Investigation into the empirical distributions of atmospheric parameters and their correspondence with the normal law

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The paper presents the results of the investigation into the empirical distributions of geopotential, temperature and orthogonal components of the wind velocity in the troposphere (up to the level of 300 hPa). Assessment is made of their correspondence with the normal distribution law.

It is well known that in solving a wide range of practical problems it is important to meet the condition that the empirical distributions of atmospheric parameters obey the normal law. In particular, such a condition should be taken into account in developing the algorithms of spatial prediction of the fields of geopotential, temperature and the wind using Kalman filter,[1] because in this case it is required to fulfill the normality of distribution of the initial (measured) and estimated parameters.

Hence, the assessment of the correspondence between the empirical distributions of different atmospheric parameters (including the above-mentioned ones) and the normal distribution law is of current interest and has great practical importance. Besides, up to now such an assessment was made only for the geopotential and temperature in the troposphere[2,3] and was not considered for the case of atmospheric boundary layer. In addition, it was not concerned with the orthogonal components of the wind velocity. Having this in mind, I present some results of my study into this problem.

The simplest method of checking the normality of empirical distributions is visual checking of the distribution histograms. As an example, Figure 1 shows the distribution histograms of the geopotential, temperature, and orthogonal components of the wind velocity plotted for the summer season based on the data of Brest station (52°07′N, 23°41′E). In this case, the abscissas show the values of the meteorological parameters studied, and the ordinates present the number of cases. The intervals of the parameter gradation chosen are: 5 dam for geopotential, 2°C for the temperature, and 2 m/s for zonal and meridional components of the wind speed. It should be noted that such histograms were constructed also for the winter season and other stations, however, because of the limitedness of the scope of this paper, these histograms are not presented here. Note that the use of only the distribution histograms will not answer the above question and to answer, it is necessary to assess the divergence of the empirical and theoretical distributions using any goodness–of–fit test. In practice, the Pearson (chi–square) compatibility test χ^2 is very often used (see Refs. 2 and 3) for this purpose. However, in recent few years it is recommended, in accordance with the All-Union State Standard RISO5479-2002,[4] to use nonparametric tests of the Kolmogorov–Smirnov type instead of this test, because this test "gives the best fit for classified data," and, in addition, "classification of data results in the information loss." Therefore, I have used the Kolmogorov–Smirnov (ks) compatibility test for the sample of data set chosen. Besides, I used, for this purpose, the coefficients of asymmetry (SR) and excess (E).[5]

In this case to assess the hypothesis on the normality of a distribution, I set the level of significance $p = 0.05$, for which, according to Ref. 6, $ks_{theor} = 1.36$ and also the condition $ks_{act} \le ks_{theor}$, which enables one to approximate the empirical distribution taken as normal.

At the same time for estimating SR (skewness) and E (kurtosis) of empirical distributions, as compared with the normal distribution, the following formulas[5] were used:

$$\sigma_{SR} = \sqrt{\frac{6}{n+4}}, \quad \sigma_E = \sqrt{\frac{24(n-5)}{n(n+7)}},$$

where σ_{SR} is the standard rms error of the asymmetry coefficient, σ_E is the same error of the excess coefficient; n is the number of realizations taken.

In this case, according to Ref. 7, for the level of significance $p > 0.05$ the following conditions should be fulfilled: $|SR| \le 2.09\sigma_{SR}$ and $-1.99\sigma_E \le E \le 2.43\sigma_E$.

In the table given below as an example, the results of assessment are given (using the above parameters) of the correspondence with the normal law of empirical distributions shown in Fig. 1. The boundary conditions at a given number of observations are: $|SR| = 0.307$, $-0.575 \le E \le 0.702$.

From analysis of data given in the table, it follows that at all the levels in the atmosphere the empirical distributions of the geopotential, temperature, zonal and meridional wind correspond to the normal law.

