PRECISION MEASUREMENTS OF OPTICAL SURFACE CURVATURE RADIUS BY USE OF DIGITAL INTERFEROMETRY

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Errors of the autocollimation method as applied to measuring a curvature radius of spherical optical surfaces with an interferometer are analyzed. A precision measurement technique ($\Delta R/R \approx 10^{-5}$) is proposed. It is based on iterative physical refinement of a test piece position by results of interferogram processing. A relation between the measurement error and characteristics of a digital interferometer is obtained.

The technique to measure a curvature radius with an autocollimation microscope[1] is well-known. It consists in measurement of a test piece displacement between two positions (Fig. 1). The first reading is taken as the test piece vertex is in the microscope *1* reticle image plane. The second reading is taken at confocal arrangement of the test piece. Both reflections give an autocollimation image. Just this causes the name of this technique.

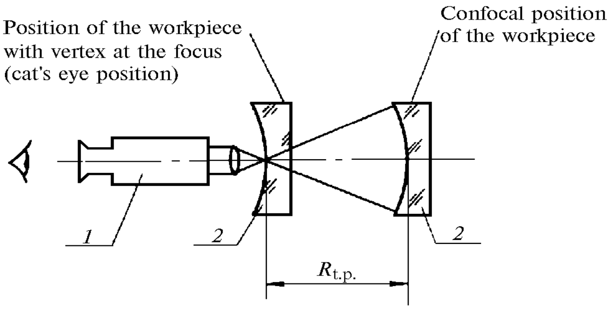

FIG. 1. Geometry of the autocollimation technique to measure the curvature radius: an instrument controlling adjustments (1); a test piece (2).

The classical technique utilizes a traditional reticle illuminated with an autocollimation eyepiece to indicate a test piece position.[2] Most interesting seems the possibility to control adjustments with a Twyman-Green interferometer or a laser interferometer by the Fizeau scheme with the external focus F'(Fig. 2). In this case, instrument *1* (Fig. 2) is the corresponding interferometer. In Ref. 3, the position of the test piece with a vertex at the focus is named the cat's eye position. For brevity, here we refer to it as a point position, and the second one is referred to as a confocal position. The point and confocal positions of the test piece correspond to most straight, parallel, and equidistant interference fringes. As compared to the autocollimation microscope, the interference scheme is more sensitive.

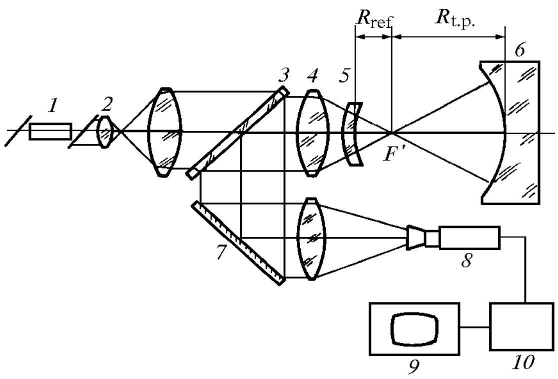

FIG. 2. Optical arrangement of the interferometer by the Fizeau scheme: laser (1), widener of a laser beam (2), beamsplitter (3), illuminating objective lens (4), aplanatic meniscus (5), test piece (6), beam-turning mirror (7), TV camera (8), video controller (9), interferogram processing device (10); F′ is the interferometer focus.

Errors of the interference autocollimation method are analyzed in detail in Ref. 3. The error of radius measurements is shown there to be ~ 0.01%. However, this accuracy is insufficient in some engineering problems. For example, the measurement error in the attestation of sample glasses[1] and the testing of spherical bearings of current gyroscopic systems[4] must be 0.001% and even less. The largest contribution into the error of these measurements is made by the Abbe error[2] and the error of defocusing.

The Abbe error ΔR_A is caused by displacement of the instrument axis and the length to be measured. The latter in this case is the axis passing through the test piece positions or the interferometer optical axis (Fig. 3). This error can be corrected for by use of

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either two measurement scales positioned on different sides from the interferometer or such instruments that can be structurally arranged on the interferometer optical axis, e.g., a laser meter of displacements (Fig. 3*b*).

