Statistics of structural state fluctuations of a laser beam disturbed by a jet of aircraft engine

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It has been shown that when a laser beam passes through a strongly turbulent aircraft engine jet, the pulses with the quasiregular spatial structure and high angular radiation concentration can be observed. The probabilities of occurrence of such "quasiregular" pulses have been determined for different experimental conditions and various radiation wavelengths (λ = 1.06 and 0.53 μm). Analysis of experimental data has shown that depending on the jet flow regime the statistics of random sequence of the quasiregular pulses can obey both Poisson's law and the binomial law. The averaged angular width and the dispersion of random shifts of centroids for beams crossing the jet close to its axis or side boundary are determined.

Introduction

Investigations of fluctuations of a laser beam structural state in the atmosphere are important from the viewpoint of both the development of theoretical models of the turbulent atmosphere impact on laser radiation[1] and practical implementation, because the obtained results would enable us to assess and optimize the conditions of profitable operation of laser information systems under noise conditions.[2,3] Sporadic change of stochastic state of a disturbed beam divided into the multitude of speckles, by another quasiregular state characterized by its relatively weak perturbations, depends upon the effect of intermittence of vortex formation structure in the zone crossed by a beam. Such a behavior of the beam was observed repeatedly in the natural atmosphere.[1,2] In this paper we consider similar structure fluctuations of a beam crossing a jet of an aircraft turbojet engine, i.e., the zone with a turbulence level characterized by a structural characteristics $C_n^2 \sim 10^{-9}$ m$^{-2/3}$, i.e., by 4 or 6 orders of magnitude exceeding the level of the atmospheric turbulence.

1. Experimental investigations

Conditions and the complete optical scheme of the experiment are described in detail in our previous paper,[4] therefore here we give only some important experimental details.

A turbulent jet source was a turbojet engine of P-25-300 type with a nozzle diameter of 55 cm. In the experiments, the mean measured temperature T in the jet at the engine output was ≈ 380°C and its mean axial velocity V at 1 m distance from the nozzle cut was ≈ 600 km/h. Two experimental cycles were conducted under conditions differed only by a position of laser beam crossing the jet cross section. In the first case the beam crossed the jet axis at a distance about $0.1D$ from the lateral boundary, where D is the jet diameter. In the second case the beam crossed the jet diameter. In the crossing zone the jet diameter was 0.7–0.8 m. In the experiments, the laser beam crossed the jet close to its lower lateral boundary at a distance of 5–10 cm from it. In this case the jet nozzle was directed upward at an angle of ~ 20° to a horizontal plane in order to avoid the occurrence of air streams reflected from the Earth's surface close to the investigated zone and to exclude their effects on the measurements. Note that the angular jet aperture φ was ~ 35°.

The measurements were conducted for collimated laser beams of different diameters (10 and 30 mm) with homogeneous distribution of intensity within the limits of the aperture located at a distance of ~ 2 m from the aircraft engine jet boundary, the radiation wavelengths λ were 1.06 and 0.53 μm. Laser system operated at a frequency of 12.5 Hz, and the pulse duration was 20–30 ns. The laser beam cross sections were recorded at each pulse simultaneously at the two wavelengths using CCD cameras located in the focal plane of the receiving objective with a focal distance of 270 cm and a large diameter (30 cm) to provide total interception of the disturbed beams. The distance between the aperture and the objective was about 80 m.

Note that the images recorded under such recording scheme characterize the angular spectrum of a laser beam disturbed by a turbulent jet. During measurements, for each above version of the experiment no less than 1500 frames were recorded. Such realization of measurements provided the recording of "instantaneous" images of laser beams with different wavelengths passed through the same inhomogeneities in a high-rate turbulent jet.

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Figure 1 shows the examples of the "instantaneous" photographs of laser beam cross sections recorded by the CCD camera. When analyzing the totality of frames we have discovered that at crossing the jet periphery by a 1-micron laser beam, a large number of frames demonstrate the high angular radiation concentration, i.e., the beam has a quasiregular structure of the angular distribution core at these moments. Such distributions are marked by an arrow in the frames given in Fig. 1. It is believed that this is just a consequence of the effect of intermittence of vortex formation structures in the turbulent jet.

