DIFFRACTION OF LIGHT BY A THIN FLAT SCREEN WITH A STRAIGHT EDGE. PART I

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The effect of the formation of a minimum in the center of the shadow from a narrow screen is discussed in the paper.

Experiment confirms the n phase shift between the edge wave components. In this study it was found that the amplitude of the edge wave was inversely proportional to the diffraction angle for light diffraction by a thin rectilinear screen.

It is shown that the edge wave had a phase advance of 0.69π on the illuminated side and a phase delay of 0.31π with respect to the incident wave on the shadow side.

As is well known, Fresnel originally explained the diffraction from a screen (independently of Young) by the interference of the rays reflected from its edge with the directly transmitted rays.[1] However, the location of light and dark bands turned out to be almost the opposite of those observed in the experiments. To eliminate this discrepancy it was necessary to assume that the rays reflected by the screen edge experience a phase shift close to π. Besides, according to Fresnel, a small amount of light, scattered by the screen edge and diffused over a large space, should cause only weak changes in the illumination in the observation region of the diffraction pattern. These conditions and the lack of any dependence of the band intensity on the surface curvature and the properties of the screen edge caused Fresnel to doubt the correctness of the approach and finally led him to the explanation of the phenomenon of diffraction by combining Huygens's principle with the principle of the interference of oscillations.

Nevertheless, Young's idea was confirmed by Sommerfeld's solution of the problem of the diffraction of plane waves by a semi-infinite reflecting screen.[2] According to Sommerfeld, in the geometrical shadow the light spreads out in the form of a cylindrical wave which seems to come from the screen edge, and on the illuminated side the light intensity is determined by the interference of the cylindrical wave and the incident wave.

To compensate the discontinuity in the incident wave at the shadow boundary, Sommerfeld's diffracted wave experiences a corresponding discontinuity, as a result of which its phase in the shadow coincides with the phase of the incident wave, but on the illuminated side it has the opposite phase.[3] This means that between the components of the edge wave there should be a phase shift of π.

The existence of the phase jump was confirmed in Ref. 4 by obtaining the image of the screen edge by means of the diffracted light only; the image was observed as a dark line because of the mutual suppression of the edge wave components. But the dark line also appears when the phase shift differs considerably from π. Therefore, the experiment based on the scheme shown in Fig. 1a should be more convincing. Here, the image S' of the slit S (30 μm wide), obtained with the aid of the objective, is covered by a wire W (0.2 mm in diameter). At the distance l = 24.9 mm from S' a thin screen S_1 (a blade) is introduced into the light beam (λ = 0.53 μm) up to its axis. The diffracted rays 1 and 2, coming from the edge of S_1, are diffracted again by the wire. Beams 3 and deviated into the shadow interfere with each other and form a minimum at its center. Therefore, the π phase shift really occurs between rays 1 and 2. The existence of the said minimum is evident in Fig. 2 (curve 2), which shows the distribution of the light intensity J over the screen S_2, removed a distance L = 128.2 mm from the wire, where h is the distance to the shadow center; the interval ab is the shadow region; the bands bordering the shadow are caused by the interference of rays 5 and the strongly deviated rays 3 and 4 with rays 1 and 2.

When the screen S_3 is brought up against the screen (Fig. 1b) so as to form a slit between them (say 30 μm wide), then oscillations from the two sides of the slit arrive at each side of the wire with an initial phase shift of n between them. Since there is no phase shift between oscillations 5 and 6 after the summation of the above oscillations, the phase shift is absent, neither is there any phase shift between oscillations 7 and 8, arriving at the center of the shadow from both sides of the wire. As a result, the central minimum changes into a maximum (Fig. 2, curve 1). The formation of the latter is clearly in accordance with Fresnel's ideas, while the formation of the minimum contradicts them and, therefore, demonstrates the limitations of the Fresnel approach.

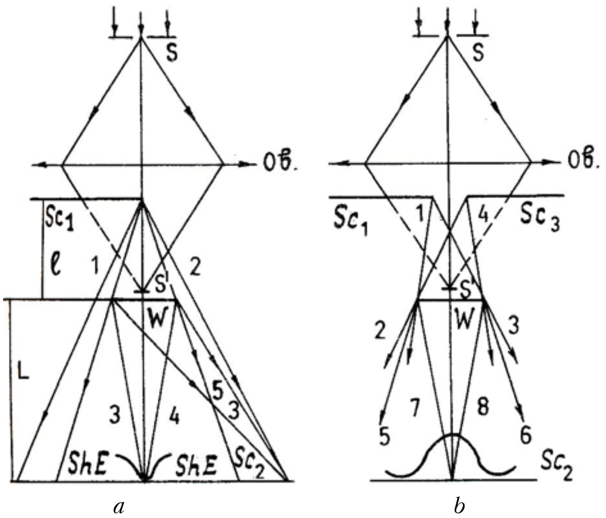

Fig. 1. Experimental schemes, illustrating the formation of the minimum (a) and the maximum (b) of the illumination at the center of the shadow of the screen.

