Optical transfer operator of the spherical atmosphere–Earth system

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The transfer of solar radiation on a global scale in the Earth's atmosphere is studied. We discuss pioneering domestic and foreign works on the mathematical simulation of terrestrial radiation field and on the methods of numerical solution of general boundary-value problem of the radiative transfer theory for a spherical shell with a reflecting underlying surface. The optical transfer operator of the spherical atmosphere–Earth system is constructed. Models of the forcing functions for the spherical problem of transfer theory are formulated.

The space-based studies make up an area in the basic and applied research whose development, and even origin, could not have been possible without computers. The space exploration has served a significant factor in computer development and creation of new research areas associated with mathematical modeling of terrestrial radiation field, image transfer theory, vision theory, theory of image processing and pattern recognition, etc. The information-mathematical software is inseparable part of any project on the space-based research.

Theoretical and numerical simulation studies at the stages of design and implementation of the first satellite devices and, in particular, systems of their navigation, positioning, and stabilization, as well as in the first satellite optical experiments, were performed by three leading research teams in (mathematical) modeling of radiative transfer in natural environments using computer. The Leningrad State University and Main Geophysical Observatory shared a few research groups headed by V.V. Sobolev and K.Ya. Kondratyev. V.V. Sobolev, I.N. Minin, and O.I. Smoktii developed first combined plane-spherical model of the Earth's atmosphere in the Sobolev's approximation.[1–8] At the Computer Center of Siberian Branch of the USSR Academy of Sciences, G.I. Marchuk and G.A. Mikhailov have developed first Monte Carlo algorithms for the model of spherical Earth.[9–11] The efficiency of these algorithms owes a lot to the mathematical methods of adjoint equations, suggested by Marchuk[12,13] and developed by Mikhailov, Nazaraliev, Antyufeev, and Darbinyan.[14–20] T.A. Sushkevich, from the Institute of Applied Mathematics of the USSR Academy of Sciences, first realized the global spherical model of radiation field in the atmosphere–Earth system (AES) on a global scale using iteration method of characteristics.[21–26] The approximate approaches were proposed by Avaste,[27,28] whereas Sobolev method was developed further on by Titarchuk.[29] The formulation of problems and discussion of the results obtained were performed jointly with T.A. Germogenova, M.V. Maslennikov, A.M. Obukhov, M.S. Malkevich, G.V. Rozenberg, A.B. Sandomirskii, A.I. Lazarev, E.O. Fedorova, V.P. Kozlov, V.N. Segeevich, I.I. Koksharov, Ch.Y. Villman, O.A. Avaste, V.E. Plyuta, G.M. Grechko, among others.

Abroad (in the USA), the solution of spherical problem was first attempted by Lenoble and Sekera,[30] who used the method of successive approximations, corresponding to expansion of solution in a series over small parameter, with solution of plane problem taken as the first approximation, and with the ratio of the effective height of homogeneous atmosphere to Earth radius assumed as a smallness parameter. Most foreign scientists utilize Monte Carlo method[31–35] or approximate numerical methods.[36–38] The method of invariant immersion still remains purely theoretical technique, without implementation into practice.[39–41]

The approaches based on analysis of characteristics in curvilinear coordinates and different methods of acceleration of convergence of iterative solutions in subregions allow one to proceed to numerical solution of 3-D inhomogeneous spherical problem, which simulates near-real terrestrial conditions.[42–47] Such a formulation of the problem is important in the context of problems of atmospheric (tropospheric and ozonospheric) radiative photochemistry under conditions of twilight, dawn, terminator, and polar regions, information support (such as that provided by refractometric methods and satellite systems operated in observations along horizontal paths) of atmospheric tomography of the Earth, remote sensing of polar regions, development of models of the Earth's spectral-radiation balance and phase brightness of Earth for space navigation instruments (spacecraft returning back to the Earth, spacecraft navigation using Earth attitude), implementation of projects on additional energy supply on spacecraft associated with the use of Earth-reflected solar radiation, etc.