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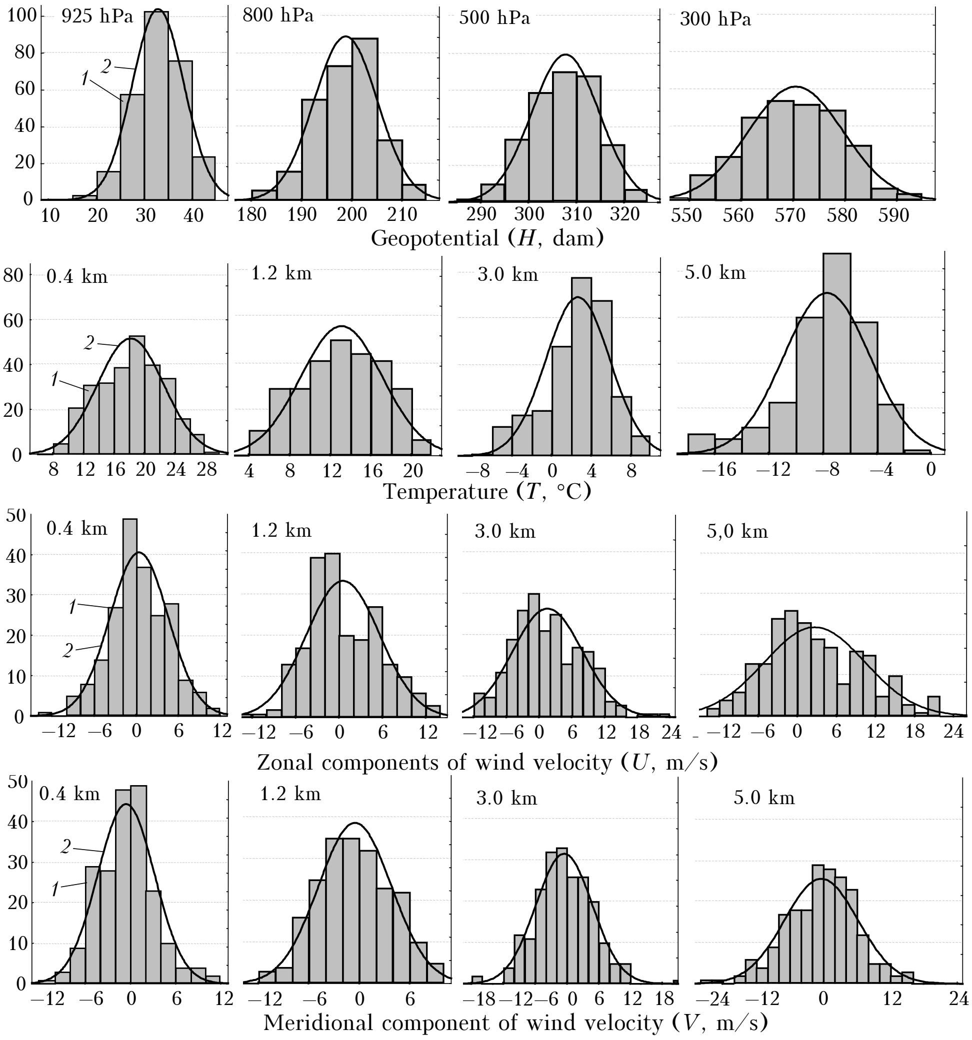

Fig. 1. Empirical (*1*) and normal (*2*) distributions of geopotential, temperature, zonal and meridional components of wind velocity for some pressure levels.

The estimated correspondence between the empirical distributions of geopotential, temperature, and orthogonal components of the wind velocity and the normal distribution law. Summer

Level, hPa	Number of observations	SR	E	ks_{act}	ks_{theor}
		Geopotential, dam			
925	275	–0.301	0.109	0.95	1.36
800	275	–0.280	–0.030	0.85	1.36
500	275	–0.070	–0.530	0.92	1.36
300	275	–0.220	–0.500	1.02	1.36
Altitude, km		*Temperature,* °C			
0.4	275	0.012	–0.521	0.85	1.36
1.2	275	–0.052	–0.573	0.82	1.36
1.6	275	–0.120	–0.532	0.79	1.36
3.0	275	–0.281	–0.016	1.34	1.36
5.0	275	–0.303	0.600	1.35	1.36
		Zonal wind, m/s			
0.4	275	0.066	0.197	1.19	1.36
1.2	275	0.272	–0.311	1.36	1.36
1.6	275	0.251	–0.294	1.34	1.36
3.0	275	0.271	0.009	1.00	1.36
5.0	275	0.300	–0.070	1.14	1.36
		Meridional wind, m/s			
0.4	275	0.249	0.498	0.85	1.36
1.2	275	0.098	–0.520	0.73	1.36
1.6	275	–0.029	–0.510	0.83	1.36
3.0	275	0.143	0.531	0.62	1.36
5.0	275	0.097	0.625	0.80	1.36

Thus, we can draw the conclusion that the empirical distributions of the above-mentioned meteorological quantities in the atmospheric boundary layer and in the troposphere can be described to sufficient accuracy by the normal distribution. Hence, those can successfully be used in solving different practical problems where the observance of this condition is necessary.

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