

# Effect of turbo-engine jet on laser radiation. Part 1. Angular spectrum of disturbed beam

V.S. Sirazetdinov, D.I. Dmitriev, I.V. Ivanova, and D.H. Titterton<sup>1</sup>

*Research Institute for Complex Testing of Optoelectronic Devices and Systems,  
Sosnovyi Bor, Leningrad Region, Russia*

<sup>1</sup> *Defense Evaluation and Research Agency (DERA), Farnborough, United Kingdom*

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The angular spectrum of laser beams ( $\lambda = 1.06$  and  $0.53 \mu\text{m}$ ) traversing a turbo-engine jet at different angles to its axis ( $90$ ,  $45$ , and  $10^\circ$ ) has been studied experimentally. The angular divergence of radiation increases by 6 to 35 times under the effect of the jet. It has been found that the angular width of the  $0.53\text{-}\mu\text{m}$  beam significantly (two to three times) exceeds that of the  $1.06\text{-}\mu\text{m}$  beam and the angular intensity distribution of radiation demonstrates azimuth asymmetry. By selecting the appropriate spectrum of the refractive index fluctuations (combination of the von Karman spectrum for turbulence and an additional high-frequency spectral function, anisotropy in the region of the outer scales of turbulence), analytical equations have been derived for estimation of the angular distribution of the disturbed beams; the obtained results are in agreement with the experimental data.

## Introduction

Propagation of laser radiation through the turbulent atmosphere is a subject of intense investigations for many years. In such investigations, the main attention is usually paid to the study of propagation of laser radiation under natural atmospheric conditions, when the level of atmospheric turbulence does not exceed the values characterized by the structure constant of the refractive index  $C_n^2 \sim 10^{-13}$ – $10^{-12} \text{ m}^{-2/3}$ . However, the situations, when zones characterized by turbulence higher three to four orders of magnitude ( $C_n^2 \sim 10^{-10}$ – $10^{-9} \text{ m}^{-2/3}$ ) arise in the ambient medium, are rather often in practice. Such zones may be the result of both natural (downward blasts, wind shears near atmospheric fronts) and artificial (turbo-engine jet or adiabatic vortical contrail) factors. The laser radiation disturbed by a turbulent layer can be an indicator of the phenomena mentioned above. This paper is devoted to the study of the effect of turbo-engine jet on the angular divergence of laser beams with the radiation wavelength  $\lambda = 1.06$  and  $0.53 \mu\text{m}$ .

## 1. Experimental studies

The optical arrangement of the experiment is depicted in Fig. 1.

As a source of the jet in the experiments, we used an R-25-300 turbo-engine 3 with the nozzle diameter of 55 cm. The temperature  $T$  of the jet at the exit from the nozzle was about  $380^\circ\text{C}$ , and its speed  $V$  at the axis was about  $600 \text{ km/h}$  at the distance of 1 m from the nozzle.

A laser 1 together with the telescopic beam expansion system 2, a beam-turning mirror  $M1$ , and a receiving system (objectives  $O1$  and  $O2$ , optical wedges  $W1$  and  $W2$ , CCD cameras 4 and 5, computers 6, and attenuators 7) were placed on vibroprotective platforms.

A turret with interchangeable aperture diaphragms  $D$  determining the diameter of the beam at the entrance to the jet was installed at the exit of the telescopic system. The duration of laser pulses with the energy of 3–5 mJ did not exceed 30 ns at the pulse repetition frequency of 12.5 Hz.

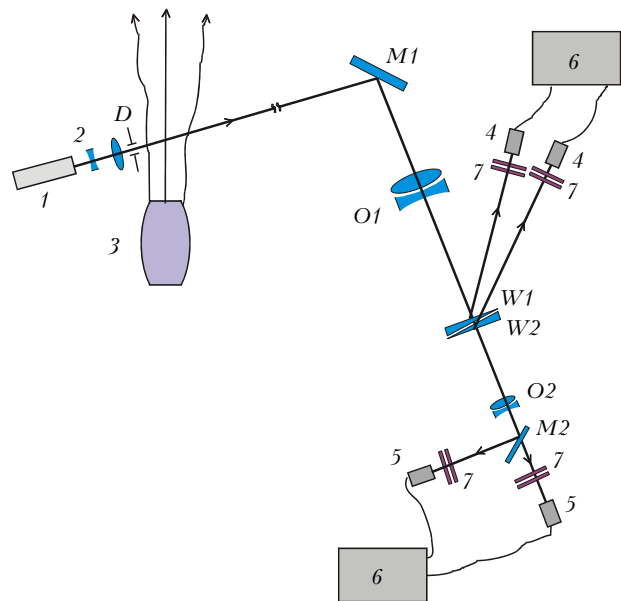


Fig. 1. Optical arrangement of the experiment.