New potentialities of the mathematical simulation of global-scale atmospheric radiation of Earth are associated with mathematical software developments for a wide range of applications on massive parallel supercomputer systems. The availability of such

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facilities makes it possible to perform benchmark computations, numerical simulation experiments, simulation modeling, verification of approximate methods and fast algorithms for mass solution of research and applied problems, as well as to refine the radiation codes for models of circulation, forecast, climate, photochemical kinetics, dynamics of ozonosphere, cross-boundary transport of air pollutants, etc.

Mathematical formulation of the problem

We shall consider below the optical (solar and terrestrial) radiative transfer in the Earth–atmosphere system (EAS) in the approximation of spherically symmetric shell, illuminated by an external parallel flux with the intensity πS_1. To account for the contribution of spatially inhomogeneous underlying surface (earth's surface, cloud top boundary, or hydrometeors) to the spherical EAS emission, we shall construct the transfer operator in the framework of linear-systematic approach, developed for plane EAS model.[48] The forcing function (FF) of the boundary-value problem of the radiative transfer theory, being the kernel of the optical transfer operator (OTO), is considered a universal quantity, invariant with respect to specific inhomogeneities of the reflecting and emitting boundary.

The OZ-axis is assumed to pass through the Earth's center along the direction opposite to extraterrestrial parallel radiative flux. The Earth and the atmosphere illumination by the Sun is symmetrical about OZ-axis. Overall, the EAS is considered as a three-dimensional system in the spherical coordinate system: the position vector $\mathbf{r}$ of any point $A(\mathbf{r})$ in the atmosphere and on the underlying surface is totally defined by the distance $r = |\mathbf{r}|$ from the Earth's center and polar ψ and azimuth η angles; that is, the three parameters (r, ψ, η), namely radius, latitude $0 \le \psi \le \pi$, and longitude $0 \le \eta \le 2\pi$ correspond to each vector value $\mathbf{r}$.

The direction of a light beam propagation $\mathbf{s}$ (with $\mathbf{s}$ assumed to be a unit vector) at the point $A(\mathbf{r})$ is described in a local spherical coordinate system with the origin at the point $A(\mathbf{r})$: zenith angle $\vartheta = \arccos(\mathbf{r}\cdot\mathbf{s})/|\mathbf{r}|$ measured from $\mathbf{r}$, and azimuth φ in the plane tangential to the sphere of radius r and passing through the point $A(\mathbf{r})$, i.e., each $\mathbf{s}$ is described by the pair (ϑ, φ). The direction $\varphi = 0$ is assumed to coincide with the azimuth of the incident extraterrestrial flux. We introduce the notation $\mu = \cos\vartheta$. Consider a cone about OZ-axis with the center at the Earth's center and with the opening angle of 2ψ. At the point $A(\mathbf{r})$, located at the cone side surface, the directions of $\mathbf{s}$ out of the cone will be assumed to lie in the range of azimuth angles $0 \le \varphi < \pi/2$, while those into the cone in the range $-\pi/2 < \varphi \le \pi$. The rays $\mathbf{s}$ with azimuth $\varphi = \pi/2$ will lie in the planes tangential to the side surfaces of these cones; while to azimuths $\varphi = 0$ and $\varphi = \pi$ there will correspond a single coordinate plane passing through the OZ-axis and position vector $\mathbf{r}$.