It is reasonable to expect that this result should be obtained in the case that the beam crosses a boundary jet area-zone where the probability of the random in time interchange of strongly- and weakly-turbulent states of the medium is high. However, as it resulted from the data processing, such effect took place when crossing the jet diameter by a narrow (10 mm) 1-micron beam. At the same time, for 1-micron beam of 30 mm diameter the frequency of realization of quasiregular structure of the angular beam spectrum is very low in this experimental situation.

In the experiments with a half-micron radiation, another pattern was observed, i.e. the angular intensity distribution of the disturbed laser beam was practically always stochastic.

Qualitative results of visual analysis of the frequency of occurrence of "quasiregular" pulses are also confirmed by statistical processing of the obtained representative ensembles of random realizations of intensity distribution in the beam cross sections. Table 1 shows the experimental data on statistical probability of the appearance of radiation pulses with quasiregular spatial core structure of the angular distribution $\alpha = N_{qr}/N_t$ where N_{qr} is the number of quasiregular pulses, and N_t is the total number of pulses processed in this experimental cycle.

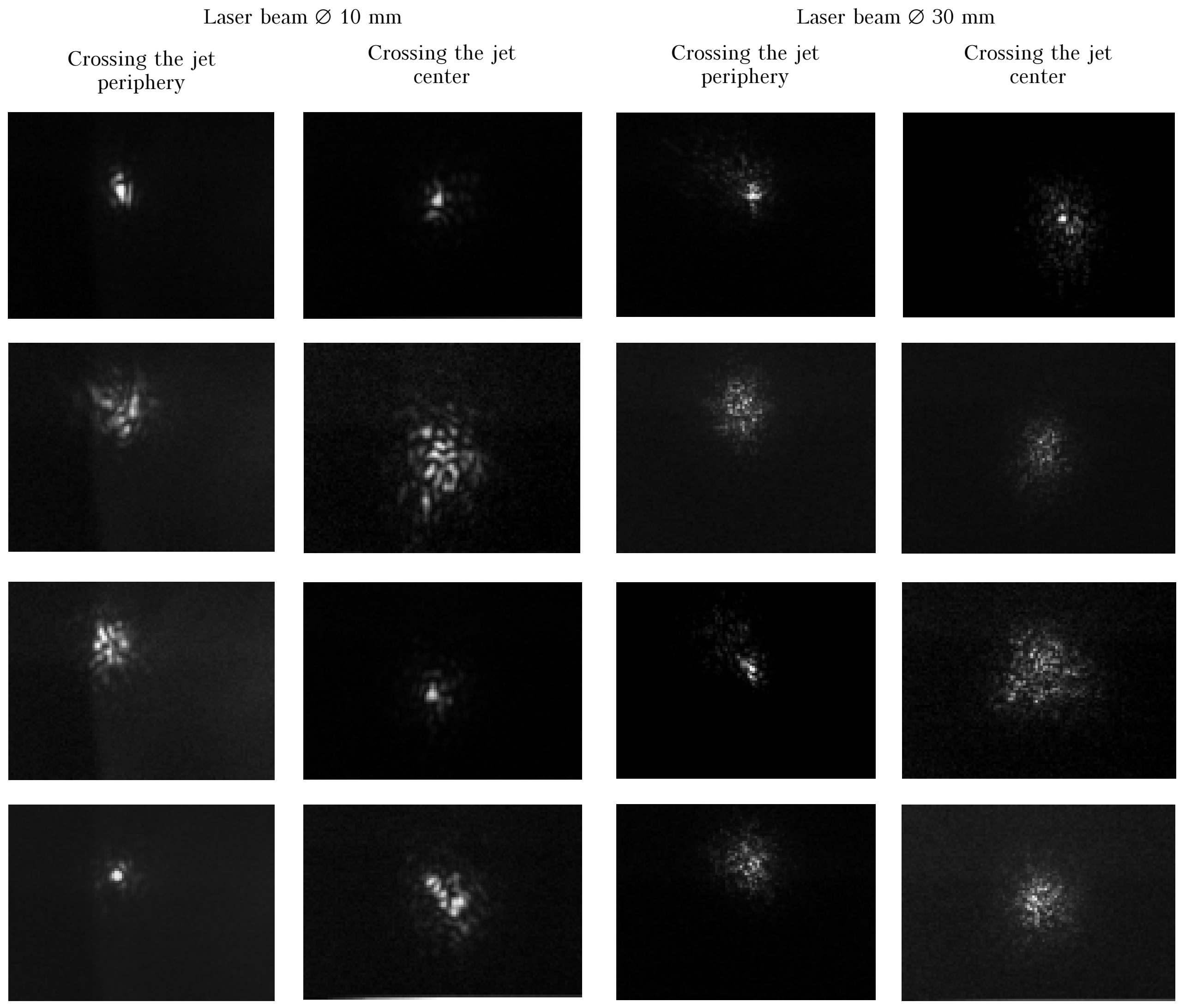

Fig. 1. "Instantaneous" images of intensity distributions of disturbed laser beams in a focal plane of an objective of the receiving system (λ = 1.06 μm). Examples of intensity distributions with "quasiregular" structure are shown by arrows.

Table 1. Statistical probability $\alpha = N_{qr}/N_t$ of crossing the jet by radiation pulses with quasiregular structure of the core of angular distribution

Beam parameters	λ = 1.06 μm		λ = 0.53 μm	
	∅ 30 mm	∅ 10 mm	∅ 30 mm	∅ 10 mm
Crossing the jet periphery	0.070	0.102	0.021	0.017
Crossing the jet center	0.010	0.071	0.004	0.007

Note that for a 10-millimeter 1-micron beam the observed number of quasiregular pulses, depending on the time of observation or, that is the same, on the total number of pulses recorded to a given moment under both geometries of jet crossing, grew practically linearly, only with small random deviations (no more than 10%) from a straight line approximating the dependence. This is indicative of the stationarity of the random process. The same takes place also for the 30-mm beam, passing through the jet periphery, when the frequency of appearance of pulses with quasiregular structure is relatively high. In other cases characterized in Table 1 by very small values of α the time dependence of the number of quasiregular pulses has a large fluctuation component (40–50%), i.e., the process, strictly speaking, cannot be considered as stationary.