The experiments were conducted as the laser pulse passed both across (the angle of jet intersection  $\varphi_1 = 90^\circ$  and  $\varphi_2 = 45^\circ$ ) and almost along ( $\varphi_3 = 10^\circ$ ) the jet axis. The distance  $L_1$  from the laser to the beam-turning mirror  $M1$  that turned the beam out of the jet zone and directed it toward the receiving system was roughly equal to 60 m. The distance  $L_2$  from the mirror  $M2$  to the entrance objective  $O1$  of the receiving system was 23 m.

The entrance objective of the receiving system had the diameter of 30 cm and the focal length  $F_1 = 265$  cm. The wedges  $W1$  and  $W2$  deflected a portion of radiation to the CCD cameras 4 placed in the focal zone of the objective  $O1$  and recording the far-field intensity distributions of the laser radiation with the wavelength  $\lambda = 0.53$  and  $1.06 \mu\text{m}$ . The objective  $O2$  retranslated, in the needed scale, the near-field image of the disturbed laser beam to the CCD cameras 5.

In the experiments, the laser beam entered the jet at the distance of about half a meter from the nozzle and crossed its axis. The axes of the turbo-engine jet and the laser beam lied in the horizontal plane at the altitude about 150 cm above the ground.

In the process of the study, control experiments were conducted to assess the influence of the vibration noise of the operating engine on the measured characteristics of the laser radiation. In these experiments, the vibrational situation and the optical path parameters were kept unchanged, but the beam did not pass the jet of the engine operating in the standard mode. The obtained data showed that vibrations excited by the engine did not disturb laser beams.

The Table below gives the measured average halfwidths of the angular distribution of laser beams traversing the engine jet at different angles to its axis. Let us emphasize an important peculiarity of the experimental studies: the laser emitted short pulses at two wavelengths simultaneously. Thus, we recorded almost instantaneous images of laser beams passed through the same inhomogeneities in the high-speed turbulent jet. No less than 1500 images were recorded in every experimental series, and this statistical array was used for averaging the angular distributions of the radiation.

**Table. Angular halfwidth of the laser beam at the level  $(1/e) I_{\max}$  in the horizontal and vertical directions:  $\theta_x$  and  $\theta_y$ , in  $\mu\text{rad}$**

Beam parameters		$\lambda = 1.06 \mu\text{m}$		$\lambda = 0.53 \mu\text{m}$	
		Diameter, mm			
		30	10	30	10
$\varphi_1 = 90^\circ$	$\theta_x$	140±20	145±20	415±50	420±50
	$\theta_y$	205±30	220±30	470±60	505±60
$\varphi_2 = 45^\circ$	$\theta_x$	210±30	225±30	530±70	560±70
	$\theta_y$	310±40	320±40	725±90	750±90
$\varphi_3 = 10^\circ$	$\theta_x$	325±40	350±40	680±90	840±100
	$\theta_y$	390±50	400±50	780±100	945±120
No jet	$\theta_x$	26±5	31±5	22±5	39±6
	$\theta_y$	26±5	42±6	22±5	46±6

From analysis of the experimental data, we have found the following.

Under the effect of the jet, the angular divergence of the  $1.06\text{-}\mu\text{m}$  beam increases roughly by 6 to 15 times, and that of the beam at  $0.53\text{-}\mu\text{m}$  wavelength by 20 to 35 times. For both of the wavelengths and diameters of the beam, the angular width increases as the angle of the jet intersection by the beam decreases. This is natural,

because as the intersection angle  $\varphi$  decreases, the length of the beam path in the zone of the turbulent jet increases: for  $\varphi = 90^\circ$  the path length in the turbulent zone  $L_t$  is 0.8 m, for  $\varphi = 45^\circ$  it is 1.4 m, and for  $\varphi = 10^\circ$  it is about 60 m.