The studied spherical shell is bounded by spherical surfaces with radii R_b at the shell bottom and R_t at shell top. The set of all points $A(\mathbf{r})$ of the spherical shell constitutes an open region G with the bottom G_b and top G_t boundaries being spherical surfaces with the radii R_b and R_t, respectively. The vector field of all directions of light beams $\mathbf{s}(A)$ at each point $A(\mathbf{r})$ is a unit sphere, namely the set $\Omega = \Omega^+ \cup \Omega^-$, where Ω^+ and Ω^- are the sets (hemispheres) of $\mathbf{s}$ directions with $\mu \in [0, 1]$ and $\mu \in [-1, 0]$, corresponding to up and down going radiative fluxes, respectively. In the studied problem, the phase volume is

$$\Gamma_{\mathrm{tot}} \equiv [G \cup G_\mathrm{b} \cup G_\mathrm{t}]\,\Omega = \\ = \{(\mathbf{r}, \mathbf{s}) : \mathbf{r} \in [G \cup G_\mathrm{b} \cup G_\mathrm{t}],\ \mathbf{s} \in \Omega\}.$$

For the convenience in writing the boundary conditions, we introduce the sets (phase regions)

$$b \equiv G_\mathrm{b}\,\Omega^+ = \{(\mathbf{r}, \mathbf{s}): \mathbf{r} = \mathbf{r}_b \in G_\mathrm{b},\ \mathbf{s} \in \Omega^+\},$$
$$t \equiv G_\mathrm{t}\,\Omega^- = \{(\mathbf{r}, \mathbf{s}) : \mathbf{r} = \mathbf{r}_t \in G_\mathrm{t},\ \mathbf{s} \in \Omega^-\},$$

where the parameters b and t, for clarity, are chosen to correspond to "bottom" and "top".

Our purpose is to determine the intensity of attenuated direct radiation from sources and stationary intensity field of singly and multiply scattered radiation inside or outside the scattering, absorbing, and emitting spherical shell G with the boundaries G_t and G_b. The approximation of stationary field is physically correct because the propagation of light is studied.

The total intensity of monochromatic (at a fixed λ) or quasi-monochromatic (at fixed λ and $\Delta\lambda$) stationary radiation $\Phi_\lambda(\mathbf{r}, \mathbf{s})$, where index λ indicates the wavelength (omitted in discussion below), at the point $A(\mathbf{r})$ with the position vector $\mathbf{r}$ along the direction $\mathbf{s}$ is determined as a solution of the general boundary-value problem (GBVP) of the transfer theory

$$K\Phi = F^{\mathrm{in}},\ \Phi\big|_t = F^t,\ \Phi\big|_b = \varepsilon R F + F^b \qquad (1)$$

in the phase region Γ with linear operators: the transfer operator

$$D = (\mathbf{s}, \mathrm{grad}) + \sigma_{\mathrm{tot}}(\mathbf{r}),$$

for 3-D spherical geometry of the problem[46]

$$(s, \nabla\Phi) = \cos\vartheta \frac{\partial\Phi}{\partial r} + \frac{\sin\vartheta\cos\varphi}{r}\frac{\partial\Phi}{\partial\psi} - \frac{\sin\vartheta}{r}\frac{\partial\Phi}{\partial\vartheta} + \\ + \frac{\sin\vartheta\sin\varphi}{r\sin\psi}\frac{\partial\Phi}{\partial\eta} - \frac{\sin\vartheta\sin\varphi\cot\psi}{r}\frac{\partial\Phi}{\partial\varphi};$$

collision integral – source function

$$B(\mathbf{r},\mathbf{s}) \equiv S\Phi = \sigma_{\mathrm{sc}}(\mathbf{r}) \int_\Omega \gamma(\mathbf{r},\mathbf{s},\mathbf{s}')\,\Phi(\mathbf{r},\mathbf{s}')\,\mathrm{d}\mathbf{s}',$$

$$\mathrm{d}\mathbf{s}' = \sin\vartheta'\mathrm{d}\vartheta'\mathrm{d}\varphi';$$

the integro-differential operator $K \equiv D - S$; the reflection operator R is, in the general case, described by the integral equation

$$[R\Phi](\mathbf{r}_b,\mathbf{s}) = \int_{\Omega^-} q(\mathbf{r}_b,\mathbf{s},\mathbf{s}^-)\,\Phi(\mathbf{r}_b,\mathbf{s}^-)\,\mathrm{d}\mathbf{s}^-, \quad \mathbf{s}\in\Omega^+.$$