Because the number of events observed and recorded in these experiments turned out to be very small, it is not necessary to analyze statistical regularities of the random process. In this connection, we analyze the statistics of random series of quasiregular pulses only in the experiments, in which the observed random process is close to a stationary one.

2. Statistics of fluctuations of laser beam structural state

In many applications it is important to know the statistical law, to which the random series of quasiregular pulses follows. Independence of the observed random events and a low statistical probability of their occurrence suggests that it is Poisson's law:

$$P(N_{qr}, \alpha\frac{t}{T}) = \frac{(\alpha\frac{t}{T})^{N_{qr}}}{N_{qr}!}e^{-\alpha\frac{t}{T}}, \qquad (1)$$

$P(N_{qr}, \alpha t/T)$ is the probability of occurrence of N_{qr} quasiregular pulses during the observation period t; T is the period of laser pulse sequence.

This assumption can be verified as follows. It is known[5] that if a series of random events obeys the Poisson statistics, then the probability density for the time interval τ between neighboring events must be subject to the exponential distribution with the parameter α:

$$P\left(\alpha, \frac{\tau}{T}\right) = \alpha e^{-\alpha\frac{\tau}{T}}. \qquad (2)$$

Based on the experimental data, the probability density was calculated for the time interval between the adjacent quasiregular pulses, and the results were compared with the theoretical dependence (2) corresponding to the experimentally measured value of α. The results of the data processing are given in Fig. 2.

A good agreement between the experimental data and the exponential distribution with the accuracy to the statistical spread is seen. Hence, it follows that the random occurrence of quasiregular pulses, caused by the effect of turbulence intermittence in the aircraft engine jet, is subject to the Poisson statistics.

All the above results were obtained for one and the same operation conditions of the aircraft engine.

Figure 3 shows how the variation of the turbulent flow of the aircraft engine jet affects the laser beam statistics. It is seen that the pulse number with quasiregular structure depends on the total number of laser pulses N, passed through the jet by a given instant of time. In the considered situation, the 1-micron laser beam of 10 mm diameter crossed the jet periphery and the images of its angular intensity distributions were recorded not only under the standard operating conditions but also after the energy supply was switched off (this moment is denoted by a point 1 on the plot). Radiation wavelength was 1.06 μm.

Let us analyze the results in the time interval between the points *1* and *2*, in which after switching off the engine the turbulent hot flow still exists due to burning down the fuel remained and the inertial rotation of the engine turbines.

