The method of asymptotic signal in the lidar sensing theory for the case of multiple scattering

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The method of lidar return representation with the account for multiple scattering as a component of the total flux into the backward looking hemisphere at unlimitedly increasing field-of-view angle of the reception is considered in terms of small-angle approximation. This approach allows us to treat the interference due to multiply scattered radiation as useful components of a signal, which can be used in the interpretation of lidar measurement data. Based on the asymptotic properties of the lidar return at large field-of-view angles of reception, the possibility is demonstrated to simplify the lidar equation by replacement of the scattering phase function for one parameter – the particles' efficient size. Based on this new representation, the iteration method for solving the lidar equation is developed.

Theory and methods of laser sounding of the atmosphere developed to date are based, as a rule, on the single scattering approximation (SS). The effects of multiple scattering (MS) become significant in the problems on sounding optically dense media. When solving the inverse problems, the contribution of MS is usually taken into account by means of corrections introduced into the traditional lidar equation. These corrections are determined from numerical solutions of the non-stationary radiation transfer equation (RTE). The problem becomes more difficult because the MS-component of a lidar return depends on the optical properties of the medium, which are unknown and are to be determined.

Great laboriousness of solving the RTE because of a significantly long computer time required is an obstacle for making an operative diagnostics of dense media by lidar methods. One can partially overcome the difficulties due to parametrization of the MS-component of the signal based on the vast model calculations at different geometry of lidar experiments and different optical properties of the medium.[1,2]

It has been shown already in early papers,[3,4] where the results were presented on calculated lidar returns based on solving the RTE by use of the Monte Carlo method, that the contribution of MS can significantly exceed (by tens or even some hundred times) the SS signal level. So it is obvious that it is expedient to more effectively use such energetically significant part of the lidar return for increasing the information capacity of lidar experiments when sounding dense media. Naturally, it is possible only in the cases when information about the optical properties of the medium along the sounding path has been "memorized" in the MS signal. Such conditions are realized in coarsely dispersed media characterized by the strong anisotropy of scattering, for which the small-angle approximation of the transfer theory[5–7] is applicable. As is shown, for example, in Refs. 8 and 9, in the frameworks of the small-angle approximation the MS component of the lidar return, different from the SS signal, contains information about not only the profiles of the extinction and backscattering coefficients, but also about the scattering phase function, hence, about the microstructure of the medium. One of the possibilities of taking into account the information contained in the MS signal when solving the inverse problems of lidar sounding is considered in this paper. The approach proposed is based on the use of asymptotic properties of the lidar returns in the case of unlimited increase of the field-of-view angle of the receiver.

1. Initial relationships

The initial analytical relationships for description of the lidar return taking into account MS in the small-angle approximation are presented in this section. Let us rely on the results from Ref. 9, where the equation was presented relating the power of the signal coming to the input of the receiving system of the lidar to the optical characteristics of the scattering medium depending on geometrical size of the sounding pattern at sending a δ-pulse of the energy W to the atmosphere.

$$P(z,R_r,\gamma_r,d) = W\frac{c}{4\pi}\beta_\pi(z)\int_0^\infty \nu J_0(\nu d)\,\tilde{D}(\nu,z\nu)F(\nu)\mathrm{d}\nu. \quad (1)$$

The following conditions and designations were accepted when writing Eq. (1). The transmitter and receiver are in the plane $z = 0$, their optical axes being parallel to the Oz axis, and the distance between the centers of the transmitter and the receiver equals to d. The function $\tilde{D}(\nu, p)$ is the Hankel transform of the sensitivity function of the receiving system $D(r, \gamma)$

with the circular symmetry on the spatial $r = |\mathbf{r}|$ and angular γ variables, where $\mathbf{r} = (x, y)$ are the cross coordinates, and γ is the angle between the given direction and the Oz axis. In the case when $D(r, \gamma)$ is a step-wise function of both variables, we have

$$\tilde{D}(\nu, z\nu) = S_r \frac{2J_1(R_r\nu)}{R_r\nu} \Omega_r \frac{2J_1(z\gamma_r\nu)}{z\gamma_r\nu}, \quad (2)$$

where $S_r = \pi R_r^2$ is the area of the receiving aperture, $\Omega_r = \pi\gamma_r^2$ is the solid angle of receiving, R_r and γ_r are, respectively, the radius of the input pupil and the half angle of the field of view of the receiver. $J_0(.)$ and $J_1(.)$ in formulas (1) and (2) are the Bessel functions of zero and first order.

