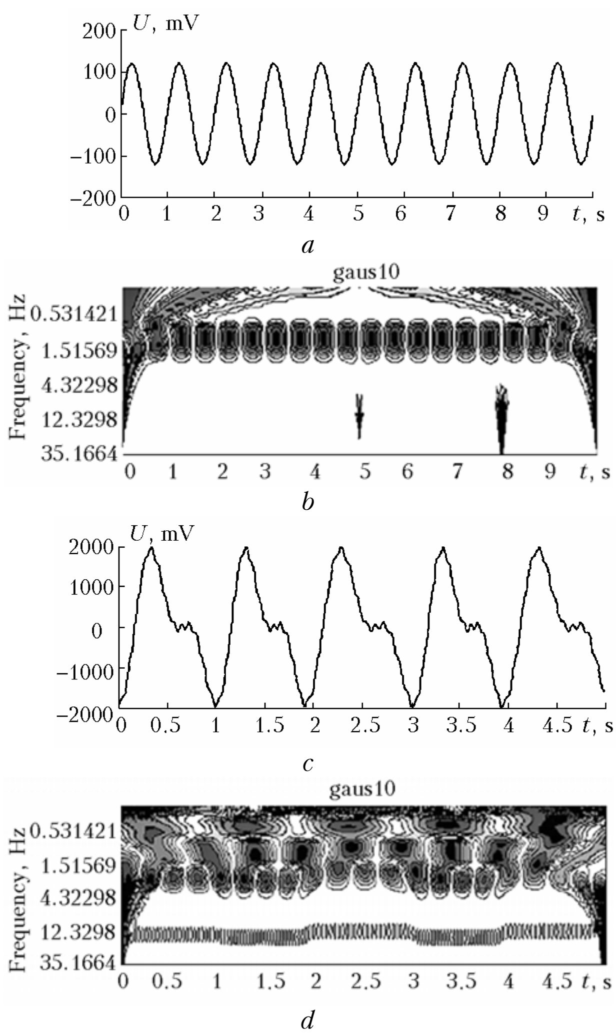

Fig. 1. Plot of sinusoid with local singularities (*a*); wavelet spectrum of sinusoid (*b*); plot of model-based pulse signal (*c*); wavelet spectrum of model signal (*d*).

Large values of wavelet coefficients correspond to signal extremes, i.e., concentration of dark regions in the wavelet spectrum; while small values of wavelet coefficients correspond to zero crossings (concentration of light regions). The vertical lines originating from the corresponding points correspond to the distortions of signal smoothness, i.e., artefacts and fluctuations (jumps and spikes) and discontinuity points. The sharper is the singularity, the easier is its discrimination against the background, and the larger are the values of wavelet coefficients. The bright white and black horizontal strip on the wavelet spectrum, corresponding to alternation of signal extremes, means the presence of periodic component, i.e., the frequency of the considered 1 Hz sinusoid, in the model signal. A certain visual complication of wavelet spectrum toward its edges, i.e., the edge discontinuities, is attributed to finiteness of signal existence in time.

The wavelet transform can reveal the changes in the signal spectral composition. Let us demonstrate this property by the example of model-based pulse signal (Fig. 1*c*), consisting of three harmonics with frequencies 1, 2, and 13 Hz, variable in period. On the wavelet spectrum of the model signal (Fig. 1*d*), the horizontal strips, corresponding to the frequencies of the signal harmonics, change their positions as functions of period variations. For the given signal, the Fourier spectrum would contain three harmonics, providing no information on their changes (evolution).

Thus, the wavelet analysis opens new possibilities in detailed analysis of nonstationary signals such as pulse wave. It allows one:

– to identify the local features of pulse signals, i.e., the characteristic points, artefacts, and fluctuations including small-amplitude ones. Large values of wavelet coefficients are located near the local singularities of the pulse signal, while small values are in the regions, where the function is locally smooth;

– to study variations of the spectral composition of pulse signal and its characteristics, not reflected in Fourier spectra.

Examples of application of continuous wavelet transform to pulse signals

Let us study the application of the continuous wavelet transform to the real pulse waves. Consider the results of continuous wavelet transform of pulse signals of a healthy human and human with functional deviations (Fig. 2*a*,*c*).