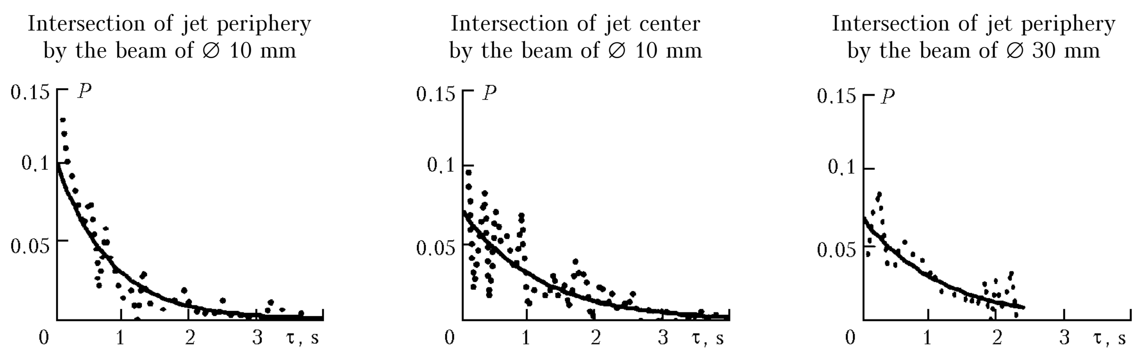

Fig. 2. The probability density P for the time interval τ between the adjacent pulses with "quasiregular" structure. ( ) is the theoretical calculation; (.....) – experimental data; λ = 1.06 μm.

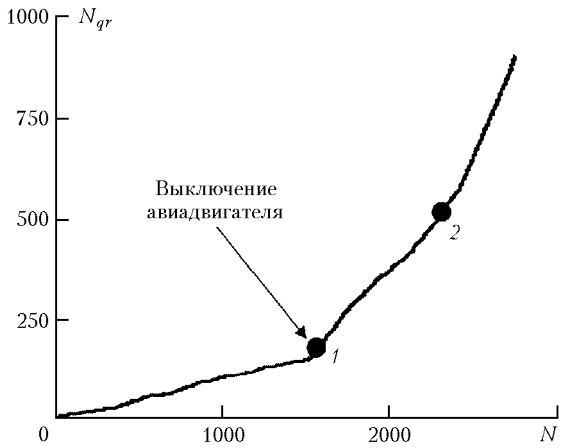

Fig. 3. The dependence of the number of pulses with the quasiregular spatial structure N_{qr} on the total number of pulses N recorded by a given instant of time.

In this interval the dependence of the number of quasiregular pulses on the number of laser pulses is almost linear, but at the point of the engine shutdown its angle of slope, characterizing the probability of occurrence of quasiregular pulses, changes sharply. Before point *1* the statistical probability $\alpha = 0.1$, between the points *1* and *2* its mean value becomes equal to 0.4. In this interval the probability density for the time intervals between adjacent pulses is already inconsistent with the exponential distribution (2) with the calculated $\alpha = 0.4$ (Fig. 4*a*).

Consequently, in these operating conditions the succession of quasiregular pulses does not obey Poisson's law.

It is believed that this succession will obey the binomial probability distribution, which limiting event is Poisson's law. To check this hypothesis, we obtained a theoretical expression for the probability density in intervals between adjacent pulses with quasiregular spatial structure in the succession following the binomial distribution:

$$P\left(\alpha, \frac{\tau}{T}\right) = \ln\left(\frac{1}{1-\alpha}\right) e^{-\left[\ln\left(\frac{1}{1-\alpha}\right)\right]\frac{\tau}{T}}. \qquad (3)$$

As is seen, this is also the exponential distribution but with the substitution $\alpha \Rightarrow \ln(1/1-\alpha)$. The probability density calculated on the basis of Eq. (3) is also shown in Fig. 4*a*. It is seen that the experimentally obtained probability density agrees well with such theoretical prediction: the probabilities of minimal interval agree closely and the probability of intervals with $\tau > T$ is very low.

One more method was also used to check the hypothesis on binomial distribution of the probability of appearance of the quasiregular pulses under given experimental conditions. The interval between the points *1* and *2* was divided into sections. In each section 10 laser pulses passed through the jet. The number of sections was 80, that was sufficient for statistical processing.