The backscattering coefficient $\beta_\pi(z)$ and optical transmission function (OTF)

$$F(\nu) = \exp\,[-2\tau(z) + g(\nu)], \quad (3)$$

bear information about the optical properties of the medium in Eq. (1). Here

$$\tau(z) = \int_0^z \varepsilon(s)\, \mathrm{d}s; \quad (4)$$

$$g(\nu) = 2\int_0^z \sigma(z - s)\, \tilde{x}(\nu s)\, \mathrm{d}s; \quad (5)$$

$\varepsilon(s)$ and $\sigma(s)$ are the extinction and scattering coefficients, $\tilde{x}(p)$ is the Hankel transform of the small-angle scattering phase function $x(\gamma)$ obeying normalization $\tilde{x}(0) = 1$.

It was assumed when deriving Eq. (1) that the scattering phase function is strongly forward peaked, the MS was taken into account in the small vicinity of the sounding pulse direction at its propagation in the forward and backward directions, and the large-angle scattering was taken into account in the single scattering approximation. It was also assumed that one could ignore the scattering phase function variations in the range of the scattering angles close to π. It was assumed that one could ignore, at least, the output aperture size and the angular deviation relatively to the analogous parameters of the receiving system.

The problem is stated, at laser sounding, of reconstructing the optical characteristics of the scattering medium, two of which (profile of the backscattering $\beta_\pi(z)$ and extinction $\varepsilon(z)$ coefficients) are present in the usual laser sounding equation in the SS approximation as unknown values, from the lidar return $P(z)$. In addition to them, Eq. (1) contains two unknown functions characterizing the optical properties of the medium – scattering coefficient $\sigma(z)$ and the small-angle scattering phase function $x(\gamma)$, the form of which is closely related to the microstructure of the medium, its disperse composition, and refractive index. So the lidar return $P(z)$ also contains the information about the microstructure of the medium.

Inverse problems for Eq. (1) can be considered in simplified statement, when only some of the optical characteristics of the medium involved in this equation appear as unknown at other optical characteristics set *a priori*.

Then, for example, for reconstructing the profiles $\beta_\pi(z)$ and $\varepsilon(z)$ from Eq. (1) it is necessary to know the scattering phase function $x(\gamma)$ and the single scattering albedo $\Lambda = \sigma/\varepsilon$. Additional *a priori* information can be attracted about the functional relations between the extinction $\varepsilon(z)$ and backscattering $\beta_\pi(z)$ coefficients, similar to what is being done in solving the usual lidar equation in the SS approximation.

Since *a priori* ideas on the optical and microphysical properties of the medium to be taken into account in solving the inverse problems of lidar sounding are of especial importance, let us consider this issue in a more detail. The scattering phase function can be presented with a satisfactory accuracy in the form of a linear combination of the diffraction $x^{(D)}(\gamma)$ and geometric-optical $x^{(GO)}(\gamma)$ components for the problems of sounding of coarse dispersed media for which $kr|m - 1| \gg 1$, where r and m are the size and the complex refractive index of the particle, respectively, $k = 2\pi/\lambda$, λ is the wavelength. The diffraction component $x^{(D)}(\gamma)$, which contains information about the microstructure parameters of the medium (size and shape of particles), is most highly variable and is described by the Airy formula[10] at scattering on large spherical particles. The geometric-optics component of the scattering phase function $x^{(GO)}(\gamma)$ does not depend on the disperse composition of the medium (for non-absorbing particles) and can be approximated, in the range of small angles, by a linear combination of the exponential ~$\exp(-a\gamma)$ and Gaussian ~$\exp(-b\gamma^2)$ functions[11] whose parameters do not depend on the refractive index of particles. It makes it possible to consider the geometric-optical component $x^{(GO)}(\gamma)$ as known *a priori* in the inverse problems of lidar sounding taking into account the MS contribution. One can also consider the single scattering albedo $\Lambda = \sigma/\varepsilon$ as known *a priori*, the value of which for the scattering phase function in diffraction approximation is equal to 1/2 and is the function of the refractive index of particles if additionally taking into account the geometric-optics part of the scattering phase function.