On the wavelet spectrum of human with functional deviations (Fig. 2*d*), the appearance of additional local features in the range 6–23 Hz is discerned, which is absent in the wavelet spectrum of the healthy human (Fig. 2*b*).

Let us analyze in more detail the low-frequency composition of the considered signals in the region 0.01 Hz and at higher frequencies; in particular,

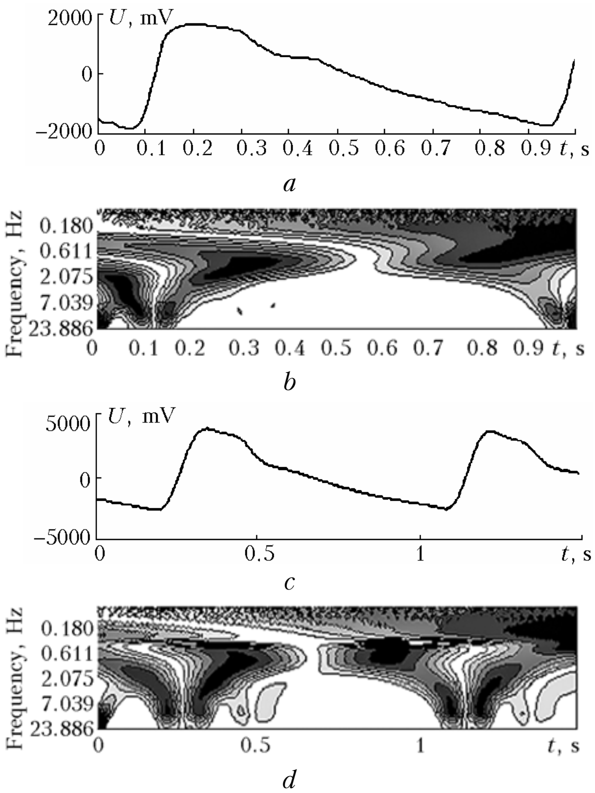

Fig. 2. A fragment of pulse signal of a healthy human (*a*); its wavelet spectrum (*b*); a fragment of pulse signal of human with functional deviations (*c*); its wavelet spectrum (*d*).

long, 100-s records of pulse waves. Below, we present the results of wavelet transform of the considered pulse signals (Fig. 3*a*, *d*).

On wavelet spectrum of the healthy human (Fig. 3*b*), the strip, corresponding to the main signal frequency, shows strong variations; whereas on wavelet spectrum of human with functional deviations (Fig. 3*e*), the corresponding strip is almost uniform, suggesting the attenuation of the heart rate variability. There appear additional features on wavelet spectrum (Fig. 3*e*) in the region 0.022 Hz, which are periodic in character, suggesting the presence of an additional harmonic, which disappears in the region $t = 35$ s. A change in the signal frequency composition in the region 0.02–1 Hz is also noticeable on the global wavelet spectrum of signal energy distribution over frequency E (Fig. 3*c*,*f*).

Thus, the considered examples make it possible to conclude that the patients, differing in health, have different structures of wavelet spectra of pulse waves: there takes place alteration of the signal in the low-frequency region, as well as alteration in terms of local features in the region 6–23 Hz of wavelet pattern.

The appearance of additional local features in wavelet spectrum is associated with a change of the pulse signal shape, which is reflected in parameters of the characteristic points of the signal. The determination of the characteristic points of the signal is the key problem in determination of the functional human state.