The total aerosol-molecular scattering phase function has the normalization condition

$$\frac{1}{4\pi}\int_{\Omega}\gamma(\mathbf{r},\mathbf{s},\mathbf{s}')\,\mathrm{d}\mathbf{s}' = \frac{1}{2}\int_{-1}^{1}\gamma(\mathbf{r},\cos\chi)\,\mathrm{d}\cos\chi = 1;$$

$\sigma_{tot}(\mathbf{r})$ and $\sigma_{sc}(\mathbf{r})$ are the spatial distributions of the total cross section of interaction (extinction) between optical radiation and substance filling the region G and of the total aerosol plus molecular scattering coefficient; the scattering angle χ is determined from the formula

$$\cos\chi = \cos(\mathbf{s}\cdot\mathbf{s}') = \cos\vartheta\cos\vartheta' + \sin\vartheta\sin\vartheta'\cos(\varphi-\varphi'),$$

if $\mathbf{s} = (\vartheta, \varphi)$, $\mathbf{s}' = (\vartheta', \varphi')$. The function $F^{in}(\mathbf{r}, \mathbf{s})$ is the density of radiation sources located within the region G; $F^b(\mathbf{r}_b, \mathbf{s}^+)$ and $F^t(\mathbf{r}_t, \mathbf{s}^+)$ are the radiation sources at the boundaries of the spherical shell, determined for beams $\mathbf{s}$ directed into the region G.

The operator R describes the law of reflection of radiation from the underlying surface located at the level of bottom boundary G_b; the parameter $0 \le \varepsilon \le 1$ characterizes the type of interaction of radiation with the underlying surface. When $R \equiv 0$ (or when $\varepsilon = 0$), we deal with the first boundary-value problem (FBVP) of the transfer theory

$$K\Phi_0 = F^{in},\ \Phi_0|_t = F^t,\ \Phi_0|_b = F^b \qquad (2)$$

for a spherical shell with transparent, nonreflecting, absolutely black boundaries, or with the problem having "vacuum" boundary conditions.

The scalar function with the vector arguments $\Phi(\mathbf{r}, \mathbf{s}) = \Phi(A, \mathbf{s}(A)) = \Phi(r, \psi, \eta, \vartheta, \varphi)$ is determined as a solution of GBVP (1) or FBVP (2) in the phase region

$$\Gamma \equiv G\,\Omega + G_t\,\Omega^+ + G_b\,\Omega^-,\ \Gamma_{tot} = \Gamma \cup t \cup b.$$

The boundary-value problem for the stationary transfer equation is solved by the successive orders of scattering (SOS) method, using simple iterations in different orders of collision or modified iterations with inclusion of accelerating procedures and subregions, representing different media (atmosphere, ocean, clouds).

General boundary-value problem (1) is linear (with respect to sources), and its solution can be sought in the form of superposition (arguments $(\mathbf{r}, \mathbf{s})$ omitted) $\Phi = \Phi_0 + \Phi_q$. The background radiation Φ_0 is determined as a solution of FBVP (2) and may consist of up to three background components: $\Phi_0 = \Phi_0^t + \Phi_0^b + \Phi_0^{in}$, each of which can be calculated separately as a solution of FBVP with the sources F^t, F^b, and F^{in}, respectively. The determination of background Φ_q due to reflection from the underlying surface is the following GBVP

$$K\Phi_q = 0,\ \Phi_q|_t = 0,\ \Phi_q|_b = \varepsilon R\Phi_q + \varepsilon E, \qquad (3)$$

where the insolation source is surface illumination (brightness, irradiance) due to background radiation: $E(\mathbf{r}_b, \mathbf{s}^+) \equiv R\Phi_0$.