2. Division of the lidar return into single and multiple scattered components

In connection with solving the inverse problems, significant attention of the specialists is paid to the study of the role of MS and ways of taking its contribution to lidar returns into account. Historically, the initial essential progress was achieved in the development of the methods for inversion of the traditional lidar equation when reconstructing the profiles of the extinction and backscattering coefficients from the SS signal. So, when solving the lidar sounding equation taking into account MS, one usually presents it in such a form that the part due to SS is selected in an explicit form. The expression for the power of the SS signal $P_1(z)$ follows from the general formulas (1) and (3) if assuming $g(\nu) = 0$. Then, under condition that $\gamma_r > (R_r + d)/z$, determining the far zone of receiving, one can derive the well-known relationship

$$P_1(z) = W \frac{c}{2} z^{-2} S_r \beta_\pi(z) e^{-2\tau(z)}, \quad (6)$$

and represent the initial equation (1) in the form

$$P(z) = P_1(z) [1 + m(z)], \quad (7)$$

where the function

$$m(z) = \frac{2z\gamma_r}{R_r} \int_0^\infty \nu^{-1} J_0(\nu d) J_1(\nu R_r) J_1(\nu z \gamma_r)(e^{g(\nu)} - 1)\, d\nu \quad (8)$$

determines the relationship between MS and SS components of the lidar return. The behavior of the ratio $m(z, \gamma_r)$ as a function of the receiver field-of-view angle γ_r was studied in detail in Ref. 9. As the angle γ_r increases, the function $m(z, \gamma_r)$ monotonically increases and at $\gamma_r \to \infty$ it tends to the limit

$$m_\infty = \exp(2\Lambda\tau) - 1. \quad (9)$$

As was shown in Ref. 9, the MS interference in the lidar return at quite large scattering angles can become dominating even at the optical thickness on the order of 1, and, as the optical thickness increases, the value of the SS signal $P_1(z)$ can form only a fraction of percent of the total signal and be at the level of the measurement error. It negatively affects the accuracy of interpreting the experimental data based on the analysis of the SS signal and imposes limitations on the selection of permissible values of the angle γ_r.

Formally, Eq. (7) is equivalent to Eq. (1). However, the form (7) is attractive because it allows to consider the problem on reconstruction of the extinction coefficient profile $\varepsilon(z)$ from the SS signal $P_1(z)$ (6) at the presence of the MS interference. It is reasonable to apply such an approach when the MS interference does not exceed the useful signal level. As is seen from formula (8), this interference depends on all optical characteristics of the medium involved in Eq. (1) excluding the backscattering coefficient profile $\beta_\pi(z)$, and so it itself turns out to be undetermined. There are the algorithms for iteration correction of the MS interference based on the data on the profile of the extinction coefficient $\varepsilon(z)$, which the SS signal $P_1(z)$ bears.[1,2] Application of such algorithms requires *a priori* knowledge on the scattering phase function $x(\gamma)$.

3. Asymptotic properties of the lidar return

Yearning for selection of the SS signal $P_1(z)$ from the total lidar return is caused by essential simplification of the equation when passing to this component and by the wide set of the methods for its inversion available to date. One can keep the noted advantages without resorting to selection of the SS signal. For this purpose let us consider the structure of the lidar return $P_\infty(z)$ at unlimited angle of the field of view of the receiver. It follows from the relationships (7) and (9) at $\gamma_r \to \infty$ that

$$P_\infty(z) = \frac{c}{2} z^{-2} S_r \beta_\pi(z) e^{-2\tau(z)(1-\Lambda)}. \quad (10)$$

It is seen from Eq. (10) that the lidar return at $\gamma_r \to \infty$ ceases depending on the scattering phase function $x(\gamma)$ and on the distance d between the transmitter and the receiver of radiation. The signal $P_\infty(z)$ is determined as in the case of SS approximation by two optical characteristics of the medium – extinction $\varepsilon(z)$ and backscattering $\beta_\pi(z)$ coefficients completed by one more parameter – single scattering albedo Λ. At the prescribed single scattering albedo Λ, Eq. (10) has the structure similar to the structure of the equation for the SS signal, $P_1(z)$, [Eq. (6)] for the far zone of receiving. The same methods and algorithms can be applied to solving Eqs. (6) and (10). Different from the SS signal $P_1(z)$, the contributions of all higher orders of scattering are summed in the signal $P_\infty(z)$. The relation between the signal $P_\infty(z)$, $P(z)$, and $P_1(z)$ is determined by the relationships

$$P_\infty(z) = P_1(z)\, e^{2\tau(z)\Lambda}, \quad (11)$$

$$P(z) = P_\infty(z)[1 - \Delta(z)]. \quad (12)$$

It is seen from Eq. (11) that the excess of the total lidar return $P_\infty(z)$ over the SS signal $P_1(z)$ is greater, the greater is the optical thickness $\tau(z)$ and the single scattering albedo Λ. The signal $P_\infty(z)$ falls off more slowly than $P_1(z)$ when penetrating into the scattering medium.

