Laser beam in a ring interferometer with delay: the structuring simulation

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A system of ordinary differential equations with delay is used as a mathematical model of the processes in a nonlinear ring-shaped interferometer. The boundaries of the stability domains are calculated. Based on the bifurcation diagram, phase pattern, and Fourier spectrum, the nonlinear dynamics of structuring in the beam cross section has been identified.

This paper continues earlier investigations into the much complicated dynamics in the model of the structure forming processes. These processes manifest themselves in the structure of a laser beam cross section propagating through a nonlinear ring-shaped interferometer (NRI) containing several Kerr media with the coupled optical fields.[1] The peculiarity of this model is in the possibility of accounting the time delay T of the optical field in the feedback loop of the interferometer. This possibility was first discussed in Ref. 2, but was left unstudied. Practically, it is interesting to reveal the joint influence of the nonlinearity K, delay T, and optical field turn by an angle Δ in the feedback loop of the interferometer on the dynamics of the structure forming processes. This is urgent because of NRI application in adaptive atmospheric optics.[3]

According to Ref. 2, the turn of the optical field by the angle $\Delta = 2\pi/N$, where N is the number of feedback loops in the interferometer (Ref. 1, Fig. 2). For $\Delta = 120°$ ($N = 3$) the dynamics of nonlinear phase modulation in the ring interferometer is described by three equations:

$$\tau\, \mathrm{d}u_j(t)/\mathrm{d}t + u_j(t) = K[1 + \gamma \cos (u_i(t - T))],$$

where $j = 1, 2, 3$; $i = 2, 3, 1$; $u_j(t) > 0$ is the phase difference in the jth channel; τ is the relaxation time; T is the delay time; K is the nonlinearity parameter; γ is contrast.

The equation was solved using the fourth-order Runge–Kutta method. The analysis of stationary solutions for stability has allowed construction of the bifurcation diagrams on the plane: stationary solutions u_{1*} – nonlinearity parameter K with regard for the normalized delay time $\nu = T/\tau$, where τ is the relaxation time of the nonlinear part of the refractive index (Fig. 1).

Comparison of the bifurcation diagram with the case of $\nu = 0$ allows the following conclusions to be drawn:

1) Delay of the field inside a ring-shaped interferometer initiates the appearance and/or shift (along K axis) of bifurcation points in the stability of stationary states. However, the delay does not influence positions of the stationary states in the structure of the bifurcation diagram.

2) As the field delay ν increases, the number of stable stationary states decreases, and the intervals of the nonlinearity parameter values, at which stationary states loose their stability, become wider.

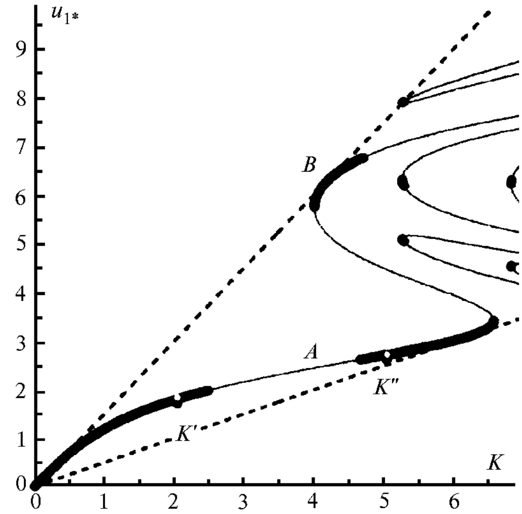

Fig. 1.

Let us consider the interval of values of the nonlinearity parameter K (from 0 to 6.62), within which all u_j are identical, that is, the monostability-like structure is formed in the cross section of a laser beam. The effect of the delay ν manifests itself in the appearance of an unstable state in the lower branch A at $\nu = 0.44$ (see Fig. 1). The distance along the K axis between the pair of the stability bifurcation points formed in such a way increases with the increase of ν. This increase is limited by the values $K' = 2.05$ and $K'' = 5.07$, which the bifurcation parameter K approaches asymptotically with the increasing ν.