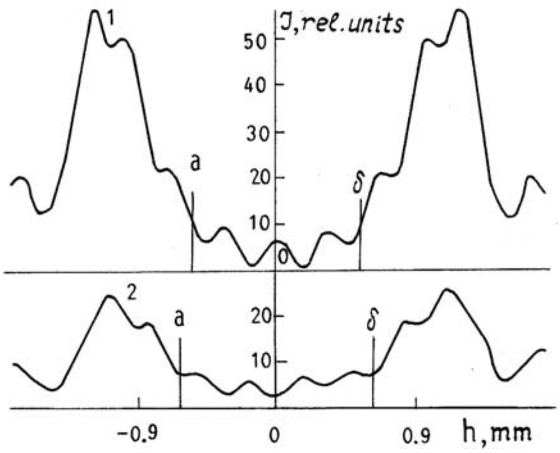

Fig. 2. Distribution of the light intensity in the shadow from a narrow screen.

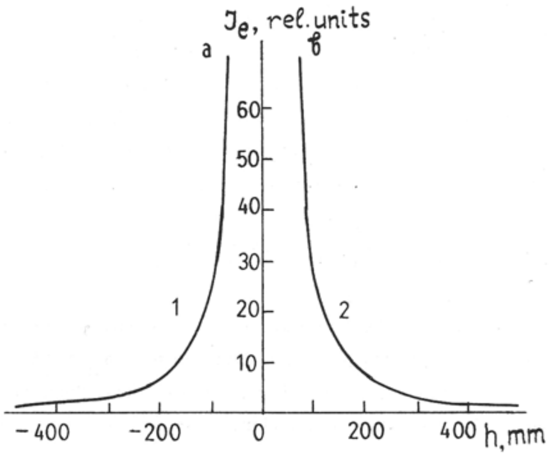

Fig. 3. Distribution of the light in the components of the edge wave.

Figure 3 shows the intensity distribution in the edge wave J_e from S_1 (Fig. 1a) in the shadow (curve 1) and outside it (curve 2), obtained by scanning the diffracted light with a slit 34 μm wide in the plane S' (with the wire removed and $l = 21.9$ μm) outside the interval ab, which is equal to the sum of the width of S' and twice the width of the scanning slit. The graph is symmetrical with respect to the vertical coordinate axis; which indicates that the fluxes of both components are equal.

As the analysis shows, the intensity of the edge wave is determined by the equation

$$J_e = A/h^2, \tag{1}$$

where A is the value which depends on the parameters of the experimental scheme and on the intensity of the incident light and h is the distance from the shadow boundary. This equation has a rather simple form in comparison with Sommerfeld's equation for the edge wave[6] and demonstrates the linear dependence of the amplitude of the diffracted light on h and on the beam diffraction angle. To ensure agreement between the experimental values of the intensity of the edge wave and the calculated ones, it is necessary to carefully attenuate the background caused by light diffraction on the iris of the objective, aberrations of the objective, light scattering in the objective, and secondary reflections of light beams from the scheme elements.

Proceeding from the interference of the edge wave with the directly transmitted light, the location of the bands in the diffraction pattern from the screen with the straight edge, in the case of cylindrical incident waves (Fig. 4), is described by the equation:

$$h = \sqrt{(1 + k)\frac{\lambda L(L + l)}{l}}, \tag{2}$$

where $1 + k$ is the number of half waves $\lambda/2$ in the geometrical path difference $\Delta_{21} = {}_2(\Delta_1 - \Delta)$ between rays 1 and 2; h is the distance from the bands to the shadow boundary; the term equal to the unity takes into account the phase shift of π between the diffracted and the direct rays; $k = 0,2,4\ldots$ correspond to the maxima, and $k = 1,3,5,\ldots$ correspond to the minima in the diffraction pattern.

