# Formation of the interference patterns in diffuse scattered fields at spatial filtering of the diffraction field of a doubleexposure hologram of a focused image of the scatterer

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The holographic interferometer sensitivity to the transverse or longitudinal motions of a plane surface diffusely scattering incident light is analyzed. It is shown, that the interference patterns are localized in the hologram plane and in the plane of the pupil image formed by a positive lens, with the help of which the hologram recording is being carried out. To record the holograms, a spatial filtering of the diffraction field is needed. The experimental results well agree with the theory.

 $(x_1, y_1)$ 

As shown in Ref. 1, in performing a doubleexposure recording of the Fresnel hologram when before the re-exposure of a photographic plate the plane surface diffusely scattering the incident light is shifted along the transverse or longitudinal direction, the interference patterns incorporating the properties of the objective speckles are localized, at the stage of the hologram reconstruction, in two planes. As in the case of Fresnel hologram recording,<sup>2</sup> it is considered in Ref. 3, that for the hologram of the diffuser infocus image, localization of the interference patterns in two planes is possible only at combination of homogeneous and inhomogeneous motions of the diffuser points. In their turn, the results of indirect researches, for example in Refs. 4-6, connected with the diffuser cross motion at the hologram recording of the in-focus image of a plane surface diffusely scattering the incident light, have shown, that the interference patterns are localized in two planes. In this connection, there is a necessity for a more complete and unambiguous definition of the interferometer sensitivity to the diffuser cross or longitudinal motions with regarding the properties of the subjective speckles.

In this paper, conditions and specific features of the formation of interference patterns are analyzed at the double-exposure recording of the hologram infocus image of a plane diffuse surface aimed at determining the interferometer sensitivity to the diffuser cross or longitudinal motions.

According to Fig. 1, a matte screen 1, that is in the plane  $(x_1, y_1)$ , is illuminated by a coherent radiation of a diverging spherical wave with the radius of curvature R. Its image is formed, with the help of a thin positive lens L with the focal length f, in the plane  $(x_3, y_3)$  of a photographic plate 2, on which the hologram is recorded of the diffuser's infocus image with the use of an off-axis plane reference wave is being done during the first exposure.

Before the re-exposure, the matte screen is moved in its plane by a distance a along, for instance, the *x*-axis.

(ξ, η)

 $(x_3, y_3)$ 

 $(x_2, y_2)$ 

**Fig. 1.** Diagram of the diffuser hologram recording: p is the aperture diaphragm;  $\theta$  is the angle between the reference beam and the normal to the plane of a photographic plate.

In the Fresnel approximation with regard for the diffraction boundedness, the distribution of the complex field amplitude, corresponding to the first exposure, in the object channel in the plane of a photographic plate, is written, following Refs. 5 and 6, as follows

$$u_{1}(x_{3}, y_{3}) \sim \iiint \int_{-\infty}^{\infty} t(x_{1}, y_{1}) \exp\left[\frac{ik}{2R}(x_{1}^{2} + y_{1}^{2})\right] \times \\ \exp\left\{\frac{ik}{2l_{1}}\left[(x_{1} - x_{2})^{2} + (y_{1} - y_{2})^{2}\right]\right\} p(x_{2}, y_{2}) \times \\ \exp\left[\frac{-ik}{2f}(x_{2}^{2} + y_{2}^{2})\right] \exp\left\{\frac{ik}{2l}\left[(x_{2} - \xi)^{2} + (y_{2} - \eta)^{2}\right]\right\} \times \\ \times \exp\left\{\frac{ik}{2(l_{2} - l)}\left[(\xi - x_{3})^{2} + (\eta - y_{3})^{2}\right]\right\} dx_{1} dy_{1} dx_{2} dy_{2} d\xi d\eta,$$

$$(1)$$

where k is the wave number;  $t(x_1, y_1)$  is the complex amplitude of the transmission of a matte screen, being a random function of the coordinates;  $(x_2, y_2)$  is the principal plane of a lens;  $(\xi, \eta)$  is the plane of the Fourier image formation of the function  $t(x_1, y_1)$ , at the distance l from the principal plane of the lens;  $p(x_2, y_2)$  is the pupil's function<sup>7</sup>;  $l_1$  and  $l_2$  are the distances between the planes. $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_2, y_2)$ ,  $(x_3, y_3)$ , respectively. V.G. Gusev

Since  $\frac{1}{R+l_1} + \frac{1}{l} = \frac{1}{f}$ , the expression (1) takes the allowing form

following form

$$u_{1}(x_{3}, y_{3}) \sim \exp\left[\frac{ik}{2(l_{2}-l)}(x_{3}^{2}+y_{3}^{2})\right] \times$$

$$\times \left\{ \exp\left[-\frac{ikl}{2l_{2}(l_{2}-l)}(x_{3}^{2}+y_{3}^{2})\right] \otimes p\left(\frac{l}{l_{2}-l}x_{3}, \frac{l}{l_{2}-l}y_{3}\right) \times$$

$$\times \left\{ \exp\left[\frac{ikRl^{2}}{2l_{1}(R+l_{1})(l_{2}-l)^{2}}(x_{3}^{2}+y_{3}^{2})\right] \otimes$$

$$\otimes t\left[-\frac{Rl}{(R+l_{1})(l_{2}-l)}x_{3}, -\frac{Rl}{(R+l_{1})(l_{2}-l)}y_{3}\right] \right\} \right\}, \quad (2)$$

where  $\otimes$  is the symbol of the convolution operation.

Because of the integral representation, in Eq. (2), of the convolution operation and having in mind that  $1/l_1 + 1/l_2 = 1/f$ , we obtain

$$u_{1}(x_{3}, y_{3}) \sim \exp\left[\frac{ik}{2l_{2}}\left(x_{3}^{2} + y_{3}^{2}\right)\right] \times \left\{t\left(-\mu x_{3}, -\mu y_{3}\right)\exp\left[\frac{ik\mu(R+l_{1})}{2Rl_{2}}\left(x_{3}^{2} + y_{3}^{2}\right)\right] \otimes P(x_{3}, y_{3})\right\},$$
(3)

where  $\mu = l_1/l_2$  is the scale conversion factor;  $P(x_3, y_3)$  is the Fourier image of the function  $p(x_2, y_2)$  with the spatial frequencies of  $x_3/\lambda l_2$ ,  $y_3/\lambda l_2$ ;  $\lambda$  is the wavelength of the coherent light, used for the hologram recording and reconstruction.

From Eq. (3) it follows, that in the plane  $(x_3, y_3)$  each point of the object is broadened to the size of a subjective speckle, determined by the function  $P(x_3, y_3)$  width, which is determined by the diffraction on the pupil of lens L of a plane wave.

