Spatial and angular distribution of a light field within an ensemble of particles with a strongly forward-peaked scattering phase function

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The Monte Carlo method is applied to investigation of the propagation of radiation from a point unidirectional radiation source through the blood layer. The authors have obtained the distribution of energy flux in the layer within the beam cross section, and have constructed scattering phase functions for the blood layer for various depths of the radiation penetration.

The data on transformation of light beam over angular and radial variables[1–5] are urgent for the wide class of problems connected with propagation of radiation through the scattering media (atmosphere, ocean, biological objects). The scattering phase function of many natural objects (seawater, clouds, blood, etc.) is strongly asymmetric. The analytical formulas for angular and spatial structure of the beam were obtained[6] for such media based on solution of the radiation transfer equation in the small-angle approximation. However, if one deals with an ensemble of densely packed particles, some alternative methods are used for radiation transfer calculations. To study the regularities of the radiation propagation through the media with dense packing, it is convenient to consider blood. On the one hand, propagation of radiation through blood is interesting for clinical medicine as a tool for diagnostics, on the other hand, the developed techniques for calculation can be applied to other densely packed media with similar optical properties.

In this paper we consider peculiarities of spatial and angular distribution of light energy in a blood layer. Blood consists of plasma and suspended erythrocytes (red blood corpuscles), thrombocytes, and leukocytes. The main objects in the blood that scatter light are erythrocytes, because their number density significantly exceeds the number densities of two other blood elements and reaches the value of $n = 5{\cdot}10^6$ mm^{-3}. Depending on the physicochemical conditions, the shape of erythrocytes can change from a disk-like shape to the spherical one of the same volume. The disk-shaped erythrocytes have the following characteristic linear size: the diameter is 7 to 9 μm, the thickness in the center is 1 μm and that at the edge is 2 μm. The red portion of the visible range is used for diagnostics of illnesses. The scattering and absorption cross sections of an individual erythrocyte at the wavelength $\lambda = 662$ nm are $\sigma_{sca} = 57.2$ μm^2 and $\sigma_{abs} = 0.06$ μm^2, respectively.[5] Linear sizes of the erythrocyte are one order of magnitude greater than the wavelength, so the erythrocyte is strongly anisotropic scatterer with the mean cosine of the scattering angle $g = \langle\cos\theta\rangle = 0.995$. The entire energy of light incident on the erythrocyte is mainly scattered in the forward direction. The scattering phase function of erythrocytes is approximated in optics of biological tissues by the Henyey–Greenstein scattering phase function[5]

$$p(\theta) = \frac{1}{4\pi}\frac{1 - g^2}{(1 + g^2 - 2g\cos\theta)^{3/2}}, \qquad (1)$$

where the angle θ is counted from the direction of the incidence of the light beam on the particle.

The complex shape of erythrocyte is replaced in the approximation (1) by spherical one. Erythrocytes occupy 40% of the blood volume. So dense package of red blood bodies leads to the necessity of taking into account the multiple scattering of light. Such problems are often solved by the method of statistical modeling.

Let us specify the statement of problem. Let the photon flux of the wavelength λ from the unidirectional point source be incident on the blood surface along the normal. Let the initial energy of the pulse be equal to W_0. It is required to determine the spatial and angular distribution of the intensity of laser radiation depending on the depth of penetration h into the blood. The point of incidence of the photon on the blood layer was accepted as the origin of the coordinate. The photon motion trajectory was modeled by the Monte Carlo method.[7] The acts of scattering and absorption occur with the probabilities

$$p_{sca} = \sigma_{sca}/\sigma_{ext}, \quad p_{abs} = \sigma_{abs}/\sigma_{ext},$$

where $\sigma_{ext} = \sigma_{sca} + \sigma_{abs}$ is the cross section of extinction. As the erythrocytes are chaotically distributed, the free path length of the photon between two sequent collisions is a random value with the probability density

$$p(\tau) = \frac{1}{\tau_0}\exp(-\tau/\tau_0),$$

where $\tau_0 = n\sigma_{ext}$ is the mean free path length. Calculation of the coordinates x_{n+1}, y_{n+1}, z_{n+1} of the next point of collision of the photon was performed by the formulas

$$x_{n+1} = x_n + a_n\tau_n;$$
$$y_{n+1} = y_n + b_n\tau_n;$$
$$z_{n+1} = z_n + c_n\tau_n.$$

Here the unit vector (a_n, b_n, c_n) determines the motion direction of the photon scattered at the point with the coordinates (x_n, y_n, z_n), τ_n is the distance passed until new collision and calculated by the formula $\tau_n = -\tau_0 \ln\gamma$, where the random value γ is set by means of the generator of random quantities. The unit vector components determining the photon motion direction after the collision are the functions of the angular variables θ and φ. The random value $\cos\theta$ is distributed with the Henyey–Greenstein probability density (1), the angle φ is isotropic. The process continues until the photon goes out of the layer or is absorbed. Statistical estimates of the values under investigation are obtained using the results of $N = 500000$ tests.

