The composite signal model in the interpretation of atmospheric-optical polarization measurements

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Application of the composite signal model to problems of the interpretation of the atmospheric-optical polarization measurements is considered. Formulas for the angle and plane of linear polarization of radiation from one of the sources are derived based on the condition of illumination of the control volume by radiation from two incoherent sources of unpolarized radiation, one of which is taken to be the background, using observations of the background and the radiation mixture.

The two parameters — degree of linear polarization and polarization angle — are used in many applied problems of sounding of the atmosphere based on the phenomenon of polarization. As is well known, the measurable intensities of natural (observed) light are formed by the superposition of many simple waves which supersede each other in rapid succession and have independent phases.[3,5] Light waves, the directions of oscillation of whose electric and magnetic fields have equal probabilities in the planes perpendicular to the beam are called natural unpolarized light. Partial polarization occurs upon scattering of natural light.

The solar radiation scattered in the atmosphere forms the background fields of brightness and degree of polarization. Situations are possible when monitoring the optical state of the atmosphere, in which the scattered radiation field in the daytime is formed not only by the main source, the Sun, but also by random sources of optical radiation – flashes, explosions, fires, etc. Additional sources may be located both on the ground and in the atmosphere, both in the field of view and beyond the horizon. These additional sources can be short-term or continuous.

Let us consider the problem, where the control volume is illuminated by two sources of unpolarized monochromatic radiation separated by a prescribed angle in the source plane. It is required to infer the degree of polarization and the angle of polarization of the radiation from each of the sources on the basis of an analysis of a sequence of polarization measurements.

Simplifying the conditions, we assume that the radiation sources and the observer are located in the same plane. We note that the majority of observed situations can be reduced to two variants of the conditions of the experiment. The first is where the volume is illuminated alternatively by radiation from the two sources, and the second is where one of the sources operates continuously and the time of switching on and duration of operation of the second source are random variables. We define two classes of readings in the observed sequence: "signal" is radiation from one of the sources and "background" is radiation from the second source. Thus, signals of both classes are present in the observed sequence, but selection of the classes "background" and "signal" is possible in the first variant, whereas selection of the classes "background" and "signal + background" is possible in the second variant.

To interpret the sequence of measurements of the total radiation $I_t(t)$, we use the composite signal model, whose essence lies in representing combined signal processes [in this case, $I_s(t)$ and $I_h(t)$] by a unit switching function[2] $q(t)$. If the switching function is random, the distribution of the times of "switching on" $q(t)$ can be described, for example, by the Poisson distribution, and the function $q(t)$ is equal to 1 at each of these times with some probability $m = P[q(t) = 1]$.[2]

For the first variant of the conditions of the problem we can represent the observed sequence in the form

$$I_t(t) = q(t)\, I_s(t) + [1 - q(t)]\, I_h(t) = \\ = I_h + q(t)(I_s - I_h), \tag{1}$$

where the control volume is illuminated either by radiation from the "signal" source ($q(t) = 1$) or by radiation from the "background" source ($q(t) = 0$).

For the second variant the form is

$$I_t(t) = I_h + q(t)\, I_s(t). \tag{1a}$$

Here the radiation from the "signal" source is added to the radiation from the continuously operating source.

Assuming the combined processes to be stationary and statistically independent, we average over an interval of values of the parameter t.

Since the subsequent discussion deals only with values averaged over the parameter t, we omit the symbol of averaging and use capital letters to designate the mean values. The mean value of the switching function is equal to m – the probability of the event in which the function $q(t)$ takes the value 1. We represent the sequence of readings for the first variant, averaged over the parameter t, in the form

