Dynamical probabilistic modeling of climate processes of admixture transport on a regional scale

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For a given local region of the Northern Hemisphere, a dynamical probabilistic modeling is used to construct a local climate ensemble of six-hour spatiotemporal realizations of the fields of meteorological quantities. This ensemble is matched with the corresponding background regional climate ensemble using a variational method of information assimilation. For construction of ensemble of realizations, a combined statistical/hydrodynamic modeling is used involving real information on the statistical structure of the corresponding fields of meteorological variables. Based on this ensemble, we propose a multistep method of climate modeling.

Introduction

The main approach used in this work is a well known representation of the climate of the atmosphere in terms of the ensemble of possible realizations of the corresponding multidimensional hydrometeorological fields for chosen time interval and for a particular geographic region:

$$\{\xi^i_{(n)},\ i = 1,\ 2,\ \ldots\}, \qquad (1)$$

where

$$\xi^i_{(n)} = [\mathbf{U}^i(\mathbf{X}_j,\ t_k), T^i(\mathbf{X}_j,\ t_k), H^i(\mathbf{X}_j, t_k),\ \ldots]^{\mathrm{T}}$$

is the vector of realizations of the fields of velocity, temperature, geopotential, etc., at space and time points $(\mathbf{X}_j,\ t_k)$ of the considered grid domain of n dimensionality.

For constructing an ensemble of realizations, we use a combined statistical/hydrodynamic modeling incorporating real information on the statistical structure of the corresponding fields of meteorological quantities. The combined modeling involves matching of hydrodynamic numerical regional- and local-scale models with the corresponding probabilistic models, as well as with real data obtained based on variational data assimilation. The result of a numerical modeling is the corresponding climatic ensemble of realizations for a local-scale area.

It is worthy to note that the dynamical model specified using a system of differential equations serves an interpolant into the space–time points of the fields considered. It insures mutual matching and filtering out of non-physical components; while the statistical model provides a given probabilistic structure of the process at hands and assigns the corresponding ensemble of independent realizations of these fields. The application of variational method of data assimilation makes it possible to optimize the process of combined use of dynamical and statistical methods of numerical simulation, as well as dynamical models of varying the scale and physical content.

The idea of this method is as follows. It assumes that the statistical structure of the fields under consideration is approximately known. In accordance with this structure, an ensemble of realizations is constructed using approximate methods of statistical simulation.[1] Realizations of the fields of this ensemble, made using the dynamical model, are used as inputs for the problem of variational data assimilation. As a result, we obtain a new ensemble, in which every realization satisfies the dynamical model, while the statistical structure of the newly obtained ensemble is close to the initial within the accuracy of assimilation problem.

In this paper, the above-mentioned approach is used for a combined modeling of ensemble of climatic fields for local and regional domains.[2]

1. Variational assimilation

The method of variational assimilation is based on solution of the problem on minimizing the quality functional[1,2] using the method of gradient descent. The quality functional characterizes the differences between realization of stochastic field and the corresponding field obtained by solving the system of dynamical equations. To determine the gradient of quality functional, we use solutions of the main and adjoint problems, corresponding to the dynamical model considered here. In the end the mean value of quality functional, seemingly, determines the measure of difference between the initial statistical structure and statistical structure obtained by means of a combined modeling.

The idea of the method of variational assimilation is as follows. We consider a numerical model,[1–4] written in the operator form as

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$$\frac{\partial \mathbf{\Phi}}{\partial t} + A(\mathbf{Y}, \mathbf{\Phi})\mathbf{\Phi} = 0, \qquad (2)$$

where $\mathbf{\Phi}$ is the vector of the state; $\mathbf{Y} = \mathbf{\Phi}|_{t=0}$ is the vector of parameters; $A(\mathbf{Y}, \mathbf{\Phi})$ is the nonlinear finite-difference operator, determined by the system of equations of the process considered above and by the corresponding boundary conditions in the region $G_t = G \otimes [0, \hat{t}]$. The system of equations (2) engenders a set of solutions which depend on the vector of parameters $\mathbf{Y}$. Among this set of solutions, it is necessary to find the one most close to the prescribed vector of measurements in the sense of some quality functional J_0, which will be written as