In all experimental situations, the angular divergence of the beams  $0.53\text{-}\mu\text{m}$  exceeds by two to three times the angular divergence of the  $1.06\text{-}\mu\text{m}$  beams. This proved to be rather unexpected, because the theoretical estimates do not give that strong spectral dependence of the angular widening of a laser beam having passed through a turbulent zone characterized by the standard spectrum of the refractive index fluctuations. The simplest assessment of the power spectrum gives the following equation. The angular width of the beam disturbed by the turbulent medium is  $\theta \sim \lambda/\rho_c$ , where the coherence length  $\rho_c \sim [1.45 C_n^2 (2\pi/\lambda)^2 L_t]^{-3/5}$ . Then the angular divergence can be assessed as

$$\theta \sim \lambda (1.45 C_n^2 (2\pi/\lambda)^2 L_t)^{3/5} \sim (5.8 \pi^2 C_n^2 L_t)^{3/5} \lambda^{-1/5} \quad (1)$$

From Eq. (1) for  $\lambda_1 = 1.06$  and  $\lambda_2 = 0.53 \mu\text{m}$  we have the difference in the angular divergence much smaller than the experimental value:  $\theta(\lambda_2)/\theta(\lambda_1) \sim (\lambda_1/\lambda_2)^{1/5} \approx 1.15$ .

Another point to be noted is the azimuth asymmetry of the angular intensity distributions of the beams disturbed by the jet. This asymmetry is rather pronounced for the geometries with  $\varphi = 45$  and  $90^\circ$ . In this case, the average distribution is shaped as an ellipse, whose longer axis is oriented along the vertical normal to the plane including the jet and beam axes.

## 2. Comparison of experimental data with theoretically calculated for the angular spectrum of the beam disturbed by a turbulent jet

To reveal all peculiarities from the results obtained, let us analyze separately the conditions for appearance of the spectral dependence of the angular intensity distribution and the asymmetry in the distribution.

The normalized angular spectrum of the laser beam having passed through the zone of the turbulent atmosphere of the thickness  $L_t$  can be found from the following equation<sup>1</sup>:

$$I(\theta) = \frac{\int_0^1 e^{-D(r)/2} J_0(2akr\theta)\gamma(r)rdr}{\int_0^1 e^{-D(r)/2}\gamma(r)rdr}, \quad (2)$$

where  $J_0(x)$  is the Bessel function;

$$\gamma = \arccos\left(\frac{r}{2a}\right) - \left(\frac{r}{2a}\right)\sqrt{1 - \left(\frac{r}{2a}\right)^2}$$

is the initial coherence function of the beam with the radius  $a$  and the homogeneous intensity distribution over the cross section;  $D(r)$  is the structure function of the complex phase for the plane wave.

The function  $D(r)$  can be found more easily in the case that the turbulent layer is homogeneous, on the average, in the length. The experiments with beams traveling across the jet ( $\varphi = 90$  and  $45^\circ$ ) fall in this category. Therefore, below we will analyze just these cases. Besides, let us take into account that the largest vortical formations in the jet are comparable, in size, with its radius,<sup>2</sup> which in our experiments was as small as  $\sim 0.5$  m in the zone of intersection. Normally used phase structure function for the wave having passed through a homogeneous and isotropic turbulent layer characterized by the structure constant  $C_n^2$  and the outer scale of turbulence  $L_0$  has the form<sup>3</sup>:

$$D(r) = 2.77 k^2 C_n^2 L_t \times \left[ \frac{\pi L_0^{5/3}}{\Gamma(1/6)} - (r L_0 / 2)^{5/6} K_{-5/6}(r/L_0) \right], \quad (3)$$

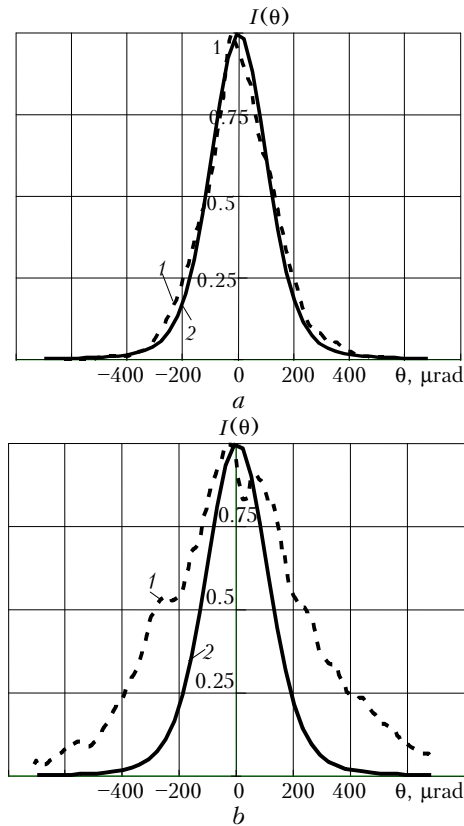
where  $K_{-5/6}(x)$  is the MacDonald function;  $\Gamma(x)$  is the gamma function.