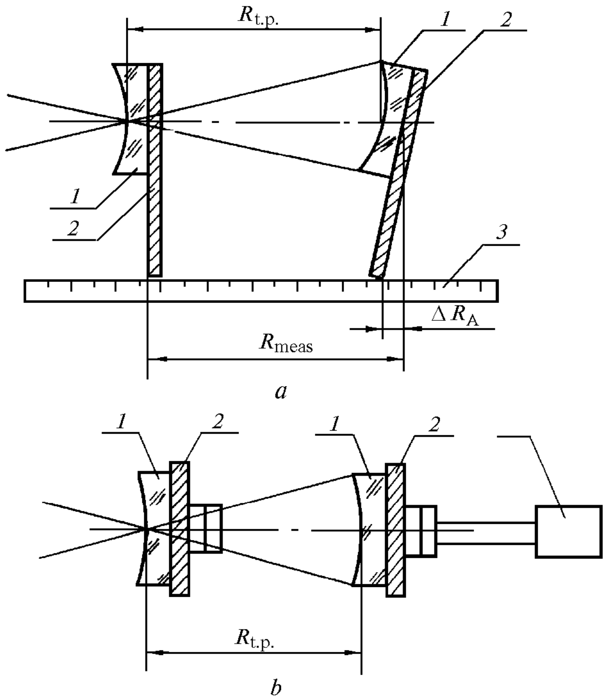

FIG. 3. The radius measurement error ΔR_A due to violation of the Abbe principle: test piece (1); test piece holder mounted through a bracket to a meter (2); meter with the measurement scale (3); laser meter of displacements (4).

The defocusing error stems from determination of the test piece position by the best interference pattern. The best interference pattern may contain some residual distortions of interference fringes corresponding to displacement of the wave front from the plane of best adjustment, ΔW_{res}. The parameter ΔW_{res} can be interpreted as wave aberration. Then, according to Ref. 6, we can write

$$\Delta W_{res} = \Delta L_{res} \cdot 0.125\ (D_{t.p}/R_{t.p})^2, \qquad (1)$$

where L_{res} is defocusing; $D_{t.p.}$ is the light diameter of the test piece; $R_{t.p.}$ is its curvature radius.

The value $\Delta R_{res} = 2\Delta L_{res}$ corresponds to the error in determination of the test piece curvature radius; ΔW_{res} is the error in determination of the best interference pattern.

It is rather evident that only high-quality test pieces (from the viewpoint of surface shape) can be subjected to precision measurements of the curvature radius in view of minimization of the influence of surface shape deviations from the reference $\Delta W_{t.p}$ on ΔW_{res}.

As shown in Ref. 5, ΔW_{res} can reach rather small values in the Fizeau interferometer at significantly high W_{ab} because of aberration subtraction. This is not true for a position of point reflection. Since mirror reflection of the object wavefront is observed in this case, wave aberration doubling occurs at non-symmetric W_{ab} in the interference equation. This requires a specific approach to be used in selection and manufacture of components of the interferometer illuminating arm (see Fig. 1).

Reference 3 considers the situation, when the test piece positions are adjusted visually, but ΔW_{res} can be determined from the results of digital processing of interferograms. Then the value of ΔR_{res} can be found from Eq. (1) with known $D_{t.p}/R_{t.p}$. Thus, this component of the radius measurement error can be reduced.

For high-quality correction, relative apertures of the reference surface D_{ref}/R_{ref} should be known accurate to ~ 0.1 – 0.2% (Ref. 3).

Since $\dfrac{d(D_{ref}/R_{ref})}{(D_{ref}/R_{ref})} = \dfrac{dD_{ref}}{D_{ref}} - \dfrac{dR_{ref}}{R_{ref}}$, we can write

$$\frac{dD_{ref}}{D_{ref}} \le 0.1\%\ , \quad \frac{dR_{ref}}{R_{ref}} \le 0.1\%\ . \qquad (2)$$

Fulfillment of conditions (2) is quite problematic, especially as for the value of dD_{ref}/D_{ref} .

The way out of this problem is not to correct numerically the test piece position in the plane of best adjustment using a calculated value of ΔW_{res}, but eliminate ΔW_{res} sequentially by physical displacement of the test piece until it reaches the position of best adjustment. Thus, information about the current state of the analyzed wavefront must serve as an initial one when deciding to generate a feedback signal for a device moving the test piece.

Certainly, this technique is time and combinationally expensive. However, it permits one to obtain good results. To speed up the convergence process, one can use the following:

– interpolation of the curve $\Delta W_{res}(l)$, where l is a displacement from a certain initial point, and determination of its minimum;

– iteration procedure of sequential determination of $\Delta R_{res} = \Delta R_{res}(\Delta W_{res})$ with the use of relation (1) at every step.

