# On transformation of the effect–dose dependence in the actual turbulent atmosphere

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The properties of the probability density function of an effect are considered based on the known probability density function of a dose in the actual turbulent atmosphere.

A wide variety of relationships for estimating the influence of environmental factors on living objects can be found in the literature.[1–13] The quantitative measure of the influence of any factor expressed in any units is called a dose, and the reaction of organisms to this factor is called an effect. At aerogenic contamination of warm-blooded organisms with aerosol and gaseous impurities of the atmosphere, the dose $D$ can be defined as the amount of a toxicant inhaled in lungs, and the effect $E$ is understood as a fraction (percentage) of a homogeneous population of organisms affected under the exposure to a fixed dose. The dependence $E = E(D)$ can be estimated theoretically and obtained experimentally in some cases.

This problem, especially as applied to human beings, is of particular complexity because no direct experiments can be conducted in this case. In such a situation, we can use the methods for estimating the effect–dose curve that are based on extrapolation of experimental data obtained for primates and other laboratory animals. Such a procedure is justified, for example, in Ref. 14. The dose considered above is a deterministic (rather than random) parameter. In the actual atmosphere, which always is turbulent, the dose becomes a random characteristic. Therefore, the effect also becomes a random parameter, and when solving particular practical problems, one needs to consider the probability that the effect exceeds some threshold value $P = P(E \geq E_0)$, where $E_0$ is the threshold value of the effect. In this paper, we consider the properties of the probability density function of the effect $g = g(E)$ based on the known probability density function of the dose in the actual turbulent atmosphere $f = f(D)$ using, as an example, a specific dependence $E = E(D)$.

The dose received by an organism for the period from $t$ to $t + T$ equals to

$$D = \int_t^{t+T} Q\, C(x)\, \mathrm{d}x, \qquad (1)$$

where $Q$ is the instantaneous value of the volume rate of air flow into lungs; $C$ is the instantaneous value of the pollutant concentration. Then we assume that the conditions of pollutant diffusion are stationary and $Q = \text{const}$ is some effective rate of a pollutant inhaling into lungs. Then Eq. (1) takes the form

$$D = Q \int_t^{t+T} C(x)\, \mathrm{d}x. \qquad (2)$$

Thus, accurate to a constant factor, the dose is the time-integrated concentration of the diffusing pollutant. It is a local characteristic determined for every point of space. Earlier we have derived theoretically the probability density function of the integral pollutant concentration and confirmed it in experiments in a wind tunnel.[15] If the dose is measured in units of its mathematical expectation $\bar{D}$, then its probability density function has the following form:

$$f(D) = [1 - \mathrm{erf}(\beta)]\, \delta(D) + \frac{\beta}{\sqrt{\pi}} \{\exp[-\beta^2 (D-1)^2] - \exp[-\beta^2 (D+1)^2]\}, \qquad (3)$$

where erf(...) denotes the probability integral; $\beta$ is the parameter; $\delta(\ldots)$ is the delta function.

In Eq. (3), the first term describes the probability of observation of zero doses. It is directly connected with the effect of concentration intermittence, i.e., the probability to observe zero concentration.[15] The parameter $\beta$ can be determined from the values of the mathematical expectation of the dose $\bar{D}$ and its variance $\sigma^2$ (see Ref. 15):

$$\frac{\sigma^2}{\bar{D}^2} = \mathrm{erf}(\beta) \left(1 + \frac{1}{2\beta^2}\right) - 1 + \frac{1}{\sqrt{\pi}\,\beta} \exp(-\beta^2). \qquad (4)$$

In the case considered here, the mathematical expectation of the dose is equal to $\bar{D} = QT\bar{C}$, where $\bar{C}$ is the mathematical expectation of the concentration. The variance of the dose is determined by the equation[16]:

$$\sigma^2 = 2Q^2 \sigma_c^2 \int_t^{t+T} (T - t_1)\, r(t_1)\, \mathrm{d}t_1, \qquad (5)$$