Forcing functions of spherical boundary-value problem of the transfer theory

By analogy with the plane EAS,[48] we introduce the "horizontal" coordinates $r_\perp = (\psi, \eta) \in \Omega$, $\mathrm{d}r_\perp = \sin\psi\mathrm{d}\psi\mathrm{d}\eta$. Let us consider the FBVP

$$K\Phi = 0,\ \Phi|_t = 0,\ \Phi|_b = f(\mathbf{s}^h;\ r_\perp,\ \mathbf{s}). \qquad (4)$$

The parameter $\mathbf{s}^h \in \Omega^+$ may not be displayed. The problem (4) refers to the linear EAS, and its generalized solution is presented by a linear functional, the integral of superposition

$$\Phi(\mathbf{s}^h; r, r_\perp, \mathbf{s}) = \mathrm{F}(f) \equiv (\Theta, f) \equiv \frac{1}{2\pi}\int_{\Omega^+}\mathrm{d}\mathbf{s}_h^+ \times$$
$$\times\frac{1}{4\pi}\int_{\Omega}\Theta(\mathbf{s}_h^+; r, r_\perp - r'_\perp, \mathbf{s})\, f(\mathbf{s}^h; r'_\perp, \mathbf{s}_h^+)\sin\psi'\mathrm{d}\psi'\mathrm{d}\eta', \qquad (5)$$

whose kernel is FF $\Theta(\mathbf{s}_h^+;\ r,\ r_\perp,\ \mathbf{s})$, being the solution of the FBVP

$$K\Theta = 0,\ \Theta|_t = 0,\ \Theta|_b = f_\delta. \qquad (6)$$

with the parameter $\mathbf{s}_h^+ \in \Omega^+$ and source $f_\delta(\mathbf{s}_h^+;\ r_\perp,\ \mathbf{s}) = \delta(r_\perp)\,\delta(\mathbf{s} - \mathbf{s}_h^+)$. In effect, FF Θ describes the radiation field in the layer with nonreflecting boundaries, formed due to processes of multiple scattering of stationary beam with the direction $\mathbf{s}_h^+$, whose source is on the boundary G_b at the point with $\psi = 0$.

If the source $f(r_\perp)$ is isotropic and horizontally nonuniform, then the solution of FBVP (4) is sought in the form of the linear functional, namely, convolution integral

$$\Phi(r, r_\perp, \mathbf{s}) = \mathrm{F}_c(f) \equiv (\Theta_c, f) \equiv$$
$$\equiv \frac{1}{4\pi}\int_{\Omega}\Theta_c(r, r_\perp - r'_\perp, \mathbf{s})\, f(r')\sin\psi'\mathrm{d}\psi'\mathrm{d}\eta' \qquad (7)$$

with the kernel

$$\Theta_c(r, r_\perp, \mathbf{s}) = \frac{1}{2\pi}\int_{\Omega^+}\Theta(\mathbf{s}_h^+; r, r_\perp, \mathbf{s})\,\mathrm{d}\mathbf{s}_h^+. \qquad (8)$$

The FF Θ_c coincides with the point-spread function and satisfies FBVP with an axial symmetry

$$K\Theta_c = 0,\ \Theta_c|_t = 0,\ \Theta_c|_b = \delta(r_\perp) f_\delta. \qquad (9)$$

In the case of anisotropic and horizontally homogeneous source $f(\mathbf{s}^h; \mathbf{s})$, the solution of FBVP (4) is determined through the linear functional

$$\Phi(\mathbf{s}^h; r, \mathbf{s}) = \mathrm{F}_r(f) \equiv (\Theta_r, f) \equiv$$
$$\equiv \frac{1}{2\pi}\int_{\Omega^+}\Theta_r(\mathbf{s}_h^+; r, \mathbf{s})\, f(\mathbf{s}^h; \mathbf{s}_h^+)\,\mathrm{d}\mathbf{s}_h^+ \qquad (10)$$

with the kernel

$$\Theta_r(\mathbf{s}_h^+; r, \mathbf{s}) = \frac{1}{4\pi}\int_\Omega \Theta(\mathbf{s}_h^+; r, r_\perp, \mathbf{s}) \sin\psi \mathrm{d}\psi \mathrm{d}\eta . \quad (11)$$