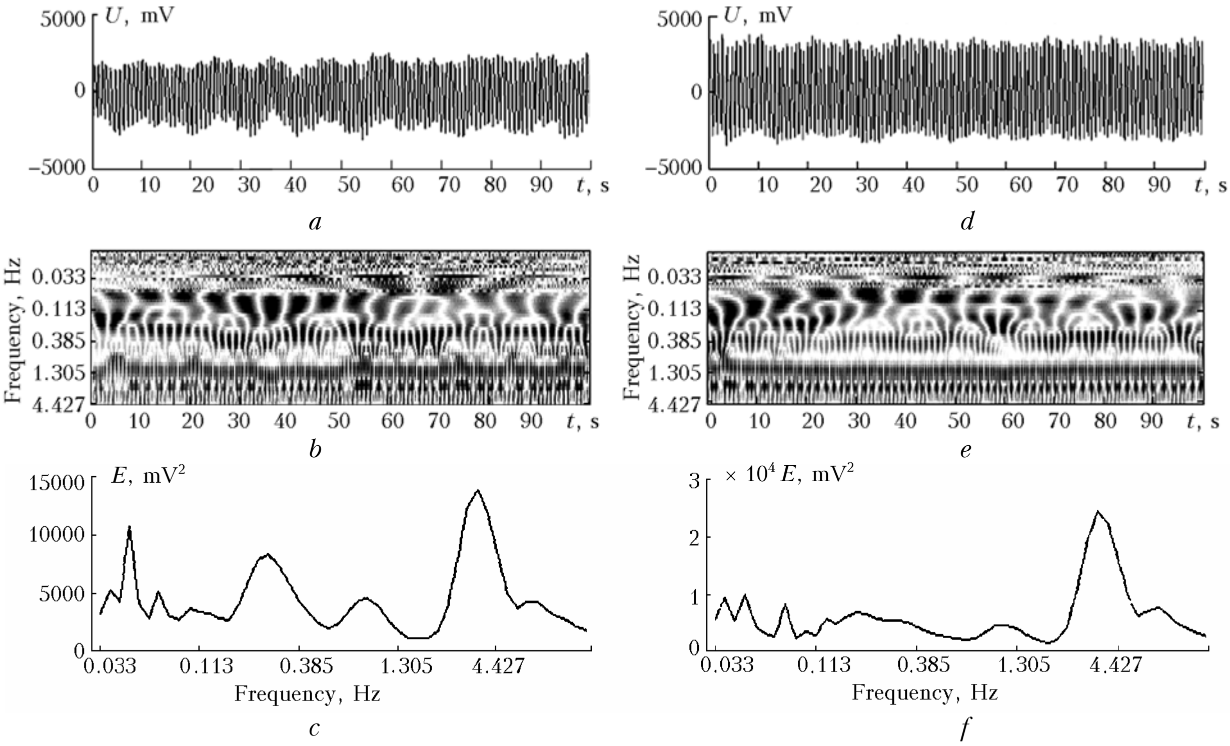

Fig. 3. Fragment of pulse signal of healthy human (long realization) (*a*); its wavelet spectrum (*b*); its global wavelet spectrum (*c*); fragment of pulse signal of human with functional distortions (long realization) (*d*); its wavelet spectrum (*e*); its global wavelet spectrum (*f*).

Application of continuous wavelet transform to determination of informative points of pulse signal

For determination of informative points of the pulse signal, it is suggested to use an analog of continuous wavelet transform for discrete signals[4]:

$$W(a,b) = \frac{1}{\sqrt{a}} \sum_k f(k) \int_k^{k+1} \psi\left(\frac{b-t}{a}\right) \mathrm{d}t,$$

where $f(k)$ is the discrete pulse signal, with use of Haar wavelet, which represents orthonormal wavelet with the compact support[5]:

$$\psi(t) = \begin{cases} 1, & 0 \le t < 0.5 \\ -1, & -0.5 \le t < 0.5 \\ 0, & t < -0.5,\ t \ge 0.5. \end{cases}$$

The possibility of using the Haar wavelet for solution of the stated problem can be demonstrated by concrete examples. The essence of the method of determination of informative points of the studied signal with the help of the Haar wavelet consists in initial expansion in wavelets of the analyzed signal (Fig. 4*a*), that gives us a pattern of absolute wavelet coefficients (wavelet spectrum); these coefficients are presented in Fig. 4*b*, where we clearly see zero values of the wavelet coefficients, shown in white.

The next step of the algorithm of identification of the informative points is to select a fixed frequency on the pattern of the wavelet coefficients, taking into account the informative scale (frequency), which makes it possible to determine the sought points. In Fig. 4*b*, the wavelet coefficients at the chosen frequency are shown by curve *1*. The value of the frequency is chosen from the viewpoint of minimization of noise effect on the accuracy of determination of the informative points.

At the concluding stage, the crossings of the curve *1* and zero values of wavelet coefficients (white-colored regions of the wavelet coefficient pattern) are used to determine the informative points of the model signal, which correspond to its extreme points, as shown in Fig. 4*b*. The method of determination of informative points of the studied signal from the values of the wavelet coefficients, equal to zero at the chosen fixed frequency (denoted by crosses), is more clearly illustrated in Fig. 4*c*.