Figure 2 depicts the angular intensity distributions for the beam of 10 mm in diameter. These distributions were obtained experimentally and calculated from Eq. (2) at the values of  $C_n^2$  and  $L_0$  determined from fitting the theoretical distribution for  $\lambda = 1.06 \mu\text{m}$ . It should be noted that it is just this form of the structure function that allowed simulation of the experimental results obtained earlier in analogous experiments for the wavelengths  $\lambda = 1.06$  and  $10.6 \mu\text{m}$  (Refs. 4 and 5).

Figure 2 depicts the averaged radiation intensity distributions in the far zone in the diagonal cross section of the beam, because at this stage we do not consider the asymmetry of the distribution. It is seen that the use of the structure function (3) corresponding to the von Karman spectrum for the refractive index fluctuations

$$\Phi(p) = 0.033 C_n^2 \left[ \left( \frac{1}{L_0} \right)^2 + p^2 \right]^{-11/6}, \quad (4)$$

in calculations does not allow simulation of the experimental results for the angular divergence of the  $0.53\text{-}\mu\text{m}$  radiation. Analogous result follows from the comparison of calculated and experimental data for beams 30 mm in diameter and different geometries of the jet intersection. Consequently, to predict the characteristics of the radiation having passed through the jet, it is insufficient to use standard model of the turbulent layer. It should be kept in mind that additional scattering mechanism to be introduced in the model of radiation propagation through the jet should possess the property of strong effect on the shape of the angular spectrum for the short wave radiation and a relatively weak effect on the spectrum of radiation with the wavelength longer than  $1 \mu\text{m}$ .



**Fig. 2.** Angular intensity distributions of the laser beam disturbed by the turbulent jet for  $\lambda = 1.06 \mu\text{m}$  (a) and  $0.53 \mu\text{m}$  (b),  $\varphi = 90^\circ$ : experiment, diagonal cross section of the beam in the far zone (1); calculation at the following parameters of the von Karman spectrum  $C_n^2 = 0.7 \cdot 10^{-9} \text{ m}^{-2/3}$  and  $L_0 = 0.5 \text{ m}$  (2).

Such a mechanism may be scattering of radiation by aerosols that are possibly present in the engine jet or by other inhomogeneities, but corresponding to high-frequency spectrum of inhomogeneities in the disturbed medium. For their simulation, we used the spectrum of fluctuations of the refractive index in the form of the combination of the initial spectrum (4) with the spectral function supplementing high-frequency components. For a comparison with the experiment, we have selected two types of functions for high-frequency component, in combination with which the complete spectra have the following forms:

$$\Phi 1(p) = 0.033 C_n^2 \left\{ \left[ \left( \frac{1}{L_0} \right)^2 + p^2 \right]^{-11/6} + B \left[ \left( \frac{2\pi}{L_s} \right)^2 + p^2 \right]^{-11/6} \right\}, \quad (5)$$

$$\Phi 2(p) = 0.033 C_n^2 \left[ \left( \frac{1}{L_0} \right)^2 + p^2 \right]^{-11/6} + \sigma_n^2 \left( \frac{L_s}{2\sqrt{\pi}} \right)^3 \exp \left[ - \left( \frac{p L_s}{2} \right)^2 \right], \quad (6)$$

where the values of the parameters  $B$  in Eq. (5) and  $\sigma_n$  in Eq. (6) are the results of fitting to the experimental data. The high-frequency component in the spectrum (6) corresponds to the so-called single-scale type of the spectrum with the characteristic size of the refractive index inhomogeneities  $L_s$ , and that in the spectrum (5) corresponds to the multiscale type, in which the parameter  $L_s$  is the outer scale of turbulence.