Distribution of the complex field amplitude, corresponding to the second exposure, in the object channel in the plane of a photographic plate is determined by the expression

which takes the following form

$$u_{2}(x_{3}, y_{3}) \sim \exp\left[\frac{ik}{2(l_{2}-l)}(x_{3}^{2}+y_{3}^{2})\right] \times \\ \times \left\{ \exp\left[-\frac{ikl}{2l_{2}(l_{2}-l)}(x_{3}^{2}+y_{3}^{2})\right] \otimes p\left(\frac{l}{l_{2}-l}x_{3}, \frac{l}{l_{2}-l}y_{3}\right) \times \right. \\ \left. \times \left\{ \exp\left\{\frac{ikRl^{2}}{2l_{1}(R+l_{1})(l_{2}-l)^{2}}\left[(x_{3}-al_{2}/l_{1})^{2}+y_{3}^{2}\right]\right\} \otimes \right. \\ \left. \otimes t\left[-\frac{lR}{(R+l_{1})(l_{2}-l)}x_{3}, -\frac{lR}{(R+l_{1})(l_{2}-l)}y_{3}\right] \right\} \right\}.$$
(5)

In this case, the integral representation in Eq. (5) of the convolution operation yields two identical expressions for the distribution of the complex field amplitude, i.e.,

$$u_{2}(x_{3}, y_{3}) \sim \exp\left[\frac{ik}{2l_{2}}\left(x_{3}^{2} + y_{3}^{2}\right)\right] \left\{t\left(-\mu x_{3} + a, -\mu y_{3}\right) \times \\ \times \exp\left[\frac{ik\mu(R+l_{1})}{2Rl_{2}}\left(x_{3}^{2} + y_{3}^{2}\right)\right] \otimes P(x_{3}, y_{3})\right\} = \\ = \exp\left[\frac{ik}{2l_{2}}\left(x_{3}^{2} + y_{3}^{2}\right)\right] \left\{t\left(-\mu x_{3}, -\mu y_{3}\right) \times \\ \exp\left\{\frac{ik\mu(R+l_{1})}{2Rl_{2}}\left[\left(x_{3} + a/\mu\right)^{2} + y_{3}^{2}\right]\right\} \otimes P(x_{3} - a/\mu, y_{3})\right\}.$$
(6)

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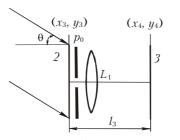
Based on the expressions (3) and (6) and assuming that the hologram recording is performed within the linear portion of the photographic material blackening curve, the distribution of the complex amplitude of its transmission, corresponding to the minus first diffraction order, takes the form

$$\tau(x_{3}, y_{3}) \sim \exp(-ikx_{3}\sin\theta)\exp\left[\frac{ik}{2l_{2}}\left(x_{3}^{2}+y_{3}^{2}\right)\right] \times \\ \times \left\{ \left\{ t\left(-\mu x_{3}, -\mu y_{3}\right)\exp\left[\frac{ik\mu(R+l_{1})}{2Rl_{2}}\left(x_{3}^{2}+y_{3}^{2}\right)\right] \otimes P(x_{3}, y_{3})\right\} + \left\{ t\left(-\mu x_{3}, -\mu y_{3}\right)\exp\left\{\frac{ik\mu(R+l_{1})}{2Rl_{2}}\left[\left(x_{3}+a/\mu\right)^{2}+y_{3}^{2}\right]\right\} \otimes \\ \otimes P(x_{3}-a/\mu, y_{3})\right\} \right\}.$$
(7)

According to Eq. (7), in the plane  $(x_3, y_3)$  the subjective speckles, corresponding to the second exposure, are displaced along the direction, opposite to the direction of the diffuser motion, by the distance which depends on the magnification factor of the optical system and does not depend on the radius of curvature R. Besides, they have a tilt by the angle  $a(R + l_1)/Rl_2$  with respect to the identical

speckles, corresponding to the first exposure, whose size depends on the radius of curvature of the spherical wave used for illuminating the diffuse scatterer. Moreover, the subjective speckle fields of both the first and second exposure are superposed with a phase distribution of the diverging spherical wave with the radius of curvature  $l_2$ .

Let the spatial filtering of the diffraction field be carried out at the stage of the hologram reconstruction in its plane on the optical axis, with the help of a round aperture in an opaque screen  $p_0$  (Fig. 2).



**Fig. 2.** Diagram of recording the interference pattern localized in a plane of the image formation of the lens's pupil: 2 is the hologram; 3 is the recording plane,  $L_1$  is the positive lens;  $p_0$  is the spatial filter.

In addition, assume that within the filtering aperture, a phase change  $k(R + l_1)ax_3/Rl_2$  does not exceed  $\pi$ . Then the distribution of the complex field amplitude at the output of the spatial filter is determined by the expression

$$u(x_{3}, y_{3}) \sim p_{0}(x_{3}, y_{3}) \exp\left[\frac{ik}{2l_{2}}(x_{3}^{2} + y_{3}^{2})\right] \times \\ \times \left\{ t(-\mu x_{3}, -\mu y_{3}) \exp\left[\frac{ik\mu(R+l_{1})}{2Rl_{2}}(x_{3}^{2} + y_{3}^{2})\right] \otimes \\ \otimes \left\{ P(x_{3}, y_{3}) + \exp\left[\frac{ik(R+l_{1})a^{2}}{2Rl_{1}}\right] \times \\ \times \exp\left[\frac{-ik(R+l_{1})ax_{3}}{Rl_{2}}\right] P(x_{3} - a/\mu, y_{3}) \right\} \right\}, \quad (8)$$

where  $p_0(x_3, y_3)$  is the transmission function of a spatial filter.<sup>8</sup>

Assume, that the positive lens  $L_1$  with the focal length  $f_1$  is in the plane  $(x_3, y_3)$  and with its help, the pupil image of a lens L (see Fig. 1) is formed in the plane  $(x_4, y_4)$ , i.e.,  $(1/f_1) = (1/l_2) + (1/l_3)$ , where  $l_3$ is the distance between the planes  $(x_3, y_3)$  and  $(x_4, y_4)$ . Besides, here and below, we shall consider for brevity, that  $l_3 = l_2$ , and do not take into account the factors, which are insignificant for the result. Then, the distribution of the complex field amplitude in the plane  $(x_4, y_4)$  takes the following form

$$u(x_4, y_4) \sim p(x_4, y_4) \left\{ 1 + \exp\left[\frac{-ik(R+l_1)a^2}{2Rl_1}\right] \exp\left(-ikax_4/l_1\right) \right\} \times$$

$$\times \left\{ F(x_4, y_4) \otimes \exp\left[-\frac{ikR}{2\mu l_2(R+l_1)} \left(x_4^2 + y_4^2\right)\right] \right\} \otimes P_0(x_4, y_4), \quad (9)$$

where  $F(x_4, y_4)$  and  $P_0(x_4, y_4)$  are the Fourier images of the functions  $t(-\mu x_3, -\mu y_3)$  and  $p_0(x_3, y_3)$ , respectively, with the spatial frequencies of  $x_4/\lambda l_2$  and  $y_4/\lambda l_2$ .