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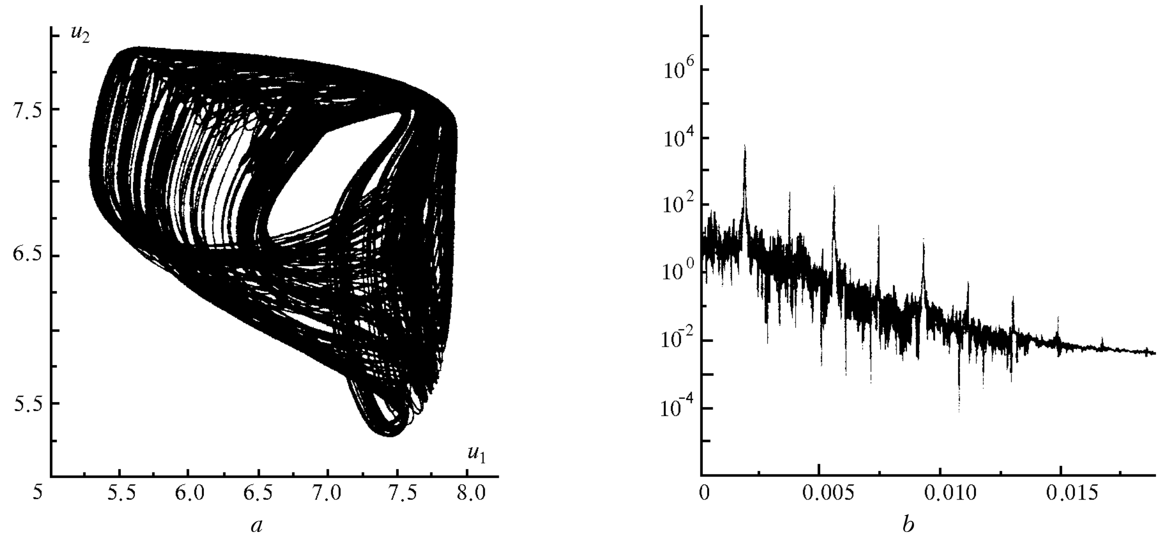

Fig. 2.

It was found that the loss of stability by the stationary states is accompanied by the appearance of a stable limiting cycle, that is, Andronov–Hopf bifurcation takes place.[4] As the bifurcation parameter K increases, only periodic motion without bifurcation in the period doubling takes place in the lower branch A. Similar transitions from the stationary state to the limiting cycle (Andronov–Hopf bifurcation) and back are observed also at sections of the diagram B situated above the branch A and corresponding to the same type of states when all u_j are identical. However, here the sequence of period doubling bifurcation points appears as the bifurcation parameter K increases. Besides, windows of periodicity are observed in the branch B, that is, simpler periodic motions appear. Therefore, one can state that the dynamics becomes more complicated when passing to the branch B in the bifurcation diagram.

The phase portrait in the plane (u_1, u_2) and the temporal energy Fourier spectrum taken together help us to reveal peculiarities in the complex motion. They are shown in Fig. 2, respectively, at $\Delta = 120°$ and $K = 5.3$. It is widely accepted to judge on the degree of a motion chaos from the presence or absence of pronounced narrow peaks (maxima) in the motion frequency spectrum. If the motion is periodic, then its Fourier spectrum has narrow peaks. As the motion approaches the chaotic mode, continuous frequency distribution appears in the spectrum.[4] If we are guided by these indicators, then based on the results of simulation we can conclude that chaos is induced by both the growth of the nonlinearity parameter K and by a decrease in the turn angle Δ.

The above results of the study of bifurcation and chaotic behavior of the processes of structuring in a NRI with delay can be used in the development and optimization of devices of atmospheric adaptive optics, which were considered in Ref. 3.

References

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