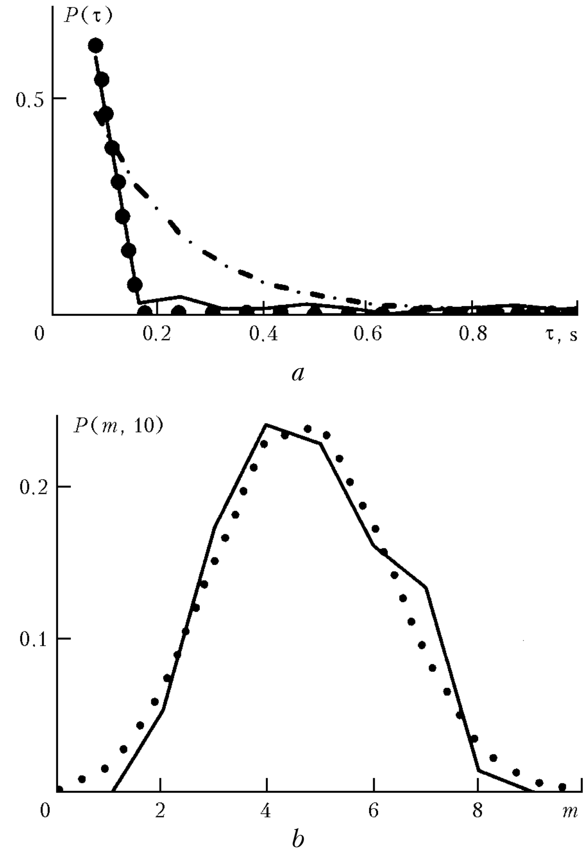

Fig. 4. The probability density $P(\tau)$ for the time interval τ between the adjacent "quasiregular" pulses: theoretical calculation for the pulse sequence following the binomial distribution (.....); theoretical calculation for the pulse sequence obeying the Poisson distribution (. — . — . —); experimental data ( ) (*a*); the probability $P(m, 10)$ of passing m "quasiregular' pulses through the turbulent jet in the sequence of 10 laser pulses: theoretical calculation (.....); experimental data ( ) (*b*).

Then a series of the probability distribution of appearance of m quasiregular pulses in a train of 10 laser pulses was calculated. Figure 4*b* shows both these experimental data and the results of theoretical calculation obtained for binomial distribution

$$P_m = C_{10}^m \alpha (1-\alpha)^{10-m},$$

where C_{10}^m is the binomial coefficient ($m = 0, 1,..., 10$). Figure 4*b* shows that the experimentally obtained probability distribution agrees well with the calculated binomial one at $\alpha = 0.4$ derived for given conditions of the aircraft engine operation.

Let us consider the averaged spatial characteristics of beams disturbed by the aircraft engine jet.

Table 2 shows the averaged over the ensemble of realizations (about 1500 patterns for every experimental case) values of angular halfwidth of laser beam in the vertical (θ_y) and horizontal (θ_x) cross sections of angular intensity distribution. Note that the coordinate axis of beam cross section OX is parallel to the jet axis and the vertical *OY* is orthogonal to it.

Table 2. Halfwidth of the beam angular spectrum by the level (1/*e*) I_{max} in horizontal (θ_x) and vertical (θ_y) directions

Beam parameters		λ = 1.06 μm		λ = 0.53 μm	
		∅ 30 mm	∅ 10 mm	∅ 30 mm	∅ 10 mm
Intersection of jet periphery	θ_x	115 ± 20	125 ± 20	360 ± 40	340 ± 40
	θ_y	155 ± 20	170 ± 20	440 ± 50	390 ± 40
Intersection of jet center	θ_x	140 ± 20	145 ± 20	415 ± 50	420 ± 50
	θ_y	205 ± 30	220 ± 30	470 ± 60	505 ± 60
The absence of jet	θ_x	26 ± 5	31 ± 5	22 ± 5	39 ± 6
	θ_y	26 ± 5	42 ± 6	22 ± 5	46 ± 6

N o t e . θ_x and θ_y are in μrad.

Table 3 shows the averaged values of variance (σ_y and σ_x) of random angular shifts of the beam centroid under the action of turbulent jet.

Note that the estimation of the structural constant value for a turbulent jet, obtained in Ref. 4 from data on the angular halfwidth of 1-micron beam perturbed by a central jet region, gave $C_n^2 \approx 1.5 \cdot 10^{-9}$ m$^{-2/3}$.

