Lidar data inversion when studying optical characteristics of a weakly turbid atmosphere

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The efficiency of methods for solving lidar equation is studied for the case of a background aerosol and *a priori* set uncertainty in lidar data. The effect of the contribution of molecular scattering and the altitude behavior of the lidar ratio on the accuracy of reconstructing the atmospheric parameters is considered in numerical experiment. Optical parameters of the atmosphere reconstructed by different methods are compared with Raman lidar data.

Interpretation of data of sounding the atmosphere with a single-frequency lidar in investigating spatiotemporal structure of atmospheric aerosol is based on inverting the lidar sounding equation (LSE) in the single scattering approximation, which relates the power of the detected signal $P(z)$ to the parameters of the medium sounded:

$$P(z) = CP_0G(z)z^{-2}[\beta_{\text{aer}}(z) + \beta_{\text{mol}}(z)] \times \times \exp\left\{-2\int_0^z[\sigma_{\text{aer}}(z') + \sigma_{\text{mol}}(z')]\text{d}z'\right\}, \quad (1)$$

where P_0 is the power of the emitted pulse, $\beta_{\text{mol}}(z)$ and $\beta_{\text{aer}}(z)$ are the molecular and aerosol backscattering coefficients, respectively, $\sigma_{\text{mol}}(z)$ and $\sigma_{\text{aer}}(z)$ are the molecular and aerosol extinction coefficients, respectively, C is instrumental constant containing characteristics of lidar transceiver and $G(z)$ is the geometric function describing the overlap of the transmitter and receiver field of view cones.

The purpose of this paper is the development of the method for stable reconstruction of the parameters of natural aerosol under conditions of *a priori* set uncertainty in lidar data.

The lidar equation can be transformed to the Bernoulli equation

$$\frac{\text{d}\sigma(z)}{\text{d}z} - \sigma(z)\frac{\text{d}\ln\psi(z)}{\text{d}z} = 2\sigma^2(z), \quad (2)$$

where

$$\psi(z) = \sigma(z)\exp\left\{-2\int_0^z\sigma(z')\,\text{d}z'\right\},$$

the solution of which is

$$\sigma(z) = \frac{X(z)}{C^*(z_0) + 2\int_z^{z_0}X(z')\text{d}z'}, \quad (3)$$

where

$$\sigma(z) = \sigma_{\text{aer}}(z) + \sigma_{\text{mol}}(z),$$

for the Klett method[4]

$$X(z) = P(z)\,z^2, \quad (3a)$$

and, assuming a two-component medium for the Fernald method[5,6]

$$X(z) = S_{\text{a}}(z)P(z)z^2 \times \times \exp\left\{2\int_z^{z_0}\left[\frac{S_{\text{a}}(z')}{S_{\text{m}}} - 1\right]\sigma_{\text{mol}}(z')\text{d}z'\right\}, \quad (3b)$$

where $S_{\text{a}}(z) = \sigma_{\text{aer}}(z)/\beta_{\text{aer}}(z)$ is the lidar ratio, $S_{\text{m}} = 8\pi/3$.

The main problem which appears in using Eqs. (3) is the determination of calibration constant $C^*(z_0)$ and (for Eq. (3*b*)) setting the vertical profile of S_{a}. The value S_{a} is not explicitly involved in formulas (3) and (3a) because of the assumption on the power-law form of the relation between β and σ at deriving the Klett formula.[4] The popularity of using Klett method is explained mainly by that the reference point z_0 used is taken from remote parts of the path, that makes it possible to avoid divergence of the solution, as well as by the fact that the method provides for satisfactory results under condition that $\sigma_{\text{aer}} \gg \sigma_{\text{mol}}$ (horizontal paths, strongly turbid atmosphere). The way of determination of the calibration constant lies in using the *a priori* data on the optical parameters of the medium whenever such data are available. The data for local calibration are taken at a certain point z_0, where $z_0 \in [z_{\min}, z_{\max}]$, then $C^*(z_0) = P(z_0)\,z_0^2/[\sigma_\Sigma(z_0)]$ (Ref. 4), or $C^{**}(z_0) = P(z_0)\,z_0^2/\beta_\Sigma(z_0)$ (Ref. 3). There are a number of papers devoted to the methods of calibration under conditions of *a priori* uncertanty.[13–15]