From Eq. (9) it follows, that within pupil image of a lens L, the identical subjective speckles of two exposures are combined. Furthermore, if a period of the function variation

$$1 + \exp\left[\frac{-ik(R+l_1)a^2}{2Rl_1}\right] \exp\left(-ikax_4/l_1\right)$$

is at least an order<sup>2</sup> of magnitude greater, than the width of the function  $P_0(x_4, y_4)$ , which determines the size of the subjective speckle in the plane  $(x_4, y_4)$ , we shall remove it from the convolution integration sign in the expression (8). Then the distribution of the illumination in the recording plane 3 (see Fig. 2) is determined by the expression

$$I(x_{4}, y_{4}) \sim \left\{1 + \cos\left[\frac{kax_{4}}{l_{1}} + \frac{k(R+l_{1})a^{2}}{2Rl_{1}}\right]\right\} |p(x_{4}, y_{4}) \left\{F(x_{4}, y_{4}) \otimes \exp\left[-\frac{ikR}{2l_{1}(R+l_{1})}(x_{4}^{2}+y_{4}^{2})\right]\right\} \otimes P_{0}(x_{4}, y_{4})|^{2}.$$
 (10)

According to Eq. (10), within the pupil image of the lens L, the subjective speckle structure is modulated by the interference fringes, which periodically change along the x axis. As a result, measuring period of the interference patterns, having known the quantities  $\lambda$ ,  $l_1$ , one can find the diffuser cross motion t (see Fig. 1).

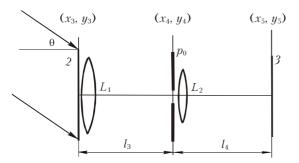
Let us now assume that at the stage of the hologram reconstruction, a spatial filtering of the diffraction field is carried out on the optical axis in the plane  $(x_4, y_4)$  (see Fig. 3) of the pupil image formation of a lens *L*. In this case, disregarding the space limitation of the diffraction field, the distribution of its complex amplitude at the output of a spatial filter, if the phase change  $kax_4/l_1$  does not exceed  $\pi$  within the filtering aperture, takes the form

$$u(x_{4}, y_{4}) \sim p_{0}(x_{4}, y_{4}) \exp\left[\frac{ik}{2l_{2}}(x_{4}^{2} + y_{4}^{2})\right] \times \\ \times \left\{F(x_{4}, y_{4}) \otimes \exp\left[-\frac{ikR}{2\mu l_{2}(R + l_{1})}(x_{4}^{2} + y_{4}^{2})\right] + F(x_{4}, y_{4}) \otimes \exp(ikax_{4}/\mu l_{2}) \times \right. \\ \left. \times \exp\left[-\frac{ikR}{2\mu l_{2}(R + l_{1})}(x_{4}^{2} + y_{4}^{2})\right]\right\}.$$
(11)

Assume, that the lens  $L_2$  (see Fig. 3) with a focal length  $f_2$  is in the plane of the spatial filter. Besides, here and further, we shall consider, for brevity, that  $l_4 = l_3 = l_2$ , where  $l_4$  is the distance between the planes  $(x_4, y_4)$  and  $(x_5, y_5)$ . Then the distribution of the complex field amplitude in the recording plane 3 (see Fig. 3), i.e., in the planes of formation of the hologram image, is determined by the expression

$$u(x_{5}, y_{5}) \sim t(\mu x_{5}, \mu y_{5}) \left\{ \exp\left[\frac{ik\mu(R+l_{1})}{2Rl_{2}}(x_{5}^{2}+y_{5}^{2})\right] + \exp\left\{\frac{ik\mu(R+l_{1})}{2Rl_{2}}[(x_{5}-a/\mu)^{2}+y_{5}^{2}]\right\} \right\} \otimes P_{0}(x_{5}, y_{5}), \quad (12)$$

where  $P_0(x_5, y_5)$  is the Fourier image of the function  $p_0(x_4, y_4)$  with the spatial frequencies of  $x_5/\lambda l_2$  and  $y_5/\lambda l_2$ .



**Fig. 3.** Diagram of recording the interference pattern localized in the hologram plane; 2 is the hologram; 3 is the recording plane;  $L_1$  and  $L_2$  are the positive lenses;  $p_0$  is the spatial filter.

From Eq. (12) it follows, that in the plane  $(x_5, y_5)$ , we have a superposition of the coinciding subjective speckles of the two exposures differed by a tilt angle. Moreover, when a period of the function variation

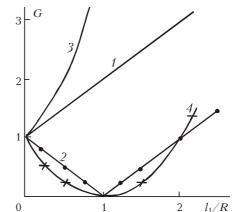
$$1 + \exp\left[\frac{ik(R+l_1)a^2}{2Rl_1}\right] \exp\left[-\frac{ik(R+l_1)}{Rl_2}ax_5\right]$$

is at least an order of magnitude greater, than the width of  $P_0(x_5, y_5)$  function, which determines the size of the subjective speckle in the recording plane 3 (see Fig. 3), we shall remove it from the convolution integration sign in the expression (12). Then, the recorded distribution of the illumination can be written in the following form

$$I(x_{5}, y_{5}) \sim \left\{ 1 + \cos \left[ \frac{kG_{1}ax_{5}}{l_{1}} - \frac{k(R+l_{1})a^{2}}{2Rl_{1}} \right] \right\} \times \\ \times \left| t(\mu x_{5}, \mu y_{5}) \exp \left[ \frac{ik\mu(R+l_{1})}{2Rl_{2}} (x_{5}^{2} + y_{5}^{2}) \right] \otimes P_{0}(x_{5}, y_{5}) \right|^{2}, (13)$$

where  $G_1 = \mu(R + l_1)/R$  is the coefficient introduced to characterize variation of the interferometer sensitivity to the diffuser cross motion.

According to Eq. (13), within the diffuser image, the subjective speckle structure is modulated by the interference fringes which periodically change along the x-axis, and measuring the period of the interference fringes, having known the quantities  $\lambda$ ,  $l_1$ ,  $l_2$ , and R, makes it possible to determine the diffuser cross motion (see Fig. 1). Thus, for the fixed quantities  $\lambda$  and  $l_1 = l_2$  the interferometer sensitivity increases with the reduction of R (see Fig. 4) because of the tilt angle's increase of the subjective speckles, corresponding to the second exposure with respect to the identical speckles in the first exposure.



**Fig. 4.** Dependences of the coefficients of the interferometer sensitivity on the radius of curvature of the spherical wave front for  $\mu = 1$ :  $G_1(t)$ ;  $G_2(2)$ ;  $G_3(3)$ ;  $G_4(4)$ .

