ON THE DETERMINATION OF REFERENCE VALUES FOR OPTICAL PARAMETERS MEASURED WITH A LIDAR

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The algorithms are proposed for determining the integral and local reference (calibration) values of the optical characteristics for three models using the information contained from the backscattering signals themselves. These algorithms do not require more simplifying and modeling assumptions about the optical properties of the medium than it is needed for the performance of known methods of determining the profiles of the optical characteristics along the path under investigation. It is shown that practically all the atmospheric and hydrospheric situations obey the applicability criteria of one of these algorithms.

The most complete review of the algorithms for backscattering signal processing relative to optical parameters may be found in Refs. 1–3. The algorithms are different in the *a priori* information used, the character of the additional experimental data, and the sequence of mathematical operations accomplished with the signals measured. Classification of the known single-lidar algorithms from one and the same point of view (revealing specific features of the algorithms) and the investigations of the condition when they are optimal are carried out in Ref. 3.

It follows from the analysis of the stability of different schemes of backscattering signal processing to noise interference that the stability of the solution is the higher, the more *a priori* information about the medium in the far ends of the sounding path is introduced.[3] The same ideology follows from Refs. 4 and 5, that assumes the use of the estimates $\varepsilon(r_k)$ or $T(r_0, r_k)$ at the far ends of the path under investigation (it is possible to obtain them from additional independent measurements. If such reference measurements are impossible, the problem on calibration at the end point of the path (and at the beginning too) is not solved. Sounding along slant and vertical paths corresponds to such a situation. The techniques currently in use for estimating the reference (calibration) values, which do not require additional independent measurements, lead to an ambiguity in the solution (method of logarithmic derivative)[3] or to a great uncertainity.[1,6]

The algorithms for obtaining the reference values of both $\varepsilon(r_0)$ and $T(r_0, r_k)$ using the information contained in the backscattering signals themselves, are presented below in the frameworks of the assumptions providing the ability of known techniques for interpreting the measured signals relative to the profiles of optical parameters for all possible atmospheric and hydrospheric situations and on different parts of the paths, including the far ends. The efficiency is considered of using the calibration value estimates obtained by the algorithms proposed for different schemes (techniques) of signal processing. The proposed theory of determining the reference (calibration) values of the optical parameters provides for avoiding additional independent measurements.

Let us take the lidar equation in the single scattering approximation as the basis (assuming that the portion of absorption in extinction is negligible)[6]

$$I_i = \int\limits_{r_i}^{r_i+\Delta r} P(r) r^2\,\mathrm{d}r = \frac{AP_0\overline{g}_\pi(r_i, r_i+\Delta r)}{2} \times T^2(0, r_i)\,[1 - \exp\{-2\overline{\varepsilon}_i((r_i+\Delta r) - r_i)\}]\,. \qquad (1)$$

Let us consider an arbitrary part of a sounding path $[r_1, r_4]$ (Fig. 1*a*). Let us write the expressions for $I_1 - I_5$ corresponding to the parts $[r_1, r_2]$, $[r_1, r_3]$, $[r_2, r_4]$, $[r_3, r_4]$ and $[r_2, r_3]$ in the form

$$\begin{aligned} I_1 &= B\,x_1\,a_0\,(1 - a_1)\,,\\ I_2 &= B\,x_2\,a_0\,(1 - a_1a_2)\,,\\ I_3 &= B\,x_3\,a_0\,a_1\,(1 - a_2a_3)\,,\\ I_4 &= B\,x_4\,a_0\,a_1a_2\,(1 - a_3)\,,\\ I_5 &= B\,x_5\,a_0\,a_1\,(1 - a_2)\,, \end{aligned} \qquad (2)$$

where

$$T^2(r_i, r_j) = \exp\left\{-2\int\limits_{r_i}^{r_j} \varepsilon(r)\,\mathrm{d}r\right\}:$$

$T^2(r_1, r_2) = a_1$, $T^2(r_2, r_3) = a_2$, $T^2(r_3, r_4) = a_3$, $T^2(0, r_1) = a_0$, $0.5\,AP_0 = B$,

$\overline{g}_\pi(r_1, r_2) = x_1$, $\overline{g}_\pi(r_1, r_3) = x_2$, $\overline{g}_\pi(r_2, r_4) = x_3$, $\overline{g}_\pi(r_3, r_4) = x_4$, $\overline{g}_\pi(r_2, r_3) = x_5$.