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For integral calibration one uses the estimate of the value of the optical transmission of the whole path $[z_{min}, z_{max}]$, then $C^{***} = T^2(z_{min}, z_{max})$, where[1]

$$T^2(z_{min}, z_{max}) = \exp\left\{-2\int_{z_{min}}^{z_{max}} \sigma_\Sigma(z')\,dz'\right\}.$$

In principle, it is possible to develop algorithms for signal processing, when the described calibrations are used simultaneously. The reference point z_0 can be taken at the initial point of the path, and it is preferable from the standpoint of independent estimation of the optical parameters of the medium by means of additional instrumentation (photometer, nephelometer, etc.).

We do not consider here the methods, which use the absolute calibration of the lidar because of the largest error in the reconstructed data[1] in this case.

To estimate the efficiency and errors in reconstructing the optical parameters of the atmosphere by different methods, let us numerically simulate the lidar signals using a model of optical parameters of the atmosphere for background aerosol by the MUSCLE working group[19] (Fig. 1). The lidar signal was analytically calculated at supposition of a two-component medium. The following algorithms were used for reconstruction of the optical characteristics: a) the Klett method[4]; b) the Fernald method[3,5]; and c) the iteration variant of the Fernald method:

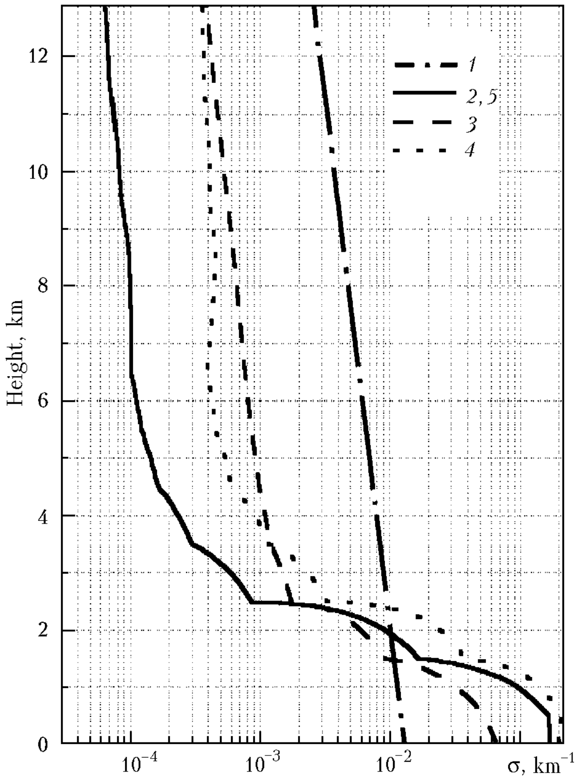

Fig. 1. Results of the numerical modeling: model profile of the molecular scattering coefficient of the atmosphere (*1*); aerosol scattering coefficient $\sigma_{aer}(z)$ (*2*); profile of $\sigma_{aer}(z)$ reconstructed by the Klett method (*3*); by the Fernald method (*4*); and by the iteration method (*5*).