The FF Θ_r is the solution of 1-D spherical FBVP with the dependence on azimuth

$$K_r\Theta_r = 0,\ \Theta_r|_t = 0,\ \Theta_r|_b = \delta(\mathbf{s} - \overset{+}{\mathbf{s}}_h). \quad (12)$$

In the case of isotropic and horizontally homogeneous source, the solution of FBVP (4)

$$\mathrm{F}(r, \mathbf{s}) = fW(r, \mathbf{s}),\ f = \text{const}, \quad (13)$$

is calculated through the FF

$$W(r,\mathbf{s}) = \frac{1}{2\pi}\int_{\Omega^+} \mathrm{d}\mathbf{s}_h^+ \frac{1}{4\pi}\int_\Omega \Theta(\mathbf{s}_h^+; r, r_\perp, \mathbf{s}) \sin\psi \mathrm{d}\psi \mathrm{d}\eta =$$

$$= \frac{1}{4\pi}\int_\Omega \Theta_c(r, r_\perp, \mathbf{s}) \sin\psi \mathrm{d}\psi \mathrm{d}\eta = \frac{1}{2\pi}\int_{\Omega^+} \Theta_r(\mathbf{s}_h^+; r, \mathbf{s})\, \mathrm{d}\mathbf{s}_h^+ , \quad (14)$$

also called the transmission function burdened by multiple scattering contribution, and determined as a solution of the 1-D spherical FVBP

$$K_rW = 0,\ W|_t = 0,\ W|_b = 1. \quad (15)$$

Formulas (8), (11), and (14) can be used for benchmark calculations of FF Θ, Θ_c, and Θ_r through solution of simpler FBVP (9), (12), and (15). The functionals (8), (11), and (14) are particular cases of the functional (5). The forcing functions Θ, Θ_c, Θ_r, W are solutions of FBVP (6), (9), (12), and (15), respectively, and represent the full set of base models of forcing functions of the first and general boundary-value problems of the radiative transfer problem in a spherical shell and invariant characteristics of the linear EAS.

Optical transfer operator

Based on the regular perturbation theory and using the series

$$\Phi_q\left(\mathbf{s}^h; \mathbf{r}, \mathbf{s}\right) = \sum_{k=1}^{\infty} \varepsilon^k \Phi_k,$$

GBVP (3) reduces to the system of recurrent FBVP of the type (4)

$$K\Phi_k = 0,\ \Phi_k|_t = 0,\ \Phi_k|_b = E_k. \quad (16)$$

with the sources $E_k = R\Phi_{k-1}$ for $k \geq 2$, $E_1 = E$. We introduce an operation describing a single interaction of radiation with the boundary in terms of FF Θ:

$$[Gf](\mathbf{s}^h; \mathbf{r}_b, \mathbf{s}) \equiv R(\Theta, f) = \int_{\Omega^-} q(\mathbf{r}_b, \mathbf{s}, \mathbf{s}^-)\,(\Theta, f)\mathrm{d}\mathbf{s}^-.$$

The solutions of FVBP system (16) are sought as linear functionals (5):

$$\Phi_1 = (\Theta, E),\ \Phi_k = (\Theta, R\Phi_{k-1}) = (\Theta, G^{k-1}E).$$

Asymptotically exact solution of GBVP (3) is obtained in the form of the linear functional (5), representing optical transfer operator

$$\Phi_q = (\Theta, Y), \quad (17)$$

where the "scenario" of the optical image or the brightness of underlying surface

$$Y \equiv \sum_{k=0}^{\infty} G^k E = \sum_{k=0}^{\infty} R\Phi_k$$

is the sum of the Neumann series in orders of surface reflection taking into account multiple scattering in the medium.