The correction coefficient $\Delta(z)$ in Eq. (12) determines the lidar return fraction formed in the scattering volume beyond the cone formed by the solid angle of receiving and is calculated by the formula[8]

$$\Delta(z) = \frac{2z\,\gamma_r}{R_r} \int_0^\infty \nu^{-1} J_0(\nu d) \times$$

$$\times J_1(\nu R_r)\, J_1(\nu z \gamma_r)[1 - e^{-2\tau\Lambda + g(\nu)}]\, d\nu. \quad (13)$$

The functions $m(z)$, Eq. (8), and $\Delta(z)$ are related by simple relationship

$$\Delta(z) = 1 - [m(z) + 1]\, e^{-2\tau(z)\Lambda}. \quad (14)$$

On the contrary to function $m(z)$, Eq. (8), the correction coefficient $\Delta(z)$, Eq. (13), is always less than 1 and tends to 0, $\Delta(z) \to 0$, at $\gamma_r \to \infty$ (Fig. 1). This property is essentially positive factor for solving the lidar equation in the form (12).

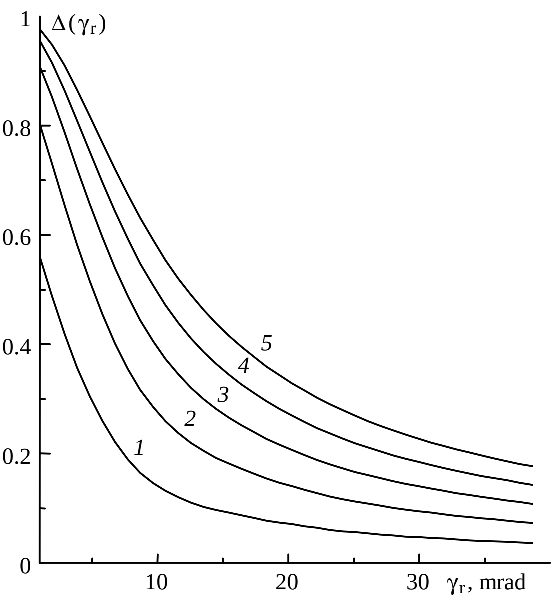

Fig. 1. Behavior of the function $\Delta(\gamma_r)$ for the homogeneous 1-km thick layer at the optical thickness τ = 1 (*1*), 2 (*2*), 3 (*3*), 4 (*4*), and 5 (*5*).

Thus, if based on the solution of the lidar equation for $P_\infty(z)$, Eq. (10), in interpreting the signal $P(z)$, the lidar return fraction which does not come to the receiver from the scattering volume situated out of the cone formed by the solid angle of receiving, plays the role of the interference.

One can simplify the formula for determination of the correction coefficient $\Delta(z)$, Eq. (13), at large receiving angles γ_r, if using the formula of expansion of the integral of the form[12] into the asymptotic power-law series

$$\int_0^\infty J_1(\xi\omega)\,h(\omega)\,\mathrm{d}\omega \sim \frac{h(0)}{\xi} + \frac{h'(0)}{\xi^2} - \frac{h'''(0)}{\xi^4} + \ldots \quad (15)$$

The peculiarity of the asymptotic series (15) is the fact that its coefficients are determined only by odd derivatives of the function $h(\omega)$ at zero point. This property has important consequences for the problem under consideration. In the case when the model of the scattering phase function has been represented in the form of linear combination of the exponential and Gaussian functions, as, for example, at approximation of the geometric-optics part of the scattering phase function, application of the expansion (15) to the integral (13) leads to the series, all coefficients of which vanish. It is related to vanishing of the derivatives of odd orders from the Hankel transform of the scattering phase function of the above-noted type at zero point.