$$\sigma_{aer}^{(n)}(z) = \frac{S_a^{(n-1)}(z)P(z)z^2 \exp\left\{2\int_z^{z_0}\left[\frac{S_a^{(n-1)}(z')}{S_m}-1\right]\sigma_{mol}(z')dz'\right\}}{C^*(z_k) + 2\int_z^{z_0} S_a^{(n-1)}(z)P(z)z^2\exp\left\{2\int_z^{z_0}\left[\frac{S_a^{(n-1)}(z')}{S_m}-1\right]\sigma_{mol}(z')dz'\right\}} - \sigma_{mol}(z), \qquad (4)$$

where $S_a^{(0)}$ = const at $n = 1$, $S_a^{(n)}$ at $n > 1$ is calculated by formula (7a) presented below. The condition of completing the iteration procedure

$$\delta = \left|\frac{\int_z^{z_0}\sigma_{aer}^{(n-1)}(z')\,dz' - \int_z^{z_0}\sigma_{aer}^{(n)}(z')\,dz'}{\int_z^{z_0}\sigma_{aer}^{(n)}(z')\,dz'}\right| \le 10^{-4}$$

was reached in 5 to 7 iterations.

Reference value for $\sigma_{aer}(z_0)$ was selected at the remotest part of the path, where $\sigma_{mol}(z_0) \gg \sigma_{aer}(z_0)$, the value of the calibration constant was set precisely. Thus, the error in reconstruction was determined only by the error in setting the vertical profile of the lidar ratio. The simulated results have shown the ambiguity of interpretation of the lidar data by different methods (Fig. 1). The Fernald methods gives the overestimated values of the reconstructed parameter σ_{aer} when using S_a = const ($S_a = \bar{S}_a(z_{min}, z_{max})$) in the whole altitude range. The greatest deviation from the model value is observed in the height range above 3 km because of the fact that the vertical behavior of $\bar{S}_a/S_a(z)$ reaches maximum values in this height range.

The use of Klett method also gives an essential error in the whole height range, the essentially underestimated values σ_{aer} are observed in the height range below 3 km because of not taking into account the contribution of the molecular component. On the contrary, the iteration algorithm provides for accurate reconstruction of the model parameters.

Comparison of the aforementioned methods with the results of inverting Raman lidar returns was realized for approbation of the used methods under field conditions. The technique for approbation was recommended in developing the methods for interpretation of the lidar returns,[9] because the data obtained by Raman lidar make it possible to obtain the molecular and aerosol components of scattering, as well as the lidar ratio separately.[2,20,21]

The solution for the backscattering coefficient in this case[2,7] takes the form

$$\beta_{\text{aer}}(\lambda_0, z) = -\beta_{\text{mol}}(\lambda_0, z) + \beta_{\text{R}}(\lambda_{\text{R}}, z)\, \frac{C_{\text{R}} P_{\lambda_0}(z)}{C_0 P_{\lambda_{\text{R}}}(z)} \times \frac{\exp\left\{-\int\limits_0^z [\sigma_{\text{aer}}(\lambda_{\text{R}}, z') + \sigma_{\text{mol}}(\lambda_{\text{R}}, z')]\, \mathrm{d}z'\right\}}{\exp\left\{-\int\limits_0^z [\sigma_{\text{aer}}(\lambda_0, z') + \sigma_{\text{mol}}(\lambda_0, z')]\, \mathrm{d}z'\right\}}, \qquad (5)$$

where $\beta_{\text{mol}}(\lambda_0, z)$ and $\beta_{\text{aer}}(\lambda_0, z)$ are respectively the coefficients of molecular and aerosol backscattering at the wavelength of sounding radiation λ_0; $\beta_{\text{R}}(\lambda_{\text{R}}, z)$ is calculated from the molecular density of nitrogen $N_{\text{R}}(z)$, $\sigma_{\text{mol}}(\lambda_0, z)$, $\sigma_{\text{aer}}(\lambda_0, z)$, $\sigma_{\text{mol}}(\lambda_{\text{R}}, z)$, and $\sigma_{\text{aer}}(\lambda_{\text{R}}, z)$ are the molecular and aerosol extinction coefficients at the wavelengths λ_0 and λ_{R}, respectively.