$$I_t = m\, I_s + [1 - m]\, I_h = I_h + m(I_s - I_h), \qquad (2)$$

and for the second variant in the form

$$I_t = I_h + mI_s. \qquad (2a)$$

In practice, the degree of polarization at different scattering angles in the atmospheric-optical measurements is found using the Fesenkov method, which consists in measuring the intensities of the scattered light at several orientations of the polaroid, for example, three orientations differing from each other by 60°.[1,4] Following Ref. 4, we require that at the second orientation the direction of greatest transmission of the polaroid is perpendicular to the scattering plane (in scattering problems one usually selects the plane containing both beams as the reference plane for the incident and scattered radiation). The intensity of light scattered in any prescribed direction, passing through the polaroid, is described by the Stokes parameters. Measurements carried out at three orientations of the polaroid give three values of the intensity I_{1t}, I_{2t}, I_{3t}, from which one can determine the three Stokes parameters S_1, S_2, and S_3:

$$\begin{aligned} S_1 &= 2\,(I_{1t} + I_{2t} + I_{3t})/3; \\ S_2 &= -2\,[2I_{2t} - (I_{1t} + I_{3t})]/3; \qquad (3) \\ S_3 &= -2(I_{3t} - I_{1t})/\sqrt{3}, \end{aligned}$$

and from these the degree of linear polarization P_n and the polarization angle (the angle between the plane of polarization of the scattered light and the scattering plane):

$$P_t = \frac{\sqrt{S_2^2 + S_3^2}}{S_1}\,; \quad \tan 2\gamma_t = \frac{S_3}{S_2}\,. \qquad (4)$$

Taking into account (2), (2a) and the additivity property of the Stokes parameters for incoherent light beams, we write in the first variant

$$S_{it} = m\, S_{is} + (1 - m)\, S_{ih} = S_{ih} + m(S_{is} - S_{ih}), \quad (5)$$

and in the second variant

$$S_{it} = S_{ih} + m\, S_{is}, \qquad (5a)$$

where $i = 1, 2, 3,$ and $m = (0 \ldots 1)$ is the probability coefficient.

Thus the value of the polarization angle for the first variant (“background” – “signal”) is given by

$$\tan 2\gamma_t = \frac{m\, S_{3s} + (1 - m)\, S_{3h}}{m\, S_{2s} + (1 - m)\, S_{2h}}\,, \qquad (6)$$

and for the second variant (“background” – “signal + background”) by

$$\tan 2\gamma_t = \frac{m\, S_{3s} + S_{3h}}{m\, S_{2s} + S_{2h}}\,. \qquad (6^*)$$

The degree of polarization is given for the first variant by

$$\begin{aligned} P_t = \{&m^2[(S_{2s} - S_{2h})^2 + (S_{3s} - S_{3h})^2] + \\ &2m[S_{2h}(S_{2s} - S_{2h}) + S_{3h}(S_{3s} - S_{3h})] + \qquad (7) \\ &S_{2h}^2 + S_{3h}^2\}^{1/2} / [m\,(S_{1s} - S_{1h}) + S_{1h}] \end{aligned}$$

and for the second variant by

$$P_t = \frac{\sqrt{m^2(S_{2s}^2 + S_{3s}^2) + 2m(S_{2s}S_{2h} + S_{3s}S_{3h}) + S_{2h}^2 + S_{3h}^2}}{m\,(S_{1s} + S_{1h})}\,. \qquad (7a)$$

Note that the radicands in the functional dependence of the degree of polarization on the probability coefficient in both variants of the conditions of the problem are polynomials of second order in m (parabolas in the coordinate system $\{P, m\}$).

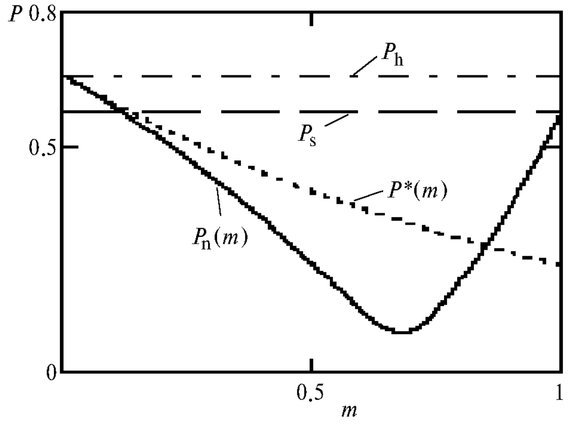

Fig. 1. Degree of polarization as a function of the probability of the presence of readings of the class “signal” in the realization. P_n is for the “signal” – “background” mixture; P^* is for the “background” – “signal + background” mixture; P_h is for the “background” class; P_s is for the “signal” class, m is the probability.