Assume, that at the stage of the double-exposure hologram recording of the in-focus image of a matte screen 1 (Fig. 1), the screen is illuminated by a coherent radiation of a converging spherical wave. Hence, two identical expressions for distribution of the complex field amplitude, corresponding to the second exposure, in the plane  $(x_3, y_3)$  take the following form

$$u_{2}(x_{3}, y_{3}) \sim \exp\left[\frac{ik(x_{3}^{2} + y_{3}^{2})}{2l_{2}}\right] \left\{ t(-\mu x_{3} + a, -\mu y_{3}) \times \left[\frac{ik\mu(R - l_{1})}{2Rl_{2}}(x_{3}^{2} + y_{3}^{2})\right] \otimes P(x_{3}, y_{3}) \right\} = \\ = \exp\left[\frac{ik(x_{3}^{2} + y_{3}^{2})}{2l_{2}}\right] \left\{ t(-\mu x_{3}, -\mu y_{3}) \times \\ \exp\left\{\frac{ik\mu|R - l_{1}|}{2Rl_{2}}\left[(x_{3} + a/\mu)^{2} + y_{3}^{2}\right]\right\} \otimes P(x_{3} - a/\mu, y_{3}) \right\}.$$
(14)

In this case, at the stage of the hologram reconstruction, the interference patterns are also localized in the two above-mentioned planes and for their recording, spatial filtering of the diffraction field is necessary. Thus, the variation of sensitivity to the diffuser cross motion for an interference pattern localized in the hologram plane, makes up the value  $G_2 = \mu |R - l_1| / R$ . Therefore, for example, at the fixed quantities  $\lambda$  and  $l_1 = l_2$  with the radius of curvature R decreasing in the limits of  $l_1 \leq R \leq \infty$ , the interferometer sensitivity (see Fig. 4) to the diffuser cross motion decreases down to zero, when  $R = l_1$ .

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At  $R = l_1$ , the coherent transfer function of a positive lens, with the help of which the diffuser image is formed, is uniform up to the maximum spatial frequency.<sup>9</sup> In addition, in the plane of the pupil image formation of a lens L, the "frozen" interference pattern is localized, the form of which and the position of the interference fringes do not change with the change of observation angle. The further reduction of radius of curvature R leads to the appearance and increase in the hologram plane of a tilt angle of the subjective speckles, corresponding to the second exposure, with respect to the identical speckles in the first exposure record. Because of it, the interferometer sensitivity (see Fig. 4) to the diffuser cross motion increases at recording the interference pattern in the plane  $(x_5, y_5)$  (see Fig. 3).

Localization of the interference patterns characterizing the diffuser cross motion, generally, in two planes (in the plane of the double-exposure hologram of the in-focus image of a plane surface diffusely scattering incident light and in the plane of the pupil image formation of a positive lens with the help of which the diffuser image was formed) is caused by the following causes.

On the one hand, the subjective speckles, corresponding to the second exposure, are displaced in the hologram plane by the identical distance with respect to the speckles of the first exposure. Assume that the phase distribution of the diverging spherical wave with the radius of curvature  $l_2$  is imposed on the subjective speckle field of two exposures and each individual speckle contains the information on the corresponding phase of this wave. Therefore, a combination of the speckles of two exposures, which is accompanied by the formation of a tilt angle between them, is possible only in the plane of the pupil image formation of a positive lens, with the help of which the diffuser image was formed.

On the other hand, the presence in the hologram plane of a tilt angle of the subjective speckles, corresponding to the second exposure, with respect to the speckles of the first exposure is a necessary condition for the formation in it of an interference pattern. Thus, for reception of a superposition of the identical speckle fields of two exposures, spatial filtering of the diffraction field in the corresponding planes is required.

Let before the photographic plate re-exposure, a matte screen 1 (see Fig. 1) be displaced along the z-axis by the distance  $\Delta l = l'_1 - l_1$ . Then, in the used approximation, two identical expressions for distribution of the complex field amplitude, corresponding to the second exposure, in the plane  $(x_3, y_3)$  have the following form

$$u_2'(x_3, y_3) \sim \exp(ik\Delta l) \exp\left[\frac{ik}{2l_2}\left(x_3^2 + y_3^2\right)\right] \times$$
$$\times \left\{ P(x_3, y_3) \otimes \exp\left[\frac{ikl_1(R+l_1)}{2Rl_2^2}\left(x_3^2 + y_3^2\right)\right] \times \right\}$$

$$\times \left\{ t \left( -\mu \frac{R - \Delta l}{R} x_3, -\mu \frac{R - \Delta l}{R} y_3 \right) \otimes \left( \exp \left[ -\frac{ikl_1(l_1 + \Delta l)}{2\Delta ll_2^2} (x_3^2 + y_3^2) \right] \right\} \right\} = \left( \exp (ik\Delta l) \exp \left[ \frac{ik}{2l_2} (x_3^2 + y_3^2) \right] \left\{ t \left( -\frac{l_1 + \Delta l}{l_2} x_3, -\frac{l_1 + \Delta l}{l_2} y_3 \right) \times \exp \left[ \frac{ik(l_1 + \Delta l)(R + l_1)}{2(R - \Delta l)l_2^2} (x_3^2 + y_3^2) \right] \right\} \right\}$$
$$\otimes \exp \left[ \frac{ikl_1(l_1 + \Delta l)}{2\Delta ll_2^2} (x_3^2 + y_3^2) \right] \otimes P(x_3, y_3) \right\}.$$
(15)

Provided that  $\Delta l \ll l_1$ ,  $l_2$ , and R, the complex transmission amplitude of the double-exposure hologram, corresponding to the minus first diffraction order, will be determined by the expression

$$\tau'(x_3, y_3) \sim \exp(-ikx_3 \sin\theta) \exp\left[\frac{ik}{2l_2}(x_3^2 + y_3^2)\right] \times \\ \times \left\{ t(-\mu x_3, -\mu y_3) \exp\left[\frac{ik\mu(R+l_1)}{2Rl_2}(x_3^2 + y_3^2)\right] \otimes \\ \otimes P(x_3, y_3) + \exp(ik\Delta l)t(-\mu x_3, -\mu y_3) \times \\ \exp\left[\frac{ik\mu(R+l_1)}{2Rl_2}(x_3^2 + y_3^2)\right] \exp\left[\frac{ik\Delta l(R+l_1)^2}{2R^2l_2^2}(x_3^2 + y_3^2)\right] \otimes \\ \left[ik\mu^2(x_3^2 + y_3^2)\right] = \left[\frac{ik\mu^2(x_3^2 + y_3^2)}{2R^2l_2^2}\right] = \left[\frac{ik\mu^2(x_3^2 + y_3^2$$

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$$\otimes \exp\left[\frac{ik\mu^2\left(x_3^2+y_3^2\right)}{2\Delta l}\right] \otimes P(x_3,y_3)\bigg\}.$$
 (16)