The solution for the aerosol scattering coefficient is written in the form[2]:

$$\sigma_{\text{aer}}(\lambda_0, z) = \frac{\dfrac{\mathrm{d}}{\mathrm{d}z}\left[\ln \dfrac{N_{\text{R}}(z)}{P_{\lambda_{\text{R}}}(z) z^2}\right] - \sigma_{\text{mol}}(\lambda_0, z) - \sigma_{\text{mol}}(\lambda_{\text{R}}, z)}{1 + \left(\dfrac{\lambda_0}{\lambda_{\text{R}}}\right)^k}, \qquad (6)$$

where k is the coefficient in the ratio $\sigma_{\text{aer}}(\lambda_0,z)/\sigma_{\text{aer}}(\lambda_{\text{R}},z) = (\lambda_0/\lambda_{\text{R}})^k$, which varies within the limits from 0 to 2 depending on the type of aerosol under study.

The reference point z_0 is selected at the heights where the scattering ratio $R(z) = 1 + \beta_{\text{aer}}(z)/\beta_{\text{mol}}(z)$ reaches its minimum, i.e., $R(z) \approx 1$. The scattering ratio is determined from the relationship

$$R(z) = R(z_m)\, \frac{P(z) z^2 \beta_{\text{mol}}(z_m) T^2(z_m)}{P(z_m) z_m^2 \beta_{\text{mol}}(z) T^2(z)},$$

where z_{m} is the height at which the value $R(z)$ is known, $T^2(z)$ is the square of the atmospheric transmission along the path from z_{min} to z set from the optical model of the atmosphere.

The reconstructed results on $\beta(z)$ and $\sigma(z)$ obtained by processing the lidar data at the frequency of sounding radiation and their ratios to the signals at the frequency of vibrational-rotational Raman scattering by molecular nitrogen (Eqs. (5) and (6)) are shown in Figs. 2*a* and *b*. The data obtained using the Raman lidar by the Ansmann's group at the Institute for Tropospheric Research, Germany, in 1999 (Ref. 22) were used as the initial ones. The aerosol scattering coefficient profile was calculated using the iteration method with adaptive estimation of the range of permissible values $\sigma_{\text{aer}}(z)$ (Ref. 16).

Application of the iteration method provides for the closest approach to $\beta_{\text{aer}}(z)$ reconstructed from the data of Raman lidar. In contrast to the solution for $\sigma_{\text{aer}}(z)$, the mean error in reconstruction in the lower 8-km layer of the atmosphere reaches no more than 35% for the Fernald method and 15% for the iteration method, because the solution for $\beta_{\text{aer}}(z)$ is weakly sensitive to the error in setting the vertical profile of S_{a}.