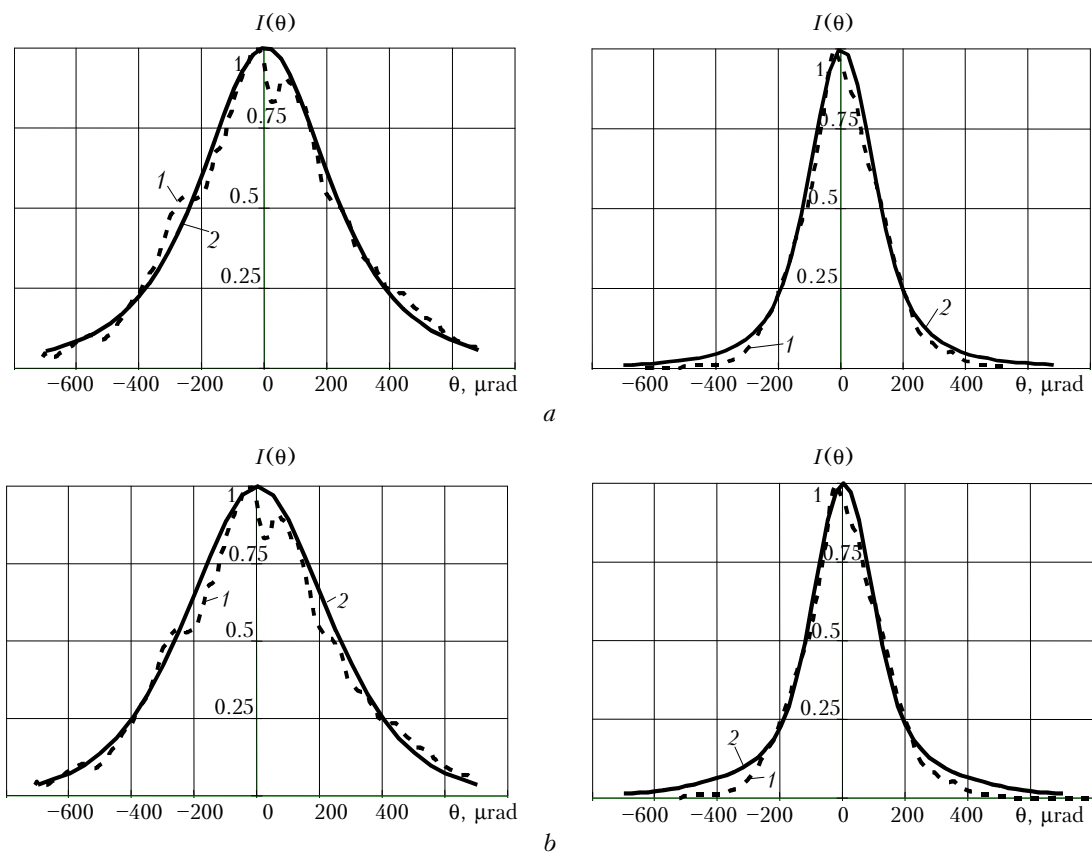
Figure 3a depicts the experimental and calculated angular spectrum of the disturbed laser beam 10 mm in diameter ( $\varphi = 90^\circ$ ). The calculations have been made by Eq. (2) with the use of the structure function corresponding to the spectrum (5) at the following values of the parameters for  $\lambda = 0.53 \mu\text{m}$ :  $C_n^2 = 0.7 \cdot 10^{-9} \text{ m}^{-2/3}$ ,  $L_0 = 0.5 \text{ m}$ ,  $B = 8$ , and  $L_s = 1 \text{ mm}$ , and for  $\lambda = 1.06 \mu\text{m}$  the values  $C_n^2 = 0.6 \cdot 10^{-9} \text{ m}^{-2/3}$  and  $B = 7$  were found from fitting. Figure 3b depicts the same angular intensity distributions calculated with the use of the spectrum (6) at the following values of the parameters for  $\lambda = 0.53 \mu\text{m}$ :  $C_n^2 = 0.7 \cdot 10^{-9} \text{ m}^{-2/3}$ ,  $L_0 = 0.5 \text{ m}$ ,  $\sigma_n = 2 \cdot 10^{-11}$ , and  $L_s = 1 \text{ mm}$ , and for  $\lambda = 1.06 \mu\text{m}$  we also used the value  $C_n^2 = 0.6 \cdot 10^{-9} \text{ m}^{-2/3}$ .