The simplicity and transparence of the method testify the efficiency of the use of the continuous Haar-based wavelet transform for identification of the informative points of the pulse signal.

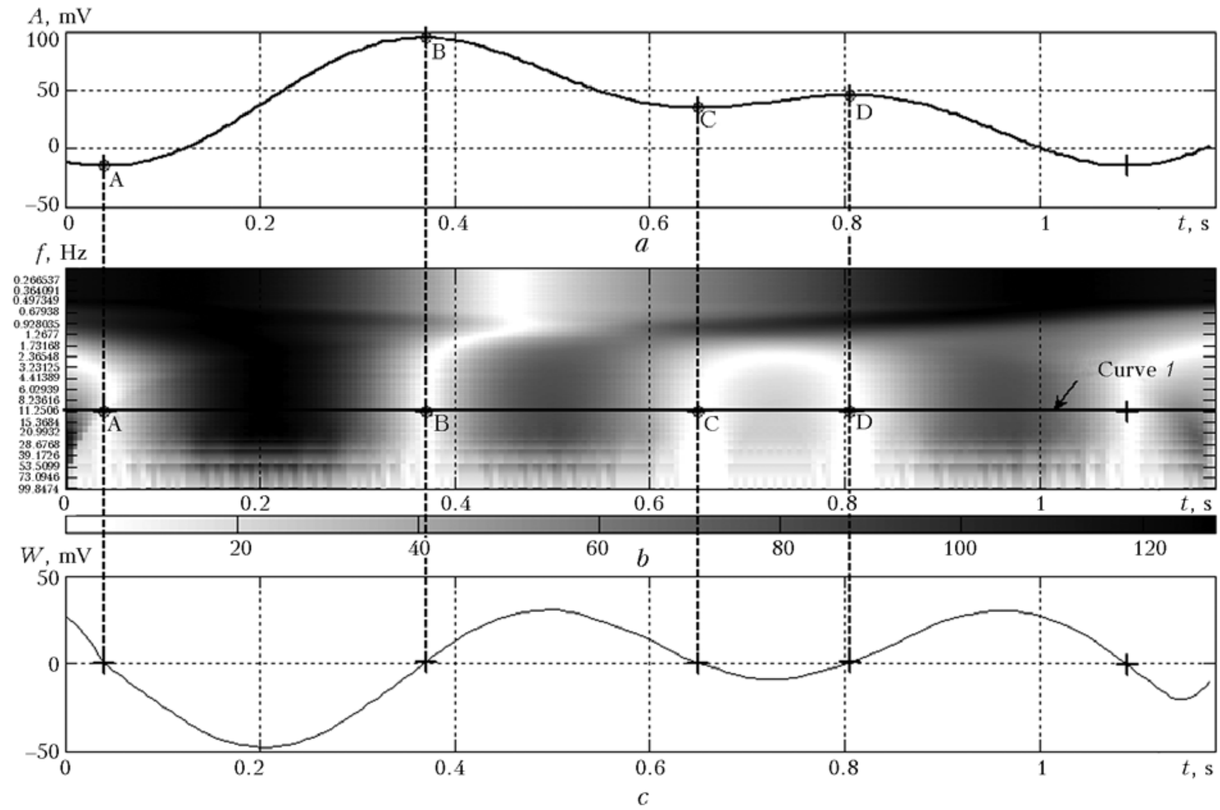

Fig. 4. Model pulse wave (*a*); wavelet spectrum (*b*); and absolute values of the wavelet coefficients at the chosen fixed frequency (scale) (*c*).

Conclusions

Study of the model signals of the pulse waves has shown that the wavelet analysis can identify the characteristic points in the small-amplitude regions, artefacts, and fluctuations of the signals, as well as discontinuity points in the case of incorrect signal recording, thereby opening new possibilities in the detailed study of the local signal features.

Based on the analysis of pulse signals of different patients, we found that the changes in human health lead to variations of the local structure of the pulse signal wavelet spectra: there appear differences in local signal features, and the differences in the frequency composition in the low-frequency region of the pulse signals. In addition, we suggested a new method of determination of informative points, based on the continuous wavelet transform.

Thus, the application of the wavelet transform to the pulse signals is a promising processing method. This method can be successively used for determination of the informative points of the pulse signal and formalization of characteristics of distortion of the organism functional state.

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