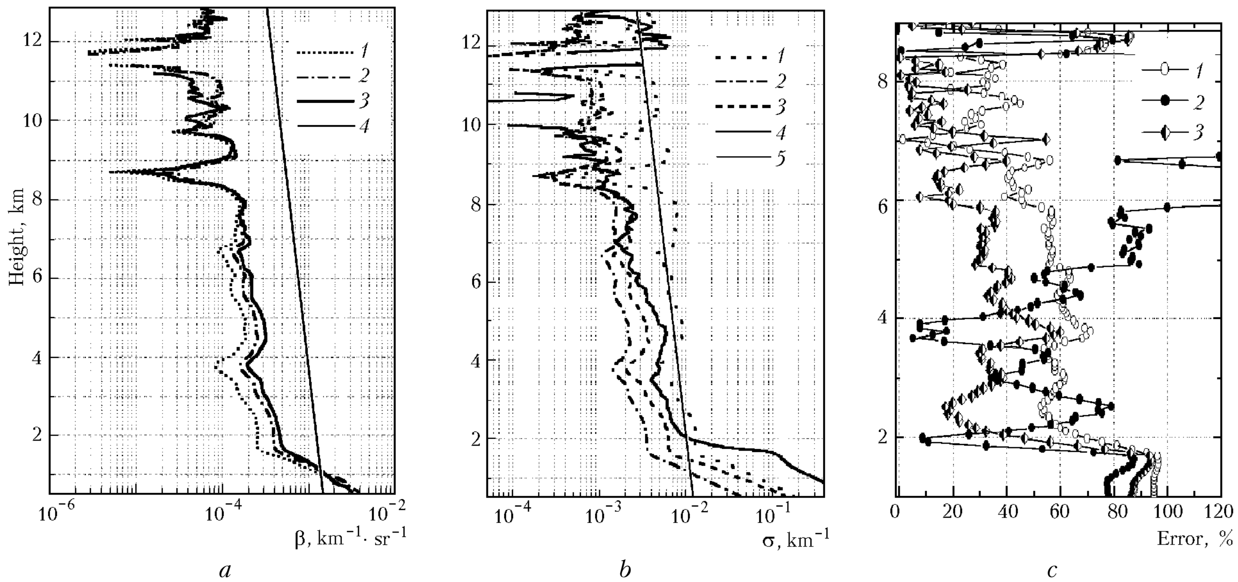

Fig. 2. Results of comparison of the profiles of optical parameters of the atmosphere: (*a*) profile of the aerosol backscattering coefficient $\beta(z)$ reconstructed by the Fernald method (*1*); profile $\beta_{\text{mol}}(z)$ reconstructed by the iteration method (*2*); the data of Raman lidar (*3*); model of $\beta_{\text{mol}}(z)$ for the molecular atmosphere (*4*); (*b*) profile $\sigma(z)$ reconstructed by the Fernald method (*1*); by the Klett method (*2*); by the iteration method (*3*); model of $\sigma_{\text{mol}}(z)$ for the molecular atmosphere (*5*); (*c*) errors in reconstruction of the vertical behavior of $\sigma_{\text{aer}}(z)$ by Klett method (*1*); by Fernald method (*2*); and by iteration method (*3*).

The mean errors in reconstruction $\sigma_{aer}(z)$ are significant for all methods, the smallest values are provided by the iteration method (35%). Mean error of the Klett method is 60%, and the maximum error characterizes the Fernald method (100%). The greatest values of the error in reconstruction $\sigma_{aer}(z)$ (Fig. 2*c*) are observed in the lower 2-km layer of the atmosphere, that is caused mainly by the errors in setting the geometric function of the lidar. Above 2 km, where, as we suppose, geometric factor is equal to 1, distribution of the error better corresponds to the errors of the method. The mean error of the iteration method here is 25%, 50% for the Klett method, and 110% for the Fernald method.

For the correct use of the iteration method it is necessary to set the functional relationship between the lidar ratio and the aerosol scattering coefficient. The published experimental dependences described by formula (7) and shown in Fig. 3 were obtained mainly for strongly and moderately turbid atmosphere.[4,17,18]

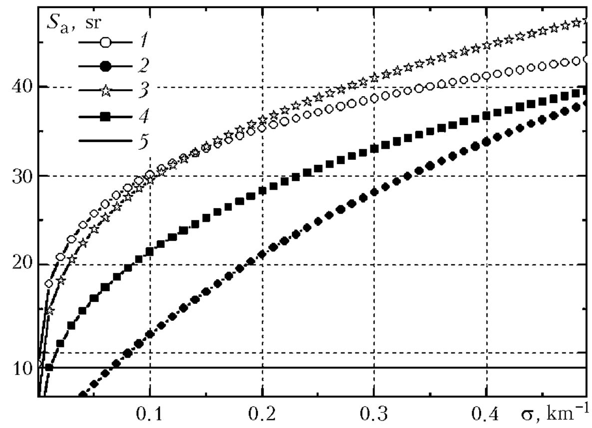

Fig. 3. Empirical dependences of $S_a(\sigma)$ according to literature data: formula (7a) (*1*); (7b) (*2*); (7c) (*3*); (7d) (*4*); and $S_m = 8\pi/3$ (*5*).