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where $\sigma_c^2$ is the variance of the pollutant concentration; $r(t)$ is the normalized autocorrelation function of concentration pulsations. In Ref. 17 we have determined the form of the correlation function $r(t)$. Therefore, Eq. (5) takes the form

$$\sigma^2 = 2Q^2\,\sigma_c^2\,\tau^2\left[\frac{T}{\tau} - 1 + \exp\left(-\frac{T}{\tau}\right)\right], \tag{6}$$

where $\tau$ is the Eulerian time scale of concentration pulsations; it estimates the characteristic length of the correlation function in time. Let us introduce the intensity of concentration pulsations as $I = \sigma_c/\bar{C}$, then from Eqs. (4) and (6) it follows that

$$2I^2\,\frac{1}{\zeta^2}\,[\zeta - 1 + \exp(-\zeta)] = \mathrm{erf}(\beta)\left(1 + \frac{1}{2\beta^2}\right) - 1 + \\ + \frac{1}{\sqrt{\pi}\,\beta}\exp(-\beta^2) = \varphi, \tag{7}$$

where $\zeta = T/\tau$. The data tabulated in Table 1 give an idea on the dependence of $I$ on $\beta$.

**Table 1. Dependence of the intensity of concentration pulsations on $\beta$**

| $I$ | 0.06 | 0.13 | 0.25 | 0.5 | 1 | 2 | 4 | 8 | 16 |
|---|---|---|---|---|---|---|---|---|---|
| $\beta$ | 11.31 | 5.66 | 2.83 | 1.41 | 0.64 | 0.23 | 0.07 | 0.02 | 0.004 |

If $\zeta \to 0$, then $\varphi \to I^2$ and corresponding $\beta$ can be found by solving Eq. (7). If $\zeta \to \infty$, then $\varphi \to 0$ as $\varphi \approx 2I^2/\zeta$, and $\beta \to \infty$ as $\beta \approx \sqrt{\zeta}/(2I)$. The asymptotic properties of the resulting probability density function of the dose are given in the first three columns of Table 2.

**Table 2. Asymptotic properties of the probability density function of the dose and the distribution function of the effect**

| | | | |
|---|---|---|---|
| $\zeta \to 0$ | $I \to 0$ | $f(D) = \delta(D - 1)$ | $G(E) = \begin{cases} 0, & D(E) < 1 \\ 1, & D(E) \ge 1 \end{cases}$ |
| $\zeta \to 0$ | $I \to \infty$ | $f(D) = \delta(D)$ | $G(E) = 1$ |
| $\zeta \to \infty$ | $I \ne 0$ | $f(D) = \delta(D - 1)$ | $G(E) = \begin{cases} 0, & D(E) < 1 \\ 1, & D(E) \ge 1 \end{cases}$ |

Note that the data given in the first row of Table 2 for $I \to 0$ do not formally refer to the problem under consideration, because we consider transformation of the effect–dose curve in the turbulent atmosphere, where $I \ne 0$.

Then we obtain the distribution function of the effect. For definiteness, take the well known form of the effect–dose dependence[18]:

$$E = 1 - \exp\left(-\frac{0.69}{D_{50}}\,D\right). \tag{8}$$

The value $D = D_{50}$ in Eq. (8) corresponds to 50% effect. Assume also that $D_{50} = \bar{D}$, then for the dimensionless parameters introduced above we have $E = 1 - \exp(-0.69D)$.

The instantaneous values of the dose and effect are connected by an equation like Eq. (8), therefore $g(E)\,\mathrm{d}E = f(D)\,\mathrm{d}D$ (Ref. 16). From this we have the equation for the distribution function of the effect in the turbulent atmosphere

$$G(E) = \int_0^{D(E)} f(x)\,\mathrm{d}x. \tag{9}$$

The last column of Table 2 gives the asymptotic values of $G(E)$ obtained according to Eq. (9) and corresponding to the asymptotic probability density functions given in the third column of Table 2. In the general case, to determine the distribution law for the effect, one has to specify the period of dose accumulation $T$, the effective volume rate of inhaling of a biological object $Q$, and the mathematical expectation of the pollutant concentration $\bar{C}$. Additionally, one has to know the Eulerian time scale of concentration pulsations $\tau$ and variance $\sigma_c^2$. For the effect–dose dependence considered here, we also should know the parameter $D_{50}$. The listed parameters are necessary and sufficient for calculation of the probability that the effect exceeds a preset threshold value $P = P(E \ge E_0)$. In this case $P(E \ge E_0) = 1 - G(E_0)$. The value of $G(E)$ should be calculated by Eq. (9) with the use of function $f(D)$ described by Eq. (3) and the dependence $D = D(E)$, which is an inverse function to the curve of the type (8).

Figure 1 shows the curves calculated for three different values of the intensity of concentration pulsations. Figure 1 corresponds to the concentration intermittence equal to 0.11, 0.84, and 1.