Let at the stage of the hologram reconstruction, a spatial filtering of the diffraction field (see Fig. 2) is performed in its plane on the optical axis, and within the filtering aperture the phase change  $k\Delta l(R + l_1)^2 (x_3^2 + y_3^2)/2R^2 l_2^2$  does not exceed  $\pi$ . Hence, the distribution of the complex field amplitude in the plane  $(x_4, y_4)$  of the pupil image formation of a lens *L* (see Fig. 1) takes the following form

$$u'(x_{4}, y_{4}) \sim p(x_{4}, y_{4}) \times \\ \times \left\{ F(x_{4}, y_{4}) \otimes \exp\left[-\frac{ikR}{2\mu l_{2}(R+l_{1})}(x_{4}^{2}+y_{4}^{2})\right] \right\} \times \\ \cdot \left\{ 1 + \exp(ik\Delta l) \exp\left[-\frac{ik\Delta l}{2l_{1}^{2}}(x_{4}^{2}+y_{4}^{2})\right] \right\} \otimes P_{0}(x_{4}, y_{4}).$$
(17)

From Eq. (17) it follows, that in the plane  $(x_4, y_4)$  a superposition of the identical subjective speckles of two exposures takes place and when a period of the function variation  $1 + \exp(ik\Delta l) \times \exp\left[-ik\Delta l(x_4^2 + y_4^2)/(2l_1^2)\right]$  is at least an order of

magnitude larger that the width of function  $P_0(x_4, y_4)$ , the distribution of the illumination in the recording plane 3 (see Fig. 2) is determined by the expression

$$I'(x_{4}, y_{4}) \sim \left\{ 1 + \cos \left[ k\Delta l - \frac{k\Delta l}{2l_{1}^{2}} (x_{4}^{2} + y_{4}^{2}) \right] \right\} \times$$
$$\times \left| p(x_{4}, y_{4}) \left\{ F(x_{4}, y_{4}) \otimes \exp \left[ -\frac{ikR}{2\mu l_{2}(R + l_{1})} (x_{4}^{2} + y_{4}^{2}) \right] \right\} \otimes$$
$$\otimes P_{0}(x_{4}, y_{4}) \right|^{2}.$$
(18)

According to the expression (18), the interference fringes of equal tilt modulate the subjective speckle structure within the pupil image of a lens L, i.e., they make up the system of concentric rings. Measuring the rings' radii in the adjacent orders of interference for the known values of  $\lambda$  and  $l_1$  allows one to determine longitudinal displacement of a plane surface diffusely scattering the incident light. Besides, it is necessary to note, that the interferometer sensitivity to longitudinal motion does not depend on radius of curvature of the diverging spherical wave, used for the diffuser illumination.

Let us now assume that at the stage of the hologram reconstruction a spatial filtering of the diffraction field is performed on the optical axis in the plane  $(x_4, y_4)$  (see Fig. 3) of the pupil image formation of a lens *L*. In this case distribution of the complex field amplitude in the plane  $(x_5, y_5)$  of formation of the hologram image, when the phase change  $k\Delta l(x_4^2 + y_4^2)/2l_1^2$  does not exceed  $\pi$ , within the filtering aperture, takes the following form

$$u'(x_{5}, y_{5}) \sim t(\mu x_{5}, \mu y_{5}) \exp\left[\frac{ik\mu(R+l_{1})}{2Rl_{2}}(x_{5}^{2}+y_{5}^{2})\right] \times \\ \times \left\{1 + \exp(ik\Delta l) \exp\left[\frac{ik\Delta l(R+l_{1})^{2}}{2R^{2}l_{2}^{2}}(x_{5}^{2}+y_{5}^{2})\right]\right\} \otimes \\ \otimes P_{0}(x_{5}, y_{5}).$$
(19)

From Eq. (19) it follows, that in the plane  $(x_5, y_5)$ , a superposition of the identical subjective speckles of two exposures also takes place, and when a period of the function variation

$$1 + \exp(ik\Delta l)\exp\left[\frac{ik\Delta l(R+l_1)^2}{2R^2l_2^2}(x_5^2+y_5^2)\right]$$

exceeds the width of function  $P_0(x_5, y_5)$  by at least an order of magnitude, the distribution of illumination in the recording plane 3 (see Fig. 3) is determined by the expression

$$I'(x_5, y_5) \sim \left\{ 1 + \cos\left[k\Delta l + \frac{kG_3\Delta l}{2l_1^2} (x_5^2 + y_5^2)\right] \right\} \times \\ \times |t(\mu x_5, \mu y_5) \exp\left[\frac{ik\mu(R+l_1)}{2Rl_2} (x_5^2 + y_5^2)\right] \otimes P_0(x_5, y_5)|^2, (20)$$

where  $G_3 = \mu^2 (R + l_1)^2 / R^2$  is the coefficient introduced to characterize the variation of the interferometer sensitivity to the longitudinal motion of the diffuser.

According to the expression (20), in the limits of the diffuser image, the fringes of equal tilt modulate the subjective speckle structure. A system of concentric rings is formed, and measuring the rings' radii in the adjacent orders of interference, having known the quantities  $\lambda$ ,  $l_1$ ,  $l_2$ , and R provides a possibility of determining the distance of longitudinal motion of a plane surface that diffusely scatters light. Thus, at fixed values  $\lambda$  and  $l_1 = l_2$ , the interferometer sensitivity (see Fig. 4) increases with the reduction of R because of the increase in the tilt angle when moving along radius from the optical axis of the subjective speckles, corresponding to the second exposure, with respect to the identical speckles, in the first exposure pattern.

At illumination of a matte screen 1 (Fig. 1) by the coherent radiation of a converging spherical wave, two identical expressions for distribution of the complex field amplitude corresponding to the second exposure in the plane  $(x_3, y_3)$  take the following form

$$u_{2}'(x_{3}, y_{3}) \sim \exp(ik\Delta l) \exp\left[\frac{ik}{2l_{2}}\left(x_{3}^{2} + y_{3}^{2}\right)\right] \times \\ \times \left\{P(x_{3}, y_{3}) \otimes \exp\left[\frac{ikl_{1}(R - l_{1})}{2Rl_{2}^{2}}\left(x_{3}^{2} + y_{3}^{2}\right)\right] \times \\ \times \left\{t\left(-\mu\frac{R - \Delta l}{R}x_{3}, -\mu\frac{R - \Delta l}{R}y_{3}\right) \otimes \\ \otimes \exp\left[\frac{ik\mu^{2}(R - \Delta l)}{2\Delta lR}\left(x_{3}^{2} + y_{3}^{2}\right)\right]\right\}\right\} = \\ = \exp(ik\Delta l) \exp\left[\frac{ik}{2l_{2}}\left(x_{3}^{2} + y_{3}^{2}\right)\right] \left\{t\left(-\frac{l_{1} + \Delta l}{l_{2}}x_{3}, -\frac{l_{1} + \Delta l}{l_{2}}y_{3}\right) \times \\ \times \exp\left[\frac{ik(l_{1} + \Delta l)|R - l_{1}|}{2l_{2}^{2}(R + \Delta l)}\left(x_{3}^{2} + y_{3}^{2}\right)\right] \right\} \\ \otimes \exp\left[\frac{ikl_{1}(l_{1} + \Delta l)}{2\Delta ll_{2}^{2}}\left(x_{3}^{2} + y_{3}^{2}\right)\right] \otimes P(x_{3}, y_{3})\right\}.$$
(21)

In this case, at the stage of the two-exposure hologram reconstruction of the in-focus image of a plane surface that diffusely scatters light, the interference patterns are also localized in the two above-mentioned planes, and for recording the hologram, spatial filtering of the diffraction field is needed. Thus, the interferometer sensitivity to the diffuser longitudinal motion does not depend on radius of curvature of the converging spherical wave, incident on the diffuser when an interference pattern is recorded in a plane of the pupil image of a lens L.