Restricting oneself to two first terms of the series (15) leads to the following asymptotic approximation of the function $\Delta(z)$:

$$\Delta(z) \cong \Delta_1(z) = -\frac{2\Lambda\tilde{x}'(0)}{z\gamma_r}\int_0^z s\varepsilon(z-s)\,\mathrm{d}s, \quad (16)$$

where the product $\Lambda\tilde{x}'(0)$ depends only on the diffraction part of the scattering phase function and is determined by the following simple formula:

$$\Lambda\tilde{x}'(0) = \Lambda^{(\mathrm{D})}\,[\tilde{x}^{(\mathrm{D})}(0)]' = -\,1/(\pi k R_{\mathrm{eff}}), \quad (17)$$

where $\Lambda^{(\mathrm{D})} = 1/2$ is the single scattering albedo in the Fraunhofer diffraction approximation; R_{eff} is the effective size of the scattering particles calculated by the formula

$$R_{\mathrm{eff}} = \left[\int_0^R r^{-1} f(r)\,\mathrm{d}r\right]^{-1}, \quad (18)$$

$f(r)$ is the size distribution function of the geometric cross sections of the particles.

Thus, in the first approximation the correction $\Delta(z) \cong \Delta_1(z)$ and Eq. (16) does not depend on the geometric–optics component of the scattering phase function $x^{(\mathrm{GO})}(\gamma)$. Besides, the correction $\Delta_1(z)$ does not depend on the parameters of the experimental pattern – radius of the receiving aperture R_r and the distance d between the source and the receiver of radiation. The effect of the parameters d and R_r, as of the geometric-optics part of the scattering phase function $x^{(\mathrm{GO})}(\gamma)$, on the function $\Delta(z)$ at its expansion into the series (15) manifests itself when taking into account the subsequent terms of this series. The geometric-optics component of the scattering phase function is present in the low-order terms of the series (15) only in combination with the diffraction component.

Summarizing, one can state that the use of the asymptotic approximation of the function $\Delta \cong \Delta_1$, Eq. (16), when describing the lidar return taking into account MS, makes it possible to replace the data on the small-angle scattering phase function $\tilde{x}(p)$ with the value of one parameter – effective size of particles R_{eff}. This result is of especial interest for solving the problems of lidar diagnostics of the size of particles from multiple scattering.

As an example, a number of typical dependences of the functions $\Delta(\gamma_r)$ calculated for different optical thickness τ for a 1-km long homogeneous layer are presented in Fig. 1. The distance H to the nearest boundary of the layer is also equal to 1 km. The dependences $\Delta(\gamma_r)$ shown in Fig. 1 were calculated for the scattering phase function in the Fraunhofer diffraction approximation at the wavelength $\lambda = 0.55$ μm. The disperse composition of the medium was described by the modified gamma-distribution with the effective size of particles $R_{\text{eff}} = 10$ μm.

Different from Fig. 1, the behavior of the functions $\Delta(\gamma_r)$ for the extinction coefficient profile increasing upon the linear law $\varepsilon(z) = a(z - H)$, $z > H$, $a = 2$, $H = 1$ km is shown in Fig. 2. The optical thickness for the depth of penetration into the layer $z - H = 1$ km is equal to 1, and that for the maximum distance $z = 3.5$ km is $\tau = 6.25$.

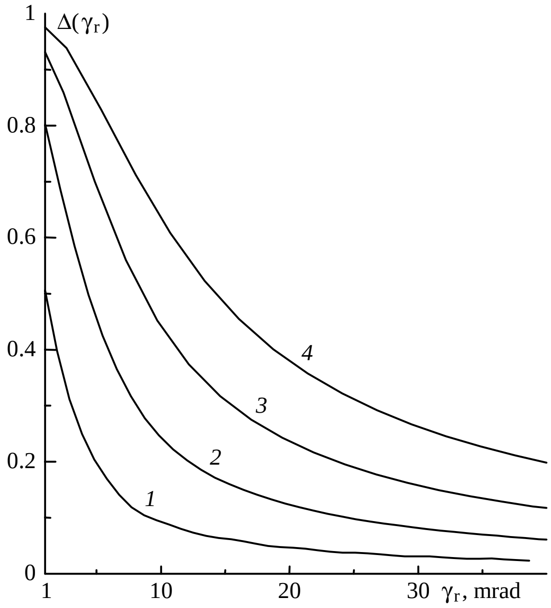

Fig. 2. Transformation of the function $\Delta(\gamma_r)$ for the linearly increasing extinction coefficient profile depending on the depth of penetration into the layer. The distance from the lidar to the nearest boundary of the layer is equal to 1 km; $z = 2$ (*1*), 2.5 (*2*), 3 (*3*), 3.5 (*4*).