For this figure, it is still assumed that transformation of the dose into the effect follows Eq. (8), $D_{50} = \bar{D}$, and the dose is measured in the units of $\bar{D} = QT\bar{C}$. We can see that at a low intensity of the concentration pulsations (Fig. 1*a*) the dependence $P = P(E \ge E_0)$ for different $\zeta$ is close to a stepwise, corresponding to the asymptotic properties of $G(E)$ presented in Table 2. Due to the increase in the intensity of concentration pulsations (Fig. 1*b*), the curves begin to differ. Note that at $E_0 < 0.5$ the values of $P = P(E \ge E_0)$ for $\zeta = 1$ are smaller than the values of $P = P(E \ge E_0)$ for $\zeta = 10$ and 100. This means that the dose accumulated at a low pollutant concentration for long time corresponds to higher probability of the effect than the same dose accumulated for a shorter time at high pollutant concentration does. This difference can reach tens percent. Just the opposite pattern is observed at $E_0 > 0.5$. And, finally, in another limiting case (Fig. 1*c*), when the intensity of concentration pulsations is very high, almost always the values of $P = P(E \ge E_0)$ at $\zeta = 1$ are less than the values of the probability that the dose exceeds a preset threshold for $\zeta = 10$ and 100. However, here we should speak about a tenfold and larger difference in the values of $P = P(E \ge E_0)$ for different $\zeta$. In this case, the curves also correspond to the asymptotic dependences $G(E)$ given in Table 2.

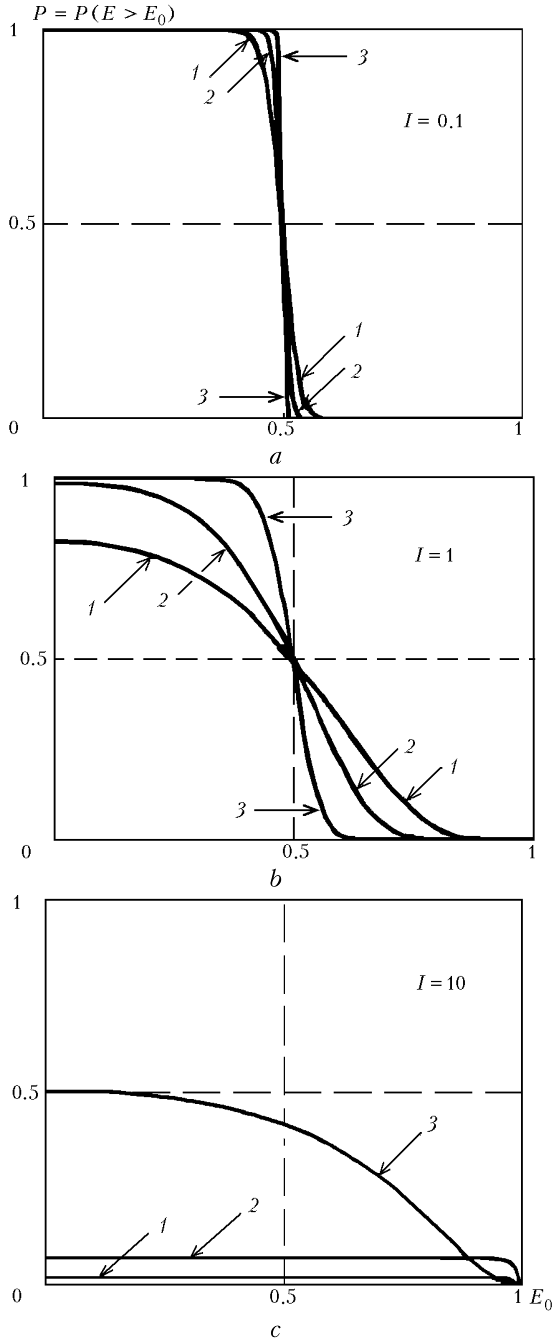


**Fig. 1.** Calculated probability that the effect exceeds a preset threshold value for different intensity of concentration pulsations. Curves *1*, *2*, and *3* correspond to dimensionless periods of dose accumulation $\zeta$ = 1, 10, and 100.

The results obtained demonstrate that pulsations of concentration of an atmospheric pollutant have a pronounced effect on the effect–dose dependence. Even insignificant variation of the pollutant concentration and the time of dose accumulation may lead to significant changes in the effect probability. All these effects should be taken into account when solving various applied and medical-biological problems.

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