First of all, note that the angular divergence of a half-micron laser beam exceeds the angular divergence of 1-micron beam 2.5–3 times, and the variance of angular shifts – 2–2.5 times.

Table 3. Rms angular shift of laser beam centroid in horizontal σ_x and vertical σ_y directions

Beam parameters*		λ = 1.06 μm		λ = 0.53 μm	
		∅ 30 mm	∅ 10 mm	∅ 30 mm	∅ 10 mm
Intersection of jet periphery	σ_x	44	69	106	166
	σ_y	65	98	92	145
Intersection of jet center	σ_x	51	85	117	170
	σ_y	80	130	134	186
The absence of jet	σ_x	7	6	16	25
	σ_y	10	7	11	12

* See Note to Table 2.

This experimental fact was analyzed in detail in Refs. 4 and 6 where it was shown that such a result is indicative of a significant difference between the spatial spectrum of inhomogeneities of the refractive index and the commonly used Karman spectrum, which should be supplemented with components in the range of high spatial frequencies $\xi \geq 1000$ m^{-1}. Inhomogeneities of such a scale (≤ 1 mm) strongly distort the angular beam distribution center at λ = 0.53 μm, and in a beam at λ = 1.06 μm the above inhomogeneities mostly contribute into the low-intensity distribution wings, which in the recording are "masked" by the CCD camera background. This can explain the experimental fact that the statistical probability of appearance of pulses with quasiregular spatial structure of the center of angular distribution for 1-micron radiation turns to be much higher than for half-micron radiation.

When comparing the corresponding averaged values of θ and the variance of σ for cases of laser beams crossing jet center and periphery, we can note a similarity of their dependence on the radiation parameters – beam diameter and wavelength.

It may appear that a single difference is somewhat smaller (by approximately 20%) level of distortions of the beam passed through the jet periphery. However this is not the case, if we consider the averaged patterns of intensity distributions given in Fig. 5.

The comet-shape intensity distribution is clearly seen in the latter case. This is the case for both wavelengths, and it is most pronounced for a beam of 30 mm diameter. In all cases "the comet tail" is directed to the beam center, and "the comet nucleus" is shifted to the side jet boundary.

It should be noted that such a pattern is indicative of essential inhomogeneity of turbulence statistical characteristics near the side jet boundary even for a narrow beam (∅ 10 mm).

Conclusion

The conducted investigations have shown that even in the case that laser beams cross a strongly turbulent jet of the aircraft engine we can observe pulses with quasiregular spatial structure and high angular radiation concentration. Such "quasiregular pulses" with λ = 1.06 μm are generated at a relatively high α ~ 0.07–0.1 when the aircraft engine jet is crossed in the vicinity of the side boundary by laser beams of 30 and 10 mm diameter. This effect depends upon the fact that in the turbulent medium the quasihomogeneous zones of certain scale are randomly formed on the way of a laser beam, and sometimes the moment of their appearance coincides with the moment of the laser pulse generation. Under the same conditions the frequency of generation of quasiregular pulses with λ = 0.53 μm is found to be many times smaller (α ≤ 0.02), because the center of angular distribution of half-micron radiation is strongly affected by a wider spectrum of inhomogeneities in the jet, which includes the small-scale inhomogeneities with spatial frequencies $\xi \geq 1000$ m^{-1}. If laser beams cross the central jet area, then the generation of quasiregular pulses has a significant statistical probability (α ~ 0.07) only for a narrow beam of 10 mm diameter at λ = 1.06 μm.

The analysis of experimental data has shown that depending on the jet flow regime the statistics of random sequence of quasiregular pulses can obey both Poisson's law and the binomial law. It is also shown that random values of time intervals between adjacent pulses, on the average, have the exponential distribution of the probability density. It should be kept in mind that in the case of validity of the binomial law the measured parameter of distribution should be substituted: $\alpha \Rightarrow \ln(1/1 - \alpha)$.