The examples allowing one to compare the behavior of the characteristics $\Delta(\gamma_r)$ calculated by the exact formula (13) and based on the asymptotic approximation (16) are shown in Fig. 3. It follows from the data shown in Fig. 3 that the field-of-view angle of the receiver at $\tau = 1$ should be not less than 3.4 mrad in order to make the error of the asymptotic approximation of the function $\Delta(\gamma_r)$ no higher than 10%. This boundary value of the angle γ_r increases to 12.5 mrad at the increase of the optical thickness up to 4. The above-noted estimates are obtained for the distance to the layer $h = 1$ km and weakly change at the movement of the layer to $H = 5$ km.

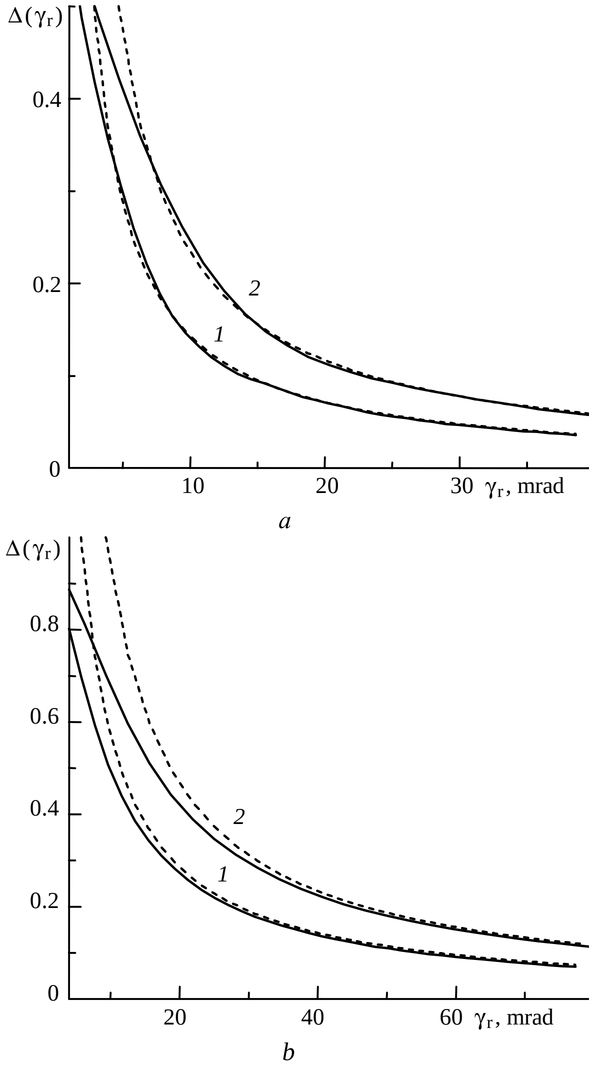

Fig. 3. Angular behavior of the functions $\Delta(\gamma_r)$ (solid lines) and $\Delta_1(\gamma_r)$ (dashed lines) for different geometric H and optical thickness $\tau = 1$ (*a*) and $\tau = 1$ (*b*): $H = 1$ km (*1*) and 5 km (*2*).

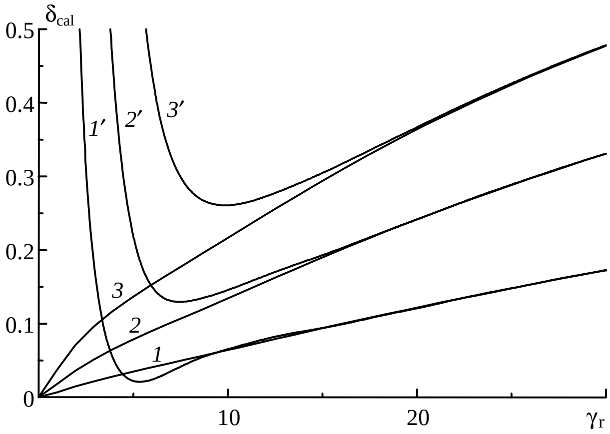

Fig. 4. Relative error δ_{cal} in calculation of the lidar return in the diffraction approximation when using the exact (13) (curves *1*–*3*) and asymptotic (16) (curves *1*′–*3*′) formulas at the change of the optical thickness of a homogeneous layer $\tau = 1$ (*1*, *1*′), 2 (*2*, *2*′), and 3 (*3*, *3*′).