The relationship (7a) was approximated on the basis of the experimental relationship between $\beta(z)$ and $\sigma(z)$ (Ref. 11) and extrapolated to the range of small $\sigma(z)$ values. The range of its applicability was calculated for the wide range of variations of $\sigma(z)$ from less than 0.01 km^{-1} to more than 20 km^{-1}. The functional dependence (7b) is to be used for application under conditions of fog and low cloudiness.[17] The relationship (7c) was presented for the first time by Klett[18] and then it was widely used for interpretation of the data of laser sounding of the atmosphere. The formula (7d) was derived by Kovalev[18] based on the analysis of published data and his own experimental observations.

$$1.\ S_a = 50\,[\sigma + 0.000415\,]^{\,0.23-0.03\sqrt{\sigma}}\,, \quad (7a)$$

$$2.\ S_a = [0.0074 + 0.055\exp\{-(\log(\sigma) - 3)/3.1)^2\}]^{-1}, \quad (7b)$$

$$3.\ S_a = 58.8\sigma^{0.3}, \quad (7c)$$

$$4.\ S_a = 50\sigma^{(0.4-0.1\sqrt{\sigma})}. \quad (7d)$$

Let us use the lidar ratio profile obtained experimentally from the data of Raman lidar as a criterion for selecting the spectral dependence of S_a. Figure 4 shows the deviations of the value $S_a(z)$ calculated by formulas (7a)–(7d) from the profile observed under the considered atmospheric conditions. The minimum value of the mean error over the lidar ratio profile (34%) is observed with the use of the dependence of the form (7a).

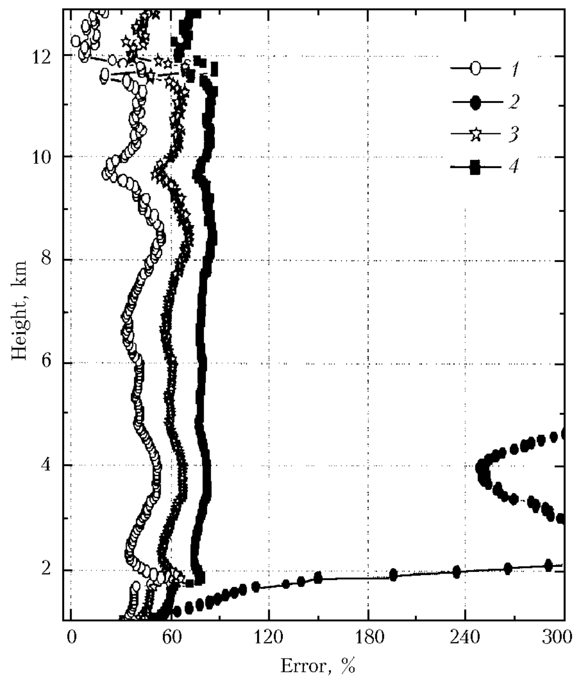

Fig. 4. Deviations of the profile $S_a(z)$ from that reconstructed from the data of Raman lidar using the relationships (7a) (*1*), (7b) (*2*), (7c) (*3*), and (7d) (*4*).

One should note that application of the iteration method for reconstructing $\beta(z)$ does not provide for noticeable advantage in comparison with the methods, which use the assumption on the constancy of the lidar ratio along the sounding path. It is enough here to introduce the physically valid value S_a = const.

The value of the lidar ratio depends on the type of aerosol under investigation and varies within the range from 5 to 30 sr for marine aerosol, from 45 to 80 sr for continental aerosol, and from 55 to 95 sr for technogenic aerosol. The ranges of variations of the lidar ratio for different aerosol types obtained by different authors[8,10,12] are shown in Fig. 5. It is recommended in Ref. 8 to use the following initial values for the lower troposphere: $S_a = 20$ sr at prevalence of the coarse fraction in the lognormal size distribution of aerosol particles, 60 sr for submicron fraction, and 40 sr for a bimodal distribution.

At the same time, the assumption that S_a = const used in reconstructing the vertical profile $\sigma(z)$ leads to the deviations of the obtained profile $\sigma(z)$ from the reference one in some parts of the height profile. It is illustrated in Ref. 12 and by our plots in Fig. 6, where the comparison of the profile $\sigma(z)$ reconstructed by the iteration method and that by Fernald method (at S_a = 50 sr) with the data of Raman lidar is shown. The difference between the profiles reconstructed by these two methods of the lidar data processing is observed starting from the height of 1.5 km, where the lidar ratio values decrease with height.