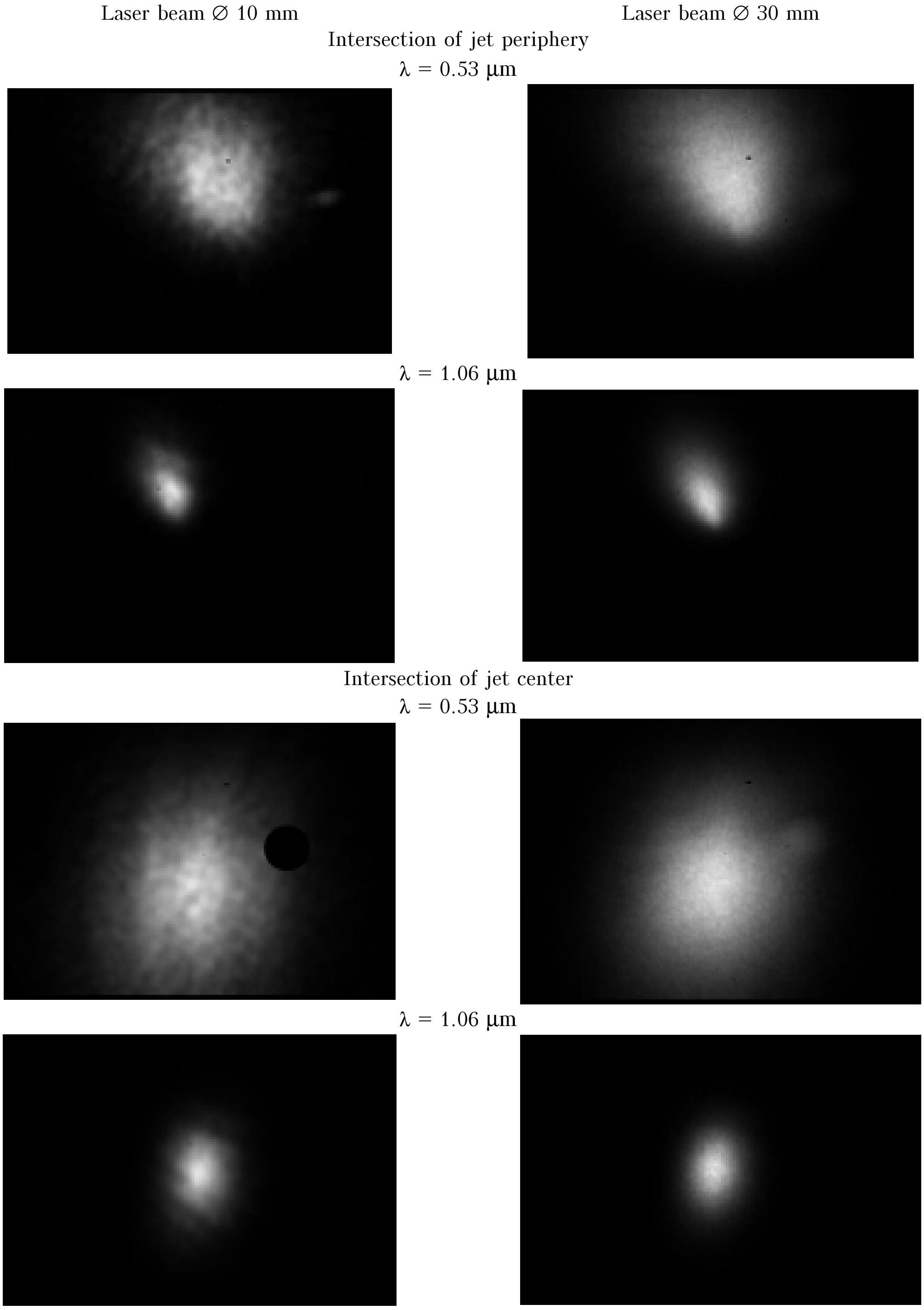

Fig. 5. Images of averaged intensity distributions of disturbed beams in the recording plane.

A comparative analysis of the numerical values averaged over an ensemble of realizations of angular halfwidth and the variance of random shifts of beam centroid for both geometries of jet intersection (near boundary or in the central area) has shown a similarity of dependences of these values on the wavelength, beam diameter, and other their peculiarities. The difference is evident only in the total level of beam disturbances, and in the periphery of a jet the disturbance is by ~ 20% less. At the same time, the direct consideration of averaged images of the disturbed beam cross sections has shown a basic distinction of the patterns for different geometries of jet intersection. In case of crossing the jet periphery an averaged angular intensity distribution is always of the comet-shaped form with a “comet tail” oriented to a jet axis. This observation is indicative of an essential inhomogeneity of statistical characteristics of the turbulence near the side jet boundary in the radial direction even on scales of several millimeters.

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