Finally, set out in the Fig. 4 is the behavior of the error $\delta_{cal} = [P(z) - P^{(D)}(z)]/P(z)$ acceptable for the use of the exact formula (13) (curves *1–3*) and the asymptotic (16) (curves *1′–3′*) for calculating the lidar return $P^{(D)}(z)$ in the Fraunhofer diffraction approximation related to the signal calculated taking into account the geometric-optical component of the scattering phase function. The optical thickness τ of the homogeneous layer changed within the limits from 1 to 3. When calculating the correction coefficient $\Delta(\gamma_r)$ by the exact formula (13), the error δ_{cal} monotonically increases in the entire range of variation of the argument due to the increase of the role of the geometric-optics component of the scattering phase function at large γ_r angles. The difference between the functions $\Delta(\gamma_r)$ and $\Delta_1(\gamma_r)$ becomes small, and the curves *1–3* asymptotically approach the corresponding curves *1′–3′*. The accuracy of asymptotic approximation increases for small γ_r angles, however, the difference between the functions $\Delta(\gamma_r)$ and $\Delta_1(\gamma_r)$ increases (see Fig. 3). Resulting from the opposite effect of the noted factors, the dependences δ_{cal} obtained using the asymptotic formula (16) (curves *1′–3′*) acquire a non-monotonic behavior.

4. Algorithm for reconstruction of the extinction coefficient profile

Based on the lidar equation taking into account MS in the form (12) one can construct the iteration procedure for reconstruction of the extinction coefficient profile $\varepsilon(z)$ in the following form. Let the following functions be calculated for some kth approximation of the extinction coefficient profile $\varepsilon^{(k)}(z)$

$$\Delta^{(k)}(z) = \Delta[\varepsilon^{(k)}(z)]; \quad (19)$$

$$P_{\infty}^{(k)}(z) = P(z)/[1 - \Delta^{(k)}(z)]. \quad (20)$$

Then the $(k + 1)$th approximation $\varepsilon^{(k+1)}(z)$ is determined for $P_{\infty}(z)$, Eq. (10). One can do that by applying different methods. As an example, let us consider the algorithm for reconstruction of the extinction coefficient profile $\varepsilon(z)$ based on one of the modifications of the integral accumulation method,[13] or Klett method[14] with prescribing the boundary value $\varepsilon^* = \varepsilon(z^*)$ at some point $z = z^*$ of the sounding path:

$$\varepsilon^{(k+1)}(z) = \frac{S^{(k)}(z)}{S^{(k)}(z^*)/\varepsilon(z^*) + 2(1 - \Lambda)\int_z^{z^*} S^{(k)}(x)\mathrm{d}x}, \quad (21)$$

where $S^{(k)}(z) = P_{\infty}^{(k)}(z) z^2$. As a zero approximation one can assume $\varepsilon(0) = 0$. Iterations stop when the values $\varepsilon^{(k)}(z)$ and $\varepsilon^{(k+1)}(z)$ at two subsequent steps has become quite close. Numerical estimates show that, as a rule, 3–6 steps are enough for convergence of the process. The iteration procedure, Eqs. (19) and (21), is written at the assumption that the ratio $\beta_{\pi}(z)/\varepsilon(z)$ is constant along the sounding path.

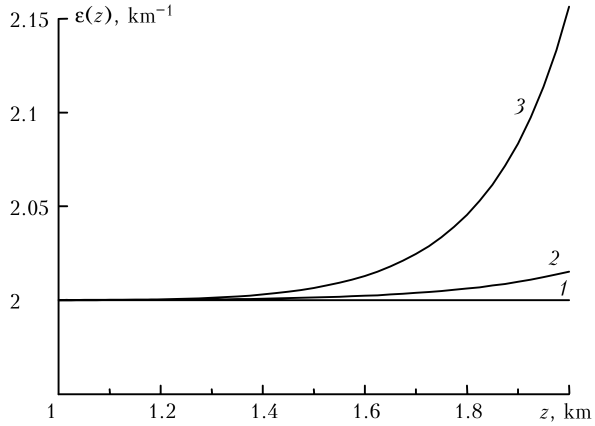

Fig. 5. Results of inversion of the lidar equation by the algorithm (19)–(21) in the numerical experiment for the constant model profile of the extinction coefficient (*1*) using the asymptotic approximation (16) for the correction coefficient $\Delta_1(\gamma_r)$ at the field-of-view angles $\gamma_r = 10$ (*2*) and 20 mrad (*3*).