Counting of the photons is performed within the 10-μm wide ring zones, into which the planes perpendicular to z-axis are divided, at different depths h from 25 to 850 μm. Except for the initial part of the path, the total normalized energy flux W exponentially decreases as a function of the coordinate z (Fig. 1). The losses of energy due to absorption and backscattering are small near the point of the radiation incidence on the blood layer, so the decrease of energy is proportional to the passed distance. The effective depth of the radiation penetration determined from the decrease of the energy flux by e times, is equal to $z_{\text{eff}} = 850$ μm. The fraction of absorbed energy is 39%, and the fraction of backscattered energy is 25% of the energy incident onto the blood layer. The energy is mainly transferred within the narrow near-axis area. Multiple scattering transfers the energy from the beam axis to the near-axis area and redistributes it over the cross section.

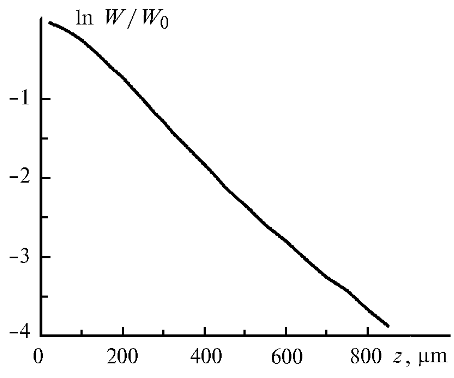

Fig. 1. Logarithmic dependence of the normalized energy flux W/W_0 on the depth z.

Broadening of the beam is shown in Fig. 2, where the energy flux is shown as a function of the radial variable $\rho = (x, y)$ in a cross section at different depths h. The maximum of the energy flux is displaced in the cross direction by the distance of 10–20 μm from the beam axis. Seventy percent of the forward scattered light energy at the depth of 500 μm is concentrated in the beam of the size 40 μm.

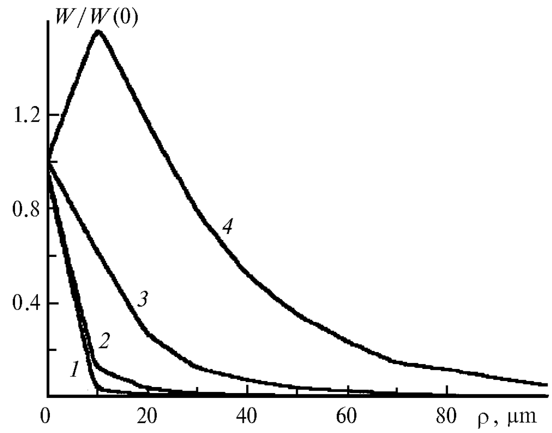

Fig. 2. Dependence of the normalized energy flux W/W_0 on the radial variable ρ in the cross section of the beam at different depths h: 50 (*1*), 100 (*2*), 250 (*3*), and 500 μm (*4*). W_0 is the energy on the beam axis at the point with the coordinate $z = h$.

The displacement of the most probable scattering angle θ_{mp} toward the side of greater values is observed in the angular structure of the scattered light as the path length h increases. The most probable angle at the blood thickness of $h = 100$ μm is $\theta_{\text{mp}} = 5.7°$, that corresponds to the characteristic scattering angle of an individual erythrocyte with the mean cosine of the single scattering $\langle\cos\theta\rangle = 0.995$. The most probable scattering angle at the depth of 850 μm is 25°. The crater is formed in the scattering phase function of the blood layer, which deepens as the layer thickness increases (Fig. 3).

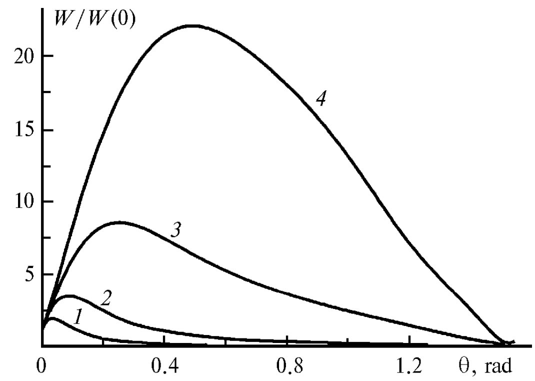

Fig. 3. Angular distribution of the normalized energy flux W/W_0 at different depths h: 50 (*1*), 100 (*2*), 250 (*3*), and 500 μm (*4*).

When affecting the blood by radiation, it is necessary to know the time t of penetration of the laser beam into the blood layer (Fig. 4). Illumination of the blood part lying at the depth h occurs with the delay $\Delta t = (s_f - h)/c$, where s_f is the distance passed by the photon to the point with the coordinate $z = h$, c is the speed of light. Then, the time of reaching the depth $h = 850$ μm by radiation increases by 1.6 times because of the multiple scattering.

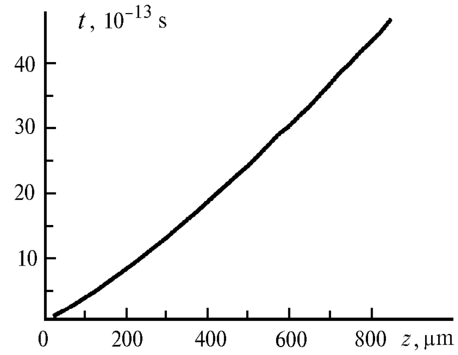

Fig. 4. Dependence of the time t of penetration of radiation into the depth of the layer on the layer thickness z.

Propagation of radiation from a point unidirectional source in the blood layer is studied in this paper by the Monte Carlo method. The distributions of the energy flux over the layer thickness in the beam cross section are obtained. The scattering phase functions of the layers at different depths of penetration of radiation are constructed.

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