In the case of recording the interference pattern localized in the hologram plane, the interferometer sensitivity changes by the factor  $G_4 = \mu^2 (R - l_1)^2 / R^2$ . In the example chosen where the quantities  $\lambda$  and  $l_1 = l_2$  are fixed, the interferometer sensitivity to the diffuser longitudinal motion decreases with the radius of curvature decreasing in the limits  $l_1 \leq R \leq \infty$  (see Fig. 4) down to that at  $G_4 = 0$ , when  $R = l_1$ . Then, at  $R = l_1$  the "frozen" interference pattern should localize in the plane of the lens L pupil image, which does not change with variation of the observation angle. Further decrease of the radius of curvature R (see Fig. 4) causes the appearance in the hologram

plane of a tilt angle of the subjective speckles of the second exposure that increases along radius from an optical axis, as compared with that of the identical speckles of the first-exposure pattern. For this reason, the interferometer sensitivity to the diffuser longitudinal motion increases at recording of the interference pattern in the plane 3 (see Fig. 3).

The cause of localization of the interference patterns in two planes in the considered case is different than that in the case of formation of the interference patterns characterizing the diffuser cross motion.

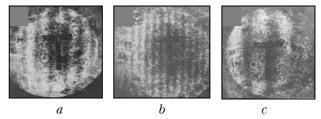
On the one hand, the tilt angle of the subjective speckles in the plane of second exposure hologram with respect to the speckles of the first exposure that changes when moving from the optical axis, is a necessary condition for formation in it of an interference pattern, as the speckles of two exposures turn out to be superposed. However, in this case the condition of the speckle identity is not satisfied. The subjective speckles, corresponding to the second exposure, have different amplitude-phase distribution determined by the function  $\exp\left[ik\mu^2\left(x_3^2+y_3^2\right)/2\Delta l\right] \otimes$ 

 $\otimes P(x_3, y_3)$ , in contrast to the distribution of the subjective speckles in the first exposure. In this connection, to obtain a superposition of identical speckle fields of the two exposures, a spatial filtering of the diffraction fields is needed.

On the other hand, isolation, by a spatial filtering of the diffraction field, of the subjective speckles corresponding to the first and second exposures, between which there is no a tilt angle, provides a superposition of the speckle fields in the plane of formation of the pupil image of the lens L caused by the diffraction of a plane wave on this lens's pupil at two positions of the diffuser.

In the experiment, the double-exposure holograms of the in-focus image of a matte screen were recorded on the photographic plates of a Mikrat-VRL type with the use of the He–Ne-laser radiation of 0.63 µm wavelength. An image with the unit magnification was formed with the help of a positive lens with the focal length f = 140 mm and the pupil's diameter d = 32 mm. The diameter of the illuminated area of a matte screen was about 35 mm. The angle between the reference beam and a normal to the plane of a photographic plate was equal to 10°. The experimental technique used consists in comparison of the holograms recorded with fixed distances of both the diffuser cross and longitudinal shifts equal, correspondingly, to  $a = 0.035 \pm 0.002$  mm and  $\Delta l = (2 \pm 0.002)$  mm. The radii of curvature of diverging spherical waves, used for illumination of a matte screen, were set to be in the ranges  $280 \text{ mm} \le R \le \infty$  and  $140 \text{ mm} \le R \le \infty$  for the converging spherical waves.

As an example, Fig. 5 presents the interference patterns localized in the hologram plane and characterizing the diffuser cross motion. The mark in the form of the letter "T" has been preliminary drawn on a matte screen. The interference patterns were recorded when applying a spatial filtering of the diffraction field on the optical axis in the plane of the pupil image formation of a positive lens (see Fig. 3) with the filtering aperture of 2 mm in diameter. Figure 5a shows the case, when at the stage of the double-exposure hologram recording, the matte screen was illuminated by a collimated beam. Figure 5brefers to illumination with a diverging spherical wave with the radius of curvature R = 280 mm and Fig. 5c to the case with a converging wave with R = 180 mm. In these cases, as well as in the subsequent ones, in which both the magnitude and a sign of the radius of curvature changed, measured periods of the interference fringes allowed the coefficients  $G_1$  and  $G_2$  to be determined (besides, they can be determined from measured values of Rand  $l_1$ ).



**Fig. 5.** The interference patterns localized in the hologram plane when at the stage of its recording, a matte screen was illuminated with a plane wave (a), with a spherical diverging wave (b), and with a spherical converging wave (c).

Thus obtained values of  $G_1$  and  $G_2$  correspond to Fig. 4 accurate to the experimental error of 10%. Besides, in all the cases of the double-exposure hologram recording the interference pattern (Fig. 6) localized in the plane of the pupil image of the positive lens L, with which the image of the matte screen *1* (see Fig. 1) is formed and used for determining the diffuser cross motion, had the same frequency of the interference fringes. The mark by the symbol " $\Pi$ " has been preliminary drawn on the lateral surface of a lens. Recording of the interference pattern localized in the plane of the pupil image of a lens was done according to Fig. 2 using spatial filtering of the diffraction field on the optical axis performed at reconstructing the hologram using a small-aperture (~2 mm) laser beam. Thus, Fig. 6 presents the particular case of the hologram reconstruction when at its recording the matte screen was illuminated with a converging spherical wave of radius of curvature  $R = l_1$ . In this case, the hologram was reconstructed using a wide collimated beam, and the interference

pattern was recorded using a 20-mm-diameter aperture diaphragm of the lens  $L_1$  (see Fig. 2), as in the recording plane 3 the "frozen" interference fringes were localized.

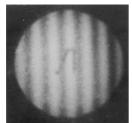


Fig. 6. The interference pattern, localized in the plane of the pupil image formation of a positive lens.