Figure 5 illustrates the effect of the error acceptable at replacing the correction term $\Delta(\gamma_r)$, Eq. (13), by its asymptotic approximation $\Delta_1(\gamma_r)$, Eq. (16), in the numerical experiment on reconstructing the extinction coefficient profile $\varepsilon(z)$ from the lidar equation (12) based on the algorithm (19)–(21).

Conclusion

We have considered the problem of laser sounding of optically dense coarsely dispersed media taking into account the multiple scattering effects in the small-angle approximation. The method is proposed for splitting the lidar equation based on the asymptotic formula for the total flux into the backward hemisphere at unlimited increase of the field-of-view angle of the receiver. This formula has a simple analytical form and involves the same optical characteristics of the medium as the single scattering signal, with one additional parameter – the single scattering albedo. But, in contrast to the single scattered signal, the scattering of all orders is summed in the total flux. The lidar return at an arbitrary finite field-of-view angle of the receiver is represented by the total flux and the correction term that describes the fraction of the flux coming from out of the cone of the solid angle of the field of view of the receiver. It is shown that as the filed-of-view angle of the receiver increases, the behavior of the correction term acquires the asymptotic behavior and its analytical description becomes significantly simpler. In this case the scattering phase function in the lidar equation is replaced with a single parameter – the value of the derivative of its Hankel transform at zero, or the effective size of particles. As a result, the bulk of *a priori* data needed for inversion of the lidar equation decreases. The numerical-analytical method is proposed for reconstruction of the extinction coefficient profile. The method provides for solving the lidar equation taking into account the multiple scattering, based on inverting the lidar return for the unlimited field-of-view angle of the receiver with the iteration correction of the term taking into account the finite angular size of the receiver.

References

1. V.A. Korshunov, Atmos. Oceanic Opt. **3**, No. 2, 101–107 (1990).

2. Q. Jinhuan, H. Quenzel, and M. Wiegner, in: *Abstracts of Reports at the 15 International Laser Radar Conference* (Institute of Atmospheric Optics, Tomsk, 1990), Part 1, pp. 345–348.

3. V.E. Zuev, G.M. Krekov, and M.M. Krekova, Izv. Vyssh. Uchebn. Zaved. SSSR, Ser. Fizika, No. 8, 13–20 (1974).

4. V.V. Belov, G.N. Glazov, and G.M. Krekov, Izv. Vyssh. Uchebn. Zaved. SSSR, Ser. Radiofizika **21**, No. 2, 275–280 (1978).

5. V.V. Ermakov and Yu.A. Il'inskii, Izv. Vyssh. Uchebn. Zaved. SSSR, Ser. Radiofizika **12**, No. 5, 694–701 (1969).

6. L.S. Dolin and V.A. Savel'ev, Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana **7**, No. 5, 505–510 (1971).

7. E.P. Zege, I.L. Katsev, and I.N. Polonskii, Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana **34**, No. 1, 45–50 (1998).

8. V.E. Zuev, V.V. Belov, and V.V. Veretennikov, *Theory of Systems in the Optics of Disperse Media* (Spektr, Tomsk, 1997), 402 pp.

9. V.V. Veretennikov, Atmos. Oceanic Opt. **12**, No. 5, 385–391 (1999).

10. M. Born and E. Wolf, *Principles of Optics* (Pergamon Press, New York, 1964).

11. E.P. Zege and A.A. Kokhanovskii, Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana **30**, No. 6, 812–818 (1994).

12. H.F. Willis, Phil. Mag. **39**, 455 (1948).

13. V.E. Zuev, G.M. Krekov, and M.M. Krekova, in: *Remote Sensing of the Atmosphere* (Nauka, Novosibirsk, 1978), pp. 3–46.

14. J.D. Klett, Appl. Opt. **20**, 211–220 (1981).