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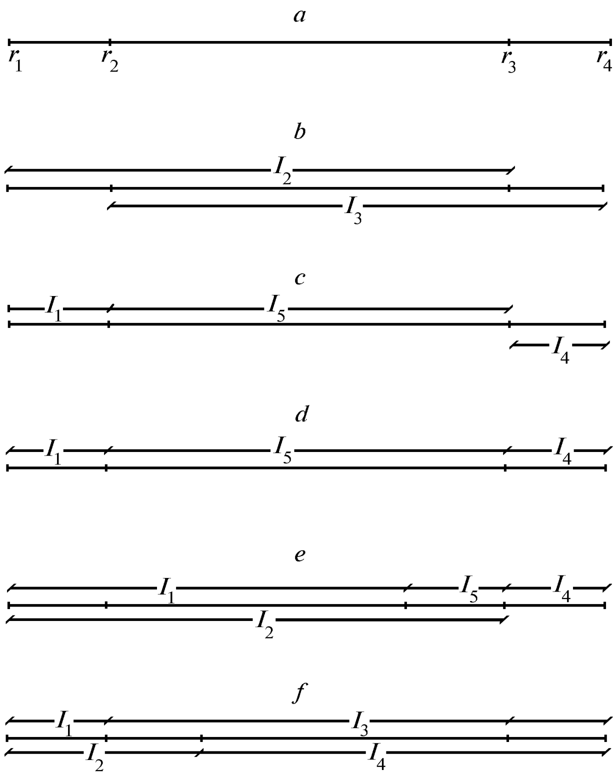

FIG. 1. The scheme of the sounding path portions where backscattering signals are accumulated for different models of scattering media used for the determination of local and integral reference values of the optical parameters.

Then we obtain the solution of system (2) for the following models of the medium.

1. Let us assume that $a_1 \approx a_3$. Then

$$I_2 = B\ x_2\ a_0\ (1 - a_1 a_2)\ ,$$
$$I_3 = B\ x_3\ a_0\ a_1\ (1 - a_2 a_3)\ . \tag{3}$$

The solution to system (3) relative to a_1 has the form

$$a_1 = I_3\ x_3 / I_2\ x_2\ . \tag{4}$$

The assumption $a_1 \approx a_3$ used means the approximate equality of the transmission of the portions $[r_1, r_2]$ and $[r_3, r_4]$ (Fig. 1*b*). This condition is practically always fulfilled for short portions ($[r_i, r_j] \to 0$). Then, if the portions $[r_1, r_2]$ and $[r_3, r_4]$ correspond to the width of the backscattering signal recording channel (strobe), usually short, the value (4) is equal to

$$T^2(r_1, r_2) = a_1 = I_3/I_2\ , \text{ or } \overline{\varepsilon}(\Delta r) = -1/2 \ln (I_3/I_2) \tag{5}$$

because the ratio $x_3/x_2 \approx 1$, i.e., the mean value of the lidar ratio for the long overlapping portions of the sounding path $[r_1, r_3]$ and $[r_2, r_4]$, $\overline{g}_\pi(r_1, r_3) \approx \overline{g}_\pi(r_2, r_4)$ in the majority of real situations. The condition $x_3/x_2 \approx 1$ and especially $a_1 \approx a_3$ is not fulfilled only in the case when either $[r_1, r_2]$ or $[r_3, r_4]$ are at the boundary of different scattering media (boundary of the sharp change in composition and concentration of the scattering substance).

The value ε on the part Δr obtained from Eq. (5) can be used as reference values (calibration) in the techniques for calculating optical parameters which need for local reference values (at $\Delta r \to 0$). The successive displacement of the functionals I_2 and I_3 by the value of the spatial resolution can be used for obtaining the profiles $\varepsilon(\Delta r)$ on a portion of a sounding path.

When using Eq. (5) for determining the transmission a_1 of a long portion $[r_1, r_2]$ (for $(r_2 - r_1) \to \infty$) the assumptions $a_1 \approx a_3$ and $x_2/x_3 \approx 1$ are more strong (less realistic) than for the case of $(r_2 - r_1) \to 0$. Obtaining of the algorithms of integral calibration (determination of the transmission of a long portion of the path) is natural when using the assumption on a_1 and a_3. Let us consider the functionals $I_1 - I_4$

$$a_2 = x_1 x_3 I_2 I_4 / x_2 x_4 I_1 I_3\ . \tag{6}$$

The obtained expression (6) is the most fulfilling the situation shown in Fig. 1*c*. At $(r_2 - r_1) \to 0$ and $(r_4 - r_3) \to 0$ $T(r_1, r_2)$ and $T(r_3, r_4)$ approach to 1 practically in any atmospheric and hydrospheric situations (that means fulfilling the condition $a_1 \approx a_3$). The ratio $(x_1 x_3)/(x_2 x_4)$ is also equal to 1 in all events corresponding to the random process with uncorrelated values ε and g_π along the path. Really, in this case

$$x_1 x_3 = \overline{g}_\pi(r_1, r_2)\, \overline{g}_\pi(r_2, r_4) = \\ = \overline{g}_\pi(r_1, r_2)\, \overline{g}_\pi(r_2, r_3)\, \overline{g}_\pi(r_3, r_4)\ ,$$

$$x_2 x_4 = \overline{g}_\pi(r_1, r_3)\, \overline{g}_\pi(r_3, r_4) = \\ = \overline{g}_\pi(r_1, r_2)\, \overline{g}_\pi(r_2, r_3)\, \overline{g}_\pi(r_3, r_