Transforming (6) and (6*), we obtain two variants of the formulas for the angle of polarization in the “signal” class. The first variant is for processing the “signal” – “background” mixture

$$\tan 2\gamma_s = \frac{\tan 2\gamma_t[m + (1 - m)(S_{2h}/S_{2s})] - (1 - m)(S_{3h}/S_{2s})}{m}\,, \qquad (8)$$

and the second variant is for processing the “signal” – “signal + background” mixture

$$\tan 2\gamma_s = \frac{\tan 2\gamma_t[m + (S_{2h}/S_{2s})]}{m}\,. \qquad (8a)$$

Here $0 < m \leq 1$.

Using formulas (3), it is easy to derive relationships for the polarization angle and degree of polarization in terms of “indexed” values of the intensity of the light scattered by the control volume and passing through the polaroid in its different orientations. We do not present these relationships here because of their cumbersomeness on the one hand, and obviousness on the other. Sample calculations of the degree of polarization and the polarization angle as functions of the probability of the presence of readings of the class "signal" in the observed sequence are plotted in Figs. 1 and 2 for two variants of the conditions of the experiment. Obviously, when processing the observed sequences, one cannot draw any conclusions about the polarization angle or degree of polarization in any of the signal classes without preliminary classification of the signal mixture.

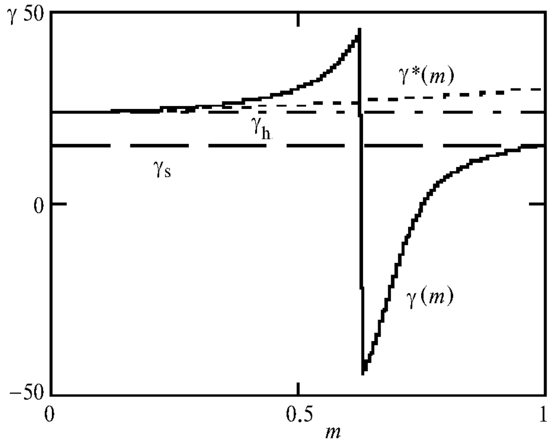

Fig. 2. The polarization angle (degrees) as a function of the probability of the presence of readings of the "signal" class in the realization. γ is for the "signal" − "background" mixture; γ* is for the "background" − "signal + background" mixture; γ_s is the polarization in the "signal" class; γ_h is the polarization in the "background" class; *m* is the probability.

Conclusion

If *a priori* data are available on radiation in one of the classes ("background") in the polarization observations of the two sources (classes "background" and "signal"), an algorithm for calculating the degree of polarization and the angle between the scattering plane and the polarization plane can be constructed in each signal class using the composite signal model.

The algorithm for classifying signals can be constructed on the condition that the observed polarization is different from the background value. This condition is fulfilled if:

– the mean values of the intensities of radiation in each class and the probability of observation of a signal mixture are different from zero;

– the angular coordinates of the sources have been determined and did not change during the measurement.

Comparison of the polarization P_t observed during some time interval $[t_1, t_2]$ and calculation of the ratio of the number of nonzero readings to the total number of observations make it possible to estimate the probability of the presence of readings of additional radiation ("signal") (additional in the sense: in addition to the background) in the sequence.

Preliminary analysis of the polarization measurements shows that the condition that the observed polarization is different from the background value is reliably fulfilled when observing scattering of radiation from two sources in the direction in which the maximum value of the degree of polarization is reached in one of the signal classes (the angle of maximum polarization of molecular scattering is equal to $\pi/2$).

Thus, using the composite signal model to interpret the results of polarization measurements of a radiation mixture allows one to determine the probability of observation of signals from different sources, as well as the polarization angle and degree of linear polarization in each signal class.

References

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