The "scenario" satisfies the Fredholm integral equation of the second kind

$$Y = R(\Theta, Y) + E,$$

which is called the equation of "near-ground photography." In the general case, $R(\Theta, Y) \neq \neq (R\,\Theta, Y)$. The total EAS radiation and "satellite photography" are described by the functional

$$\Phi = \Phi_0 + (\Theta, Y).$$

The FF $\Theta(\overset{+}{\mathbf{s}}_h; r, r_\perp, \mathbf{s})$ is used to solve GBVP (3) with the following set of pairs of source function and reflection characteristic:

$$E(\mathbf{r}_b, \mathbf{s}),\ q(\mathbf{r}_b, \mathbf{s}, \mathbf{s}');\ E(\mathbf{r}_b, \mathbf{s}),\ q(\mathbf{s}, \mathbf{s}');\ E(\mathbf{s}),\ q(\mathbf{r}_b, \mathbf{s}, \mathbf{s}');$$
$$E(\mathbf{r}_b),\ q(\mathbf{r}_b, \mathbf{s}, \mathbf{s}');\ E(\mathbf{r}_b),\ q(\mathbf{s}, \mathbf{s}');\ E,\ q(\mathbf{r}_b, \mathbf{s}, \mathbf{s}').$$

The FF $\Theta_c(r, r_\perp, \mathbf{s})$ is the kernel of functionals, when source and reflection parameter form the following pairs:

$$E(\mathbf{r}_b, \mathbf{s}),\ q(\mathbf{r}_b, \mathbf{s}');\ E(\mathbf{r}_b, \mathbf{s}),\ q(\mathbf{s}');\ E(\mathbf{s}),\ q(\mathbf{r}_b, \mathbf{s}');$$
$$E(\mathbf{r}_b),\ q(\mathbf{r}_b, \mathbf{s}');\ E(\mathbf{r}_b),\ q(\mathbf{s}');\ E,\ q(\mathbf{r}_b, \mathbf{s}').$$

With the help of FF $\Theta_r(\overset{+}{\mathbf{s}}_h; r, \mathbf{s})$ we can determine the functionals for the following sources' and reflection parameters: $E(\mathbf{s})$, $q(\mathbf{s}, \mathbf{s}')$; E, $q(\mathbf{s}, \mathbf{s}')$. The FF $W(r, \mathbf{s})$ is used to determine the solution for the pair E, $q(\mathbf{s}')$.

Conclusion

Summarizing, we can state that the initial GBVP (3) is reduced to the linear functional (17), and a linear-systematic approach to solving remote sensing problems and accounting for the contribution of reflecting and emitting spherical surface of the Earth has been formulated. Also, we clearly determined how nonlinear effects due to multiple re-reflections from surface influence the type of scenario; these effects are described by the linear transfer characteristics of isolated atmospheric layer. It is noteworthy, that the FF characteristics are efficiently calculated by the Monte Carlo method.

The representative reports and discussions on the results of solution of spherical problems have first taken place at the First Summer School on the Optics of Scattering Media (Minsk–Svityaz, June 1969), at the All-Union Conference on Light Scattering in the Atmosphere (Alma-Ata, November 1969), and at the Eighth Scientific Conference on Atmospheric Optics and Actinometry (Tomsk–Novosibirsk, June 1970).

In recent decade, there has been shown an increased interest in the study and development on the basis of multidimensional spherical models, a tendency well reflected in scientific programs and reports of International Radiation Symposium (IRS-2000), held on July 24–29, 2000 in Saint Petersburg; also, a new area of satellite studies, namely satellite monitoring of the Earth,[49,50] has evolved.

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