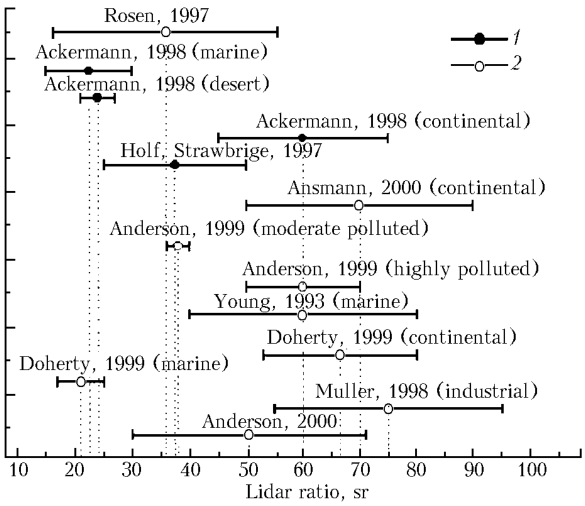

Fig. 5. Ranges of variations of the lidar ratio for different aerosol types (from the data obtained in 1993–2000): theoretically calculated data (*1*); experimental data (*2*).

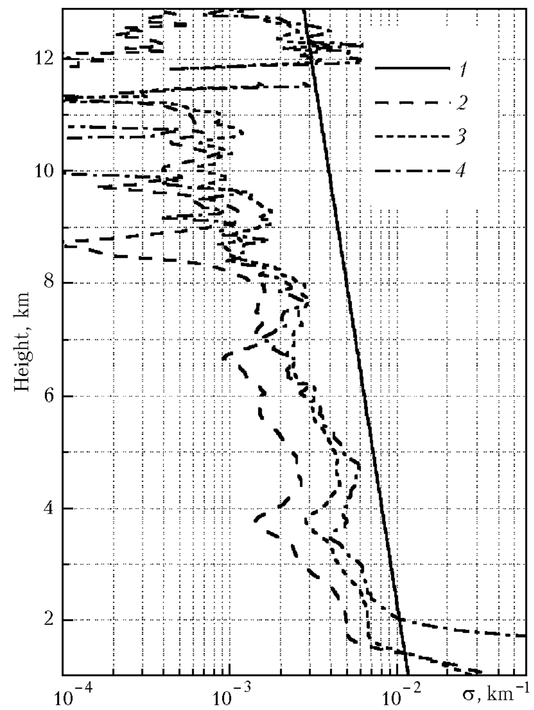

Fig. 6. Profile of the aerosol scattering coefficient $\sigma(z)$: model of $\sigma(z)$ for the molecular atmosphere (*1*); reconstructed by the Fernald method (*2*); by the iteration method (*3*); and the data of Raman lidar (*4*).

Application of the iteration method provides for quite a good approach to the actual profile $\sigma(z)$, at least, starting from 2-km height. Significant deviations of both parameters from the data of Raman lidar are observed up to this height. Most likely, the cause is the peculiarities in the data of Raman scattering that was discussed above when discussing Fig. 2. Thus, we can give the following recommendations for interpretation of the lidar return signals under conditions of *a priori* uncertainty:

1) under conditions of clean and weakly turbid atmosphere it is necessary to use the methods which take into account the vertical behavior of the molecular scattering coefficient of the atmosphere. The profile $\sigma_{mol}(z)$ should be set by the model of the molecular atmosphere, or, if possible, calculated from the aerological data.

2) the least errors in data reconstructed from the lidar returns are provided for by the iteration method described above.

3) when using the iteration method, it is most optimum to use the functional relationship between the lidar ratio and the aerosol scattering coefficient proposed by Kovalev,[11] formula (7a).

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