However, the actual position of the bands is determined by the equation

$$h = \sqrt{(0.69 + k)\frac{\lambda L(L + l)}{l}}, \tag{3}$$

obtained from the previous one by the substitution of 0.69 for unity. This means that ray 2 (Fig. 4), at the moment of deviation from the initial direction, experiences a phase shift relative to ray 1 not of π, but of $0.69\ \pi$. In addition, ray 2 runs ahead of ray 1 since $\max_1$ is formed at that value of h at which the geometric path of ray 2 is larger than the path of ray 1 by the value $0.69\ \lambda/2$.

Since a phase shift of π occurs between rays 2 and 3, and ray 2 leads ray 1 by a phase advance of 0.69 π, ray 3, which has been diffracted into the shadow, experiences (in contradiction to Sommerfeld) a phase delay of 0.31 π relative to the incident rays at the moment of deviation.

The validity of Eq. (3) is confirmed by the data from Tables I and II, where h_{exp} are the experimental values; h_c are the calculated values; and h_C are the h values derived in accordance with the Cornu spiral; and $\Delta h_{exp,c} = (h_{exp} - h_c)$; and, $\Delta h_{exp,C} = (h_{exp} - h_C)$.

TABLE I

Band	l = 12 mm;			L = 99.5mm		l = 24 mm;			L = 99.5mm	
	h_{exp}, mm	h_c, mm	$\Delta h_{exp,c}$, µm	h_C, µm	$\Delta h_{exp,C}$, µm	h_{exp}, mm	h_c, mm	$h_{exp,c}$, µm	h_C, mm	$\Delta h_{exp,C}$, µm
max_1	0.582	0.582	0	0.629	-47	0.433	0.433	0	0.468	-35
min_1	0.900	0.910	-10	0.933	-33	0.686	0.677	9	0.694	- 8
max_2	1.145	1.148	- 3	1.163	-18	0.865	0.854	11	0.866	1
min_2	1.350	1.345	5	1.356	- 6	1.006	1.0	6	1.009	- 3
max_3	1.540	1.516	24	1.510	30	1.126	1.128	- 2	1.124	2
min_3	1.670	1.670	0	1.658	12	1.249	1.243	6	1.234	15
max_4	1.830	1.811	19	1.832	- 2	1.351	1.347	4	1.363	-12
min_4	-	-	-	-	-	1.443	1.445	- 2	-	-
max_5	-	-	-	-	-	1.529	1.536	- 7	-	-
min_5	-	-	-	-	-	1.611	1.622	-11	-	-
max_6	-	-	-	-	-	1.711	1.703	8	-	-
min6	-	-	-	-	-	1.786	1.781	5	-	-

TABLE II

Band	l = 6 mm;			L = 99.5mm		l = 117 mm;			L = 376.5mm	
	h_{exp}, mm	h_c, mm	$\Delta h_{exp,c}$, µm	h_C, µm	$\Delta h_{exp.C}$, µm	h_{exp}, mm	h_c, mm	$h_{exp,c}$, µm	h_C, mm	$\Delta h_{exp,C}$, µm
max_1	0.8	0.8	0	0.865	–65	0.76	0.763	0	0.824	-64
min_1	1.261	0.252	9	1.283	-22	1.185	1.190	-5	1.223	-38
max_2	1.609	1.579	30	1.6	9	1.510	1.500	10	1.524	-14
min_2	-	-	-	-	-	1.760	1.760	0	1.777	-17
max_3	-	-	-	-	-	1.980	1.983	-3	1.979	1

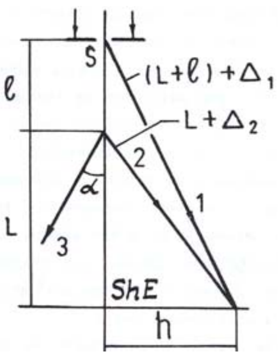

Fig. 4. Diffraction scheme with a rectilinear screen.

In the experiments the slit S (Fig. 4), which was 30 µm wide and was illuminated by a parallel beam of green light (λ = 0.53 µm), was used as the light source. The shadow boundary was initially defined by the intersection of the curves of the intensity distributions in the diffraction pattern from the diametrically opposed screens, and later, by h_c for max_1; the positions of the maxima and minima were determined by the greatest differences between the intensity of the diffraction pattern and the intensity of the incident light without the screen.

In the case of a plane incident wave $l = \infty$, and Eq. (3) has the following form:

$$h = \sqrt{(0,69 + k)\lambda L} \quad . \tag{4}$$

its agreement with experiment is demonstrated by Table III.

It is evident from the tables that the values of h_C for the first maximum and for the first minimum differ rather significantly from h_{exp}.

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