If spatial filtering of the diffraction field is performed in the hologram plane outside of the optical axis, for example at the point with the coordinates of  $x_{03}$ , 0, and the diameter of the filtering aperture does not exceed the width of an interference fringe of the interference pattern localized in the hologram plane, the distribution of the complex field amplitude at the exit of the spatial filter takes the following form

$$u(x_{3}, y_{3}) \sim p_{0}(x_{3}, y_{3}) \exp\left\{\frac{ik}{2l_{2}}\left[\left(x_{3} + x_{03}\right)^{2} + y_{3}^{2}\right]\right\} \times \\ \times \left\{t\left(-\mu x_{3} - \mu x_{03}, -\mu y_{3}\right) \exp\left\{\frac{ik\mu(R+l_{1})}{2Rl_{2}}\left[\left(x_{3} + x_{03}\right)^{2} + y_{3}^{2}\right]\right\} \otimes \\ \otimes P(x_{3}, y_{3}) + t\left(-\mu x_{3} - \mu x_{03}, -\mu y_{3}\right) \exp\left[\frac{ik(R+l_{1})ax_{03}}{Rl_{2}}\right] \times \\ \times \exp\left[\frac{ik(R+l_{1})a^{2}}{2Rl_{1}}\right] \exp\left\{\frac{ik\mu(R+l_{1})}{2Rl_{2}}\left[\left(x_{3} + x_{03}\right)^{2} + y_{3}^{2}\right]\right\} \otimes \\ \otimes \exp\left[\frac{-ik(R+l_{1})ax_{3}}{Rl_{2}}\right] P(x_{3} - a/\mu, y_{3})\right\}.$$
(22)

In the case of such a spatial filtering, distribution of the complex field amplitude in the plane  $(x_4, y_4)$ (see Fig. 2) with the account of the function  $p(x_2, y_2)$ parity (without the constant phase term  $k(R + l_1) \times x ax_{03}/Rl_2$ ) is determined by the expression

$$u(x_4, y_4) \sim \left\{ \left\{ \exp\left[-\frac{ik\mu x_{03}(x_4 - x_{03})}{l_2}\right] F(x_4 - x_{03}, y_4) \otimes \right. \\ \left. \left. \left. \left. \left. \exp\left[\frac{ikx_{03}(x_4 - x_{03})}{l_2}\right] \times \right. \right. \right. \\ \left. \left. \left. \left. \left. \left. \left. \left. \left. \frac{ikR}{2l_1(R + l_1)} \right[ \left(x_4 - x_{03}\right)^2 + y_4^2 \right] \right] \right\} \right\} \right\} p(x_4 - x_{03}, y_4) + \left. \frac{ik\mu x_{03}(x_4 - x_{03})}{l_2}\right] F(x_4 - x_{03}, y_4) \right. \right. \right. \right\} \right\} \right\} \right\} \right\}$$

$$\otimes \exp\left[\frac{-ik(R+l_{1})a^{2}}{2Rl_{1}}\right] \exp\left\{-\frac{ikR}{2l_{1}(R+l_{1})}\left[(x_{4}-x_{03})^{2}+y_{4}^{2}\right]\right\} \right\} \times \\ \times p(x_{4}-x_{03},y_{4}) \exp\left[\frac{-ika(x_{4}-x_{03})}{l_{1}}\right] \right\} \otimes \\ \otimes P_{0}(x_{4},y_{4}).$$
(23)

Based on Eq. (23), the distribution of illumination over the recording plane 3 (see Fig. 2) takes the following form

$$I(x_{4}, y_{4}) \sim \left\{1 + \cos\left[\frac{k}{l_{1}}a(x_{4} - x_{03}) + \frac{k(R + l_{1})a^{2}}{l_{1}R}\right]\right\} \times \\ \times \left|\left\{\exp\left[-\frac{ik\mu x_{03}(x_{4} - x_{03})}{l_{2}}\right]F(x_{4} - x_{03}, y_{4})\otimes\right. \\ \left.\otimes \exp\left[\frac{ikx_{03}(x_{4} - x_{03})}{l_{2}}\right] \times \\ \left.\times \exp\left\{-\frac{ikR}{2l_{1}(R + l_{1})}\left[(x_{4} - x_{03})^{2} + y_{4}^{2}\right]\right\}\right\} \times \\ \left.\times p(x_{4} - x_{03}, y_{4})\otimes P_{0}(x_{4}, y_{4})\right|^{2}.$$
(24)

As it follows from Eq. (24), at variation of  $x_{03}$ , there is a displacement caused by parallax of the pupil image of a positive lens, with the help of which the hologram recording of the diffuser in-focus image was carried out. In this case, the interference fringes also shift, while their frequency keeps unchanged. Besides, in moving interference fringes ("living" interference fringes), the phase of the interference pattern changes from 0 to  $\pi$ , when the center of the filtering aperture moves from the minimum of an interference fringe, for the interference pattern localized in the hologram plane, to its maximum.

At reconstruction of the double-exposure holograms, characterizing the longitudinal motion of the plane surface diffusely scattering the incident light using spatial filtering of the diffraction field on the optical axis (see Fig. 2) in the plane of the positive lens L pupil image, an interference pattern presented in Fig. 7*a* is formed. In addition, in all cases of the hologram recording, when a matte screen *t* in Fig. 1 was illuminated with the diverging and converging waves the interference pattern remains unchanged and involves same number of the interference orders. The measured radii of the rings in the adjacent interference orders, yield the value of the longitudinal motion  $\Delta l = 2$  mm accurate within the experimental error of 10%.

If spatial filtering in the hologram plane is performed out of the optical axis, for example at the point with the coordinates  $x_{03}$ , 0, and the diameter of a filtering aperture does not exceed the width of the interference fringe for the interference pattern, localized in the hologram plane, the distribution of the complex field amplitude at the exit of a spatial filter (without the constant phase term  $k\Delta l(R + l_1)^2 \times x_{03}^2/2R^2l_2^2$ ) is determined by the expression

$$u'(x_{3}, y_{3}) \sim p_{0}(x_{3}, y_{3}) \exp\left\{\frac{ik}{2l_{2}}\left[(x_{3} + x_{03})^{2} + y_{3}^{2}\right]\right\} \times \\ \times \left\{t(-\mu x_{3} - \mu x_{03}, -\mu y_{3}) \exp\left\{\frac{ik\mu(R + l_{1})}{2Rl_{2}}\left[(x_{3} + x_{03})^{2} + y_{3}^{2}\right]\right\} \otimes \\ \otimes P(x_{3}, y_{3}) + \exp(ik\Delta l)t(-\mu x_{3} - \mu x_{03}, -\mu y_{3}) \times \\ \times \exp\left\{\frac{ik\mu(R + l_{1})}{2Rl_{2}}\left[(x_{3} + x_{03})^{2} + y_{3}^{2}\right]\right\} \otimes \\ \otimes \exp\left\{\frac{ik\mu^{2}}{2\Delta l}\left[(x_{3} + x_{03})^{2} + y_{3}^{2}\right]\right\} \otimes P(x_{3}, y_{3})\right\}.$$
(25)

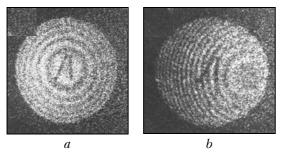


Fig. 7. The interference patterns, localized in the plane of the pupil image formation of a positive lens and recorded using spatial filtering in the hologram plane: on the optical axis (a), out of the optical axis (b).