Since random factors (e.g., photometry noise), besides test piece displacement, influence the interference pattern, it is expedient to take the rms deviation of the wavefront W_{rms} as a criterion for estimation of the test piece position. This is also expedient from the viewpoint of result resistance to random factors.

According to Ref. 6, within the Strehl criterion applicability domain, we can write

$$W_{rms}^2 = \frac{W_{20}^2}{12} + \frac{W_{20}\,W_{40}}{6} + \frac{4\,W_{40}^2}{45} + \frac{W_{11}^2}{4} +$$

$$+\frac{W_{11}W_{31}}{3}+\frac{W_{31}^2}{8}+\frac{W_{22}^2}{16}+\frac{W_{22}W_{20}}{6}, \quad (3)$$

where W_{ij} are the coefficients of serial expansion of the wave aberration; they correspond to defocusing (W_{20}), spherical aberration (W_{40}), wavefront tilt (W_{11}), coma (W_{31}), and astigmatism (W_{22}).

It is clear from Eq. (3) that defocusing can partially compensate only for spherical aberration and astigmatism, which possess mirror symmetry with respect to the instrument optical axis.

Coma can be partially reduced (rebalanced) only due to wavefront tilt. Just this operation is performed at adjustment of the reference meniscus position at point reflection from the test piece. Thus, coma is the most unpleasant aberration in the interferometric system for measuring radii. So, in the further analysis, we restrict ourselves to the situation, when coma and wavefront tilt are present in the interferometer. In the plane of best adjustment, $W_{\rm rms}$ takes the value $W_{\rm rms}^{\rm opt}$. Coma in this case is partially compensated for due to the tilt. According to Ref. 6, the tilt is optimal if

$$W_{11} = -2W_{31}/3 . \quad (4)$$

Substituting Eq. (4) into Eq. (3) for $W_{20} = 0$, we obtain the following expression for coma:

$$(W_{\rm rms}^{\rm opt})^2 = \frac{W_{31}^2}{72} = \frac{\lambda^2}{m^2} . \quad (5)$$

Here m is an integer number characterizing the residual aberration in fractions of λ.

Assume that W_{ij} are small enough. Then values of these coefficients, except for W_{20}, vary not widely at slight defocusing of the test piece. Consequently the aberration $W_{\rm rms}^{\rm d}$ of the wavefront defocused from the optimal one can be written in the form

$$(W_{\rm rms}^{\rm d})^2 = \frac{W_{20}^2}{12} + \frac{W_{31}^2}{72} . \quad (6)$$

Suppose that the interferogram processing system distinguishes variations of the rms deviation

$$\Delta W_{\rm rms} = (W_{\rm rms}^{\rm opt} - W_{\rm rms}^{\rm d}) = \frac{\lambda}{n} ,$$

where n is an integer number characterizing the accuracy of the processing system in fractions of λ. Then under the condition $\lambda/n \ll W_{\rm rms}^{\rm opt}$ we can write

$$W_{20} \approx \lambda\sqrt{24/(nm)} . \quad (7)$$

According to Eq. (1), apply Eq. (7) to determination of the radius measurement error under aberrations $W_{\rm rms}$ of the wavefront reflected from the test piece:

$$\Delta R = 16\frac{R_{\rm t.p}}{D_{\rm t.p}}\lambda\sqrt{24/(nm)} = 16\frac{R_{\rm t.p}}{D_{\rm t.p}}\sqrt{24W_{\rm rms}\Delta W_{\rm rms}} . \quad (8)$$

Relation (8) can be used in imposing requirements to technical systems when designing interferometric systems for precision measurements of curvature radii.

It can be derived from Eq. (8) that the relative error in measurements by the above-described technique is $\Delta R/R \le 6\cdot 10^{-5}$ for spherical surfaces with $R_{\rm t.p} \approx 10$ mm with an interferometer with the aperture $D_{\rm t.p}/R_{\rm t.p} = 1{:}1$, e.g., for residual aberrations $W_{\rm rms} \le \lambda/40$ and at the processing system error $W_{\rm rms} \le \lambda/200$.

Tolerances for values of residual aberrations and coma can be specified the following based on Eq. (3):

$$W_{40} \le \lambda/4 , \quad W_{31} \le \lambda/13 .$$

These requirements can be satisfied due to very careful selection of all optical components and, first of all, objective lenses.

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