It is seen that both versions of spectrum of the refractive index fluctuations allow simulation of the angular intensity distribution of the radiation disturbed by the jet at the two wavelengths assuming proper choice

of the parameters of the spectrum of inhomogeneities. The characteristic scale of inhomogeneities in the high-frequency part of the spectrum proves to be  $L_s \approx 1 \text{ mm}$ . Similar results were obtained for other experimental situations, as well.

Thus, the experimental wavelength dependence of the averaged angular intensity distribution of the laser beam disturbed by the jet is well described by usual von Karman spectral function (4) for fluctuations of the refractive index in combination with the additional high-frequency spectral function. At the same time, the comparison of the experimental and theoretically calculated angular spectra of the beams intersecting the jet do not allow us to choose unambiguously the form of the high-frequency component of the spectrum.

As noted above, there exists one more effect of the jet on the angular spectrum of the laser beam, which is not theoretically described by Eq. (2). It is the asymmetry of the angular distributions of radiation, i.e., different angular halfwidth of the beam in the horizontal ( $\theta_x$ ) and vertical ( $\theta_y$ ) directions. It should be noted that random wandering of the beam centroid along the vertical and horizontal is different, as well. These data suggest that for simulation of random inhomogeneities in the jet we should reject the assumption of their isotropic spatial spectrum.



**Fig. 3.** Angular intensity distribution of the laser beam disturbed by the turbulent jet for the wavelength  $\lambda = 0.53 \mu\text{m}$  (left) and  $1.06 \mu\text{m}$  (right): experiment, diagonal cross section along the beam in the far zone (curve 1) and calculation with the von Karman function supplemented with the function increasing the contribution of the high-frequency components (2).

When it is necessary to simulate the anisotropy of the randomly inhomogeneous medium in the region of the outer scale of turbulence, the spectrum is modified in the following way<sup>3</sup>:

$$\Phi(p_x, p_y) = \frac{A C_n^2 (p_{x0} p_{y0})^{-11/6}}{[1 + (p_x/p_{0x})^2 + (p_y/p_{0y})^2]^{11/6}}, \quad (7)$$

where  $p_{x0} = 1/L_{0x}$ ;  $p_{y0} = 1/L_{0y}$ ;  $L_{0x}$  is the outer scale of turbulence in the direction  $OX$ , and  $L_{0y}$  is the outer scale of turbulence in the direction  $OY$ .

The comparison of the experimental and calculated data obtained based on the spectral function (7) assumes that the observed asymmetry of the angular characteristics is connected with large-scale vortices.

In the approximation valid for narrow beams (beam radius  $a \ll L_{0x}, L_{0y}$ ) and not very strong anisotropy and using the spectrum (7), we can obtain the following analytical equation for the structure function of the complex phase:

$$D(x, y) = 2.92k^2 L_t C_n^2 \left\{ (x^2 + y^2)^{5/6} - \frac{3 \left[ \left( \frac{L_{0x}}{L_{0y}} \right)^2 x^2 + \left( \frac{L_{0y}}{L_{0x}} \right)^2 y^2 \right] + x^2 + y^2}{(L_{0x} L_{0y})^{1/6}} \right\}. \quad (8)$$

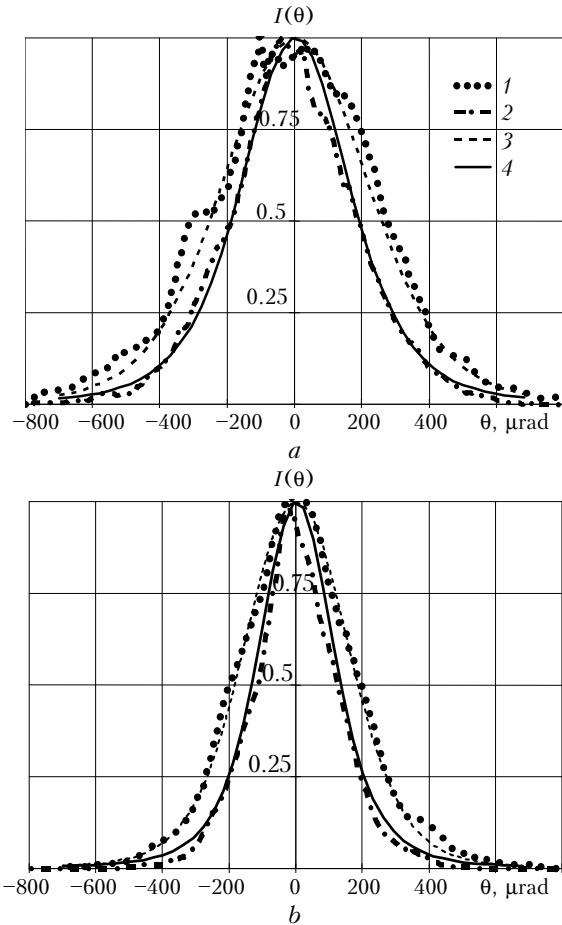
In spite of the approximate character, this equation allows qualitative consideration of the situation of a laser beam affected by the anisotropic randomly inhomogeneous medium. At  $L_{0x} = L_{0y}$ , Eq. (8) transforms into the well-known equation, which can be obtained from Eq. (3) with the allowance for the limitedness of the outer scale of turbulence in the first approximation:

$$D(x, y) = 2.92k^2 L_t C_n^2 \left[ (x^2 + y^2)^{5/6} - 0.8 \left( \frac{x^2 + y^2}{L_0^{1/3}} \right) \right]. \quad (9)$$

Figure 4 depicts both the experimental and calculated profiles of the angular intensity distribution along the orthogonal axes  $OX$  and  $OY$  for the radiation with the wavelength  $\lambda = 1.06 \mu\text{m}$  and the angle of intersection of the beam and the jet  $\varphi = 45^\circ$  and  $\varphi = 90^\circ$  (beam diameter of 10 mm). The calculations were made by Eq. (2) with the structure function in the form (8). The calculated curves were fitted to the experimental ones with the fitting parameters:  $C_n^2 \approx 1.5 \times 10^{-9} \text{m}^{-2/3}$ ,  $L_{0x} \approx 0.35 \text{m}$ , and  $L_{0y} \approx 0.7 \text{m}$ .

It should be noted that the value of the structure characteristic  $C_n^2$  proved to be half as large when fitting by the profile of the diagonal cross section of the intensity distribution in the far zone. Consequently, the transition to the anisotropic form of the spectrum of the refractive index fluctuations in the mathematical model is very

important for obtaining correct numerical estimates of the parameters of a turbulent medium.



**Fig. 4.** Averaged angular intensity distribution of radiation disturbed by the turbulent jet for the wavelength  $\lambda = 1.06 \mu\text{m}$  in the orthogonal cross sections of the beam. The intersection angle:  $\varphi = 45^\circ$  (a) and  $90^\circ$  (b). Experimental and calculated distribution of the beam in the vertical (1 and 3) and horizontal (2 and 4) cross sections, respectively.

### Conclusion

The comparison of the experimental data with known theoretical equations showed that the considered equation for the angular characteristics of radiation having propagated through the jet of a turbo-engine does not agree with the standard model of the turbulent layer. The experimentally observed wavelength dependence of the averaged angular intensity distribution of the laser beam disturbed by the jet agrees with the theoretical one if the calculations use the ordinary von Karman spectral function for fluctuations of the refractive index in combination with the additional spectral function increasing the contribution of high-frequency components ( $p \geq 10^3 \text{ 1/m}$ ). However, the agreement between the experimental and theoretically calculated angular spectra for radiation of different wavelengths can be achieved by using high-frequency spectral functions of different form, what does not allow us to select finally the form of the spectrum.

The experiments revealed the azimuth asymmetry of the angular intensity distributions of laser beams disturbed by the jet. The asymmetry is more pronounced at  $\lambda = 1.06 \mu\text{m}$ , than at  $\lambda = 0.53 \mu\text{m}$ :  $(\theta_y/\theta_x)_{1.06} \sim 1.5$  and  $(\theta_y/\theta_x)_{0.53} \sim 1.2$  for the beam-jet intersection angles  $\varphi = 45$  and  $90^\circ$ . As the laser beams propagate almost along the jet ( $\varphi = 10^\circ$ ), the asymmetry of the angular distributions proves to be somewhat smaller. In the approximation of narrow beams propagating through the jet, the equations have been obtained for the structure function of the phase. The similarity of the experimental and calculated results has been demonstrated as applied to the asymmetry of the distributions; this similarity was achieved by introducing the anisotropy in the region of the outer scales of turbulence for the von Karman spectral function of the refractive index fluctuations.

The modifications of the spectrum of the refractive index fluctuations considered in this paper are important

for correct numerical simulation of the process of laser radiation propagation through the turbulent jet using the method of random phase screens.

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