For the case of such a spatial filtering, the distribution of the complex field amplitude in the plane  $(x_4, y_4)$  (see Fig. 2) takes the following form

$$u'(x_4, y_4) \sim \left\{ \left\{ \exp\left[\frac{-ik\mu x_{03}(x_4 - x_{03})}{l_2}\right] F(x_4 - x_{03}, y_4) \otimes \right\} \right\}$$

$$\otimes \exp\left[\frac{iRx_{03}(x_4 - x_{03})}{l_2}\right] \exp\left\{-\frac{iRR}{2l_1(R + l_1)}\left[(x_4 - x_{03})^2 + y_4^2\right]\right\}\right\} \times$$

$$\times p(x_{4} - x_{03}, y_{4}) + \exp(ik\Delta l) \left\{ \exp\left[-\frac{ik\mu x_{03}(x_{4} - x_{03})}{l_{2}}\right] \times F(x_{4} - x_{03}, y_{4}) \otimes \exp\left[\frac{ikx_{03}(x_{4} - x_{03})}{l_{2}}\right] \times \exp\left\{-\frac{ikR}{2l_{1}(R + l_{1})}\left[(x_{4} - x_{03})^{2} + y_{4}^{2}\right]\right\} \right\} \times p(x_{4} - x_{03}, y_{4}) \exp\left[\frac{ikx_{03}(x_{4} - x_{03})}{l_{2}}\right] \times \exp\left\{-\frac{ik\Delta l}{2l_{1}^{2}}\left[(x_{4} - x_{03})^{2} + y_{4}^{2}\right]\right\} \right\} \otimes P_{0}(x_{4}, y_{4}).$$
(26)

As follows from Eq. (26) the distribution of illumination over the recording plane 3 (see Fig. 2) is determined by the following expression

$$I'(x_{4}, y_{4}) \sim \left\{ 1 + \cos\left\{ k\Delta l - \frac{k\Delta l}{2l_{1}^{2}} \left[ (x_{4} - x_{03})^{2} + y_{4}^{2} \right] + \frac{kx_{03}}{l_{2}} (x_{4} - x_{03}) \right\} \right\} \left\{ \exp\left[ \frac{-ik\mu x_{03} (x_{4} - x_{03})}{l_{2}} \right] \times F(x_{4} - x_{03}, y_{4}) \otimes \exp\left[ \frac{ikx_{03} (x_{4} - x_{03})}{l_{2}} \right] \times \exp\left\{ -\frac{ikR}{2l_{1}(R + l_{1})} \left[ (x_{4} - x_{03})^{2} + y_{4}^{2} \right] \right\} \right\} \times p(x_{4} - x_{03}, y_{4}) \otimes P_{0}(x_{4}, y_{4}) \right|^{2}.$$
(27)

From Eq. (27) it follows that, as in the case of recording the double-exposure hologram for determining the diffuser cross motion, the image of the positive lens pupil and the center of the axially symmetric interference pattern are displaced at the same distance because of the parallax. Besides, redistribution of the phase of an interference pattern occurs because of the linear term in Eq. (27) that is shown in Fig. 7b. Figure 7b corresponds to spatial filtering of the diffraction field in the hologram plane being done at the point with the coordinates of  $x_{03} = 15$  mm, 0. In addition, Fig. 7 also presents the case when the hologram recording was carried out using illumination of a matte screen with the converging spherical wave with radius of curvature  $R = l_1$  and demonstrates the cause why no "frozen" interference fringes exist in the plane  $(x_4, y_4)$  (Fig. 2) under such conditions of the double-exposure hologram recording. Besides, it becomes obvious, that a superposition of the interference patterns in the plane  $(x_4, y_4)$  (Fig. 3) from all the elementary areas of the diffuser holographic image makes impossible the recording of the interference pattern localized in the hologram plane, in form of the concentric rings with several orders of interference, because in this case according to expression (27) the diameter of the filtering aperture in the plane  $(x_4, y_4)$  (see Fig. 3) should be equal to the speckle size.

In its turn, at small longitudinal displacement of the diffuser, performed together with its cross shift, it is possible to record the interference pattern localized in the hologram plane when the number of interference fringes within the image of the positive lens' pupil varies insignificantly depending on position of a filtering aperture in the hologram plane. Figures 8*a* and *b* show the interference patterns localized in the plane of the image of the positive lens' pupil that correspond to the cross shift of the matte screen  $a = (0.035 \pm 0.002)$  mm and the longitudinal displacement  $\Delta l = (0.15 \pm 0.002)$  mm. Their recording was done at the spatial filtering performed in the hologram plane on the x-axis and in the edges of the diffuser image. In addition, the hologram recording was performed at illumination of the matte screen by a plane wave. At the hologram reconstruction, according to Fig. 3, the recorded interference pattern localized in its plane charactering the diffuser longitudinal and cross motions is presented in Fig. 8c.

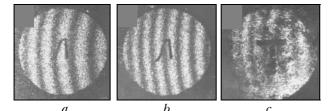


Fig. 8. The interference patterns localized in a plane of the positive lens' pupil image formation (a and b) and in the hologram plane (c).

Thus, the results of the theoretical analysis of formation of the interference patterns characterizing the cross or longitudinal motion of a plane surface diffusely scattering the incident light, at the doubleexposure hologram recording of the diffuser in-focus image and the experiments carried out have shown the following.

The interference patterns, characterizing the diffuser cross motion, are localized in the plane where the image is formed of the positive lens pupil, with the help of which the hologram recording was carried out, and in the hologram plane. For their recording, spatial filtering of the diffraction field in the corresponding planes is needed. The interferometer sensitivity, on the one hand, depends neither on the sign nor on the size of the radius of curvature of the **s**pherical wave used for the diffuser illumination at the stage of the hologram recording, in case of recording the interference pattern in the plane of formation of the positive lens' pupil image, while on the other hand, it depends on both the sign and size

of the radius of curvature of the spherical wave at recording the interference pattern localized in the hologram plane.

At the hologram reconstruction, when before the re-exposure of a photographic plate, the diffuser is moved along the optical axis and spatial filtering of the diffraction field is performed on the optical axis in the plane of formation of the positive lens' pupil image, the interference pattern of fringes of equal tilts is formed. Thus, the interferometer sensitivity depends neither on the sign nor the size of the radius of curvature of the spherical wave used for the diffuser illumination at the stage of the hologram recording. In its turn, the known properties<sup>4,10</sup> of the subjective speckles in the hologram plane of the diffuser in-focus image, which appear due to diffraction of the plane waves on the pupil of the positive lens, do not allow one to record the interference pattern localized in the hologram plane in the form of concentric rings involving several orders of interference.

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