$$J_0 = \frac{1}{2}\sum_k (L\mathbf{\Phi}^j - \mathbf{\Phi}_S^k,\, L\mathbf{\Phi}^j - \mathbf{\Phi}_S^k)_{D_S},$$

where $(,)_{D_S}$ is the scalar product in the space of measured data $\mathbf{\Phi}_S^k$; L is the corresponding interpolation operator, while $\mathbf{\Phi}^j$ is the solution of the problem (2) at the moment in time t_j. Thus, it is necessary to find a minimum of the functional J_0 with respect to the vector of parameters $\mathbf{Y}$, given limitations (2).

To solve this problem, we use iteration method of gradient descend, based on the Lagrange method and the solution of the corresponding direct and adjoint problems. We note that in the general case, the minimum of the functional J_0 is not unique, because of nonlinearity of the system (2) and degree of completeness of the assimilated data $\mathbf{\Phi}_S^k$. In this case the minimum is determined by the initial guess for the vector $\mathbf{Y}$ of the corresponding iteration process.

Thus, the algorithms considered above, determine the dynamical-probabilistic model for numerical construction of an ensemble of realizations of the multidimensional fields, which optimally takes into account both physical and probabilistic properties of the processes studied.

2. Use of dynamical-probabilistic models for solution of applied problems

By solving a sequence of assimilation problems, for each realization from Eq. (1) we obtain a new ensemble of realizations

$$\{\boldsymbol{\xi}^i_{(n)},\ i = 1, 2, \ldots\}, \qquad (3)$$

whose properties are discussed extensively elsewhere.[2]

The properties of the vertical structure of the temperature field are within the ensemble mean of the functional J_0 in the corresponding problems of variational assimilation. Below the main diagonal in Table there are the corresponding correlations calculated over ensemble (3) using a special interpolation of the correlations from calculated levels into the standard ones. These correlations well agree with the corresponding values calculated using real data.[2]

Analysis of statistical structure of the ensemble (3) shows that this ensemble can be used as a climatic one for subsequent solving applied problems and, in particular, the problem of admixture transport in the atmosphere.[3,4]

Table

P,	P, mbar							
mbar	1000	850	700	500	400	300	200	100
1000	1.00	0.78	0.62	0.52	0.46	0.23	–0.24	–0.10
850	0.67	1.00	0.75	0.55	0.43	0.16	–0.29	–0.15
700	0.56	0.74	1.00	0.68	0.52	0.16	–0.34	–0.23
500	0.45	0.66	0.78	1.00	0.77	0.26	–0.35	–0.31
400	0.40	0.50	0.64	0.90	1.00	0.51	–0.26	–0.29
300	0.30	0.20	0.38	0.46	0.66	1.00	0.21	–0.05
200	–0.24	–0.48	–0.54	–0.60	–0.58	–0.08	1.00	0.45
100	–0.13	–0.42	–0.48	–0.61	–0.71	–0.49	0.52	1.00

In modeling the climatic ensemble on a regional scale, it is reasonable to take into account the background atmospheric processes in the region comprising the local region studied, as well as the corresponding statistical characteristics of these processes. As has already been mentioned above, the statistical structure of the background atmospheric processes can be specified either as a set of different statistical characteristics or in terms of the corresponding ensemble of realizations. The latter is preferable in our case. However, the main difficulty is that the fields of hydrometeorological variables are specified with respect to the local region on quite a sparse and irregular grid.

The use of the above-mentioned variational method of data assimilation makes it possible to efficiently overcome this difficulty. At the same time, as was already noted above, the local numerical model serves as a spatiotemporal interpolant, allowing us to accomplish the so-called "telescopization" process for climatic ensemble as a whole. The numerical local-scale model is obtained through the corresponding regional model adapted to include modules for modeling the corresponding physical processes in the atmosphere.

To obtain the local climatic ensemble, we have chosen a spherical domain of 10° latitude by 20° longitude in size, with the center having coordinates $\varphi_0 = 60.56°$N and $\lambda_0 = 77.7°$E. The task was completed in the system of coordinates x, y, p in rectangle in plane, tangent at the point (φ_0, λ_0) of the Northern Hemisphere, with the resolution 24×20 along x- and y-axes and with the steps $\Delta x = 48.069$ and $\Delta y = 58.74$ km, respectively. In vertical coordinate p, the calculations were made for 10 standard atmospheric levels from regional model.[1] The rectangle comprises a projection of the chosen local spherical region onto the plane. In accordance with the above-mentioned procedure of variational assimilation for the problem (2), the input data are realizations from ensemble (3) for regional domain with the values at points belonging to the corresponding projections in the rectangle. The solution of the problem of variational assimilation by use of local numerical model yields the ensemble of realizations

$$\{\tilde{\boldsymbol{\eta}}^i_{(n)},\ i = 1, 2, \ldots\}, \tag{4}$$

some properties of which will be presented below.

Above the main diagonal in the Table there are also the inter-level correlation of temperature, which shows qualitatively close character of the obtained correlations to the corresponding correlations of the background climatic ensemble, whose values are presented below the main diagonal. The horizontal structure of correlation is most characteristically represented by the correlation function of velocity field as shown in Fig. 1. It is seen from it that the qualitative character of correlation dependences for the local region has not changed significantly considering the interpolation made into a finer grid.

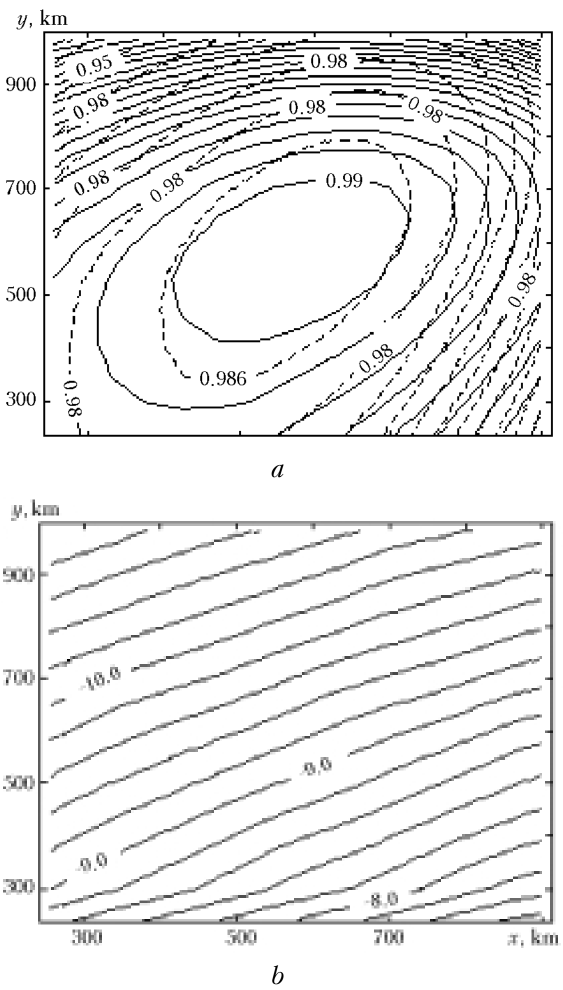


Fig. 1. Contour lines of correlations of the wind velocity components at the level of 500 mbar, calculated over climatic ensemble for a local region (*a*), and the corresponding mean temperature field, °C, at the level of 450 mbar (*b*).

Since the constructed realizations are statistically independent and make up a climatic ensemble, we can use them in describing the transport of passive tracer in the atmosphere. These also can make up the entire ensemble of realizations of spatiotemporal admixture fields on the time interval selected. The admixture transport is simulated using three-dimensional modification of the numerical transport model based on van Leer quasi-monotonic numerical scheme.[2]

As the climatic admixture distribution over a local region, we shall take the corresponding average value over the ensemble of realizations of the admixture fields obtained. In virtue of statistical independence of the realizations, this process can be iterated until the admixture perturbation, represented by the ensemble average, leaves the local region. Thus, we obtain a certain sequence of ensembles of realizations of the admixture fields in the local region

$$\{\mathbf{c}^{(i,k)}_{(n)}, i = 1, 2, \ldots;\ k = 1, 2, \ldots, M\}, \tag{5}$$

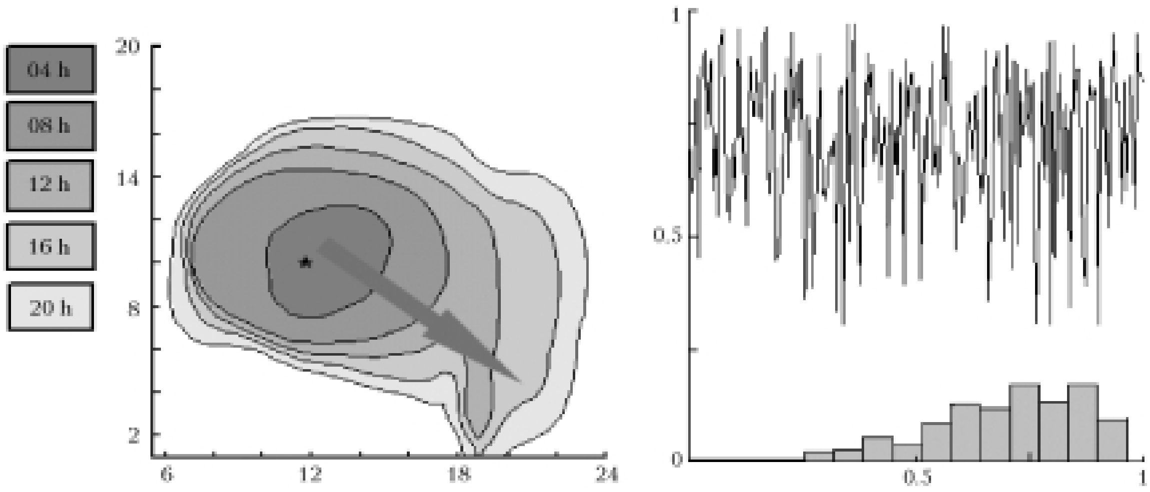
where i is the realization number of the kth ensemble; and M is the number of local ensembles. In numerical simulation of the realizations of the kth ensemble, we use, as the first-guess admixture field, either the initial admixture field ($k = 1$) or the field obtained by averaging the preceding ensemble with the number $k-1$ ($k > 1$) from Eq. (5) at the last moment in time. As a result, after averaging of all the ensembles of sequence (5) we obtain some fit of climatic trajectory of the admixture propagation from a pollution source specified by the initial distribution. For analysis of this trajectory, we use the well-known method of indicator functions. To this end, we introduce the function

$$\chi(f, lev) = \begin{cases} 1, & \text{if } f \ge lev, \\ 0, & \text{if } f < lev, \end{cases} \tag{6}$$

where f is the value of tested function, while lev is some prescribed number. Values of this function, equal unity in the considered region of averaged sequence of realizations from ensembles (5) and determine the approximate climatic trajectory Tr of the admixture at a given threshold lev value.

In numerical experiments conducted, we used the model admixture field, localized in the region of 5×5 points at $p = 850$ mbar level with the center shown by the asterisk (Fig. 2*a*) and maximum value equal to unity at this center as initial admixture field determining the instant pollution source.

The horizontal transect of this four-dimensional trajectory Tr for $lev = 0.05$ in the considered region is presented in Fig. 2*a* for 500 mbar level, at which the coordinate axes indicate the number of grid nodes across the region in horizontal. The arrow shows general climatic direction of the polluted



a *b*

Fig. 2. Statistical properties of the climatic trajectory of admixture transport.

volume transport at this level, though each individual trajectory realization from the sequence of ensembles (5) may differ from trajectory Tr. In the upper part of Fig. 2*b* we present the ratio of the number of trajectory points of an individual realization, belonging to trajectory Tr, to the total number of points in the realization of this individual trajectory. The abscissa on this plot shows the ratio of the realization number to the total number of realizations. From Fig. 2*b* it is seen that, at a quite large scatter of this quantity, its average value is close to 0.75. The histogram of the distribution of this ratio is presented in the lower part of Fig. 2*b*. These characteristics show that, in this problem formulation, the admixture transport has some preferred direction that makes it possible to identify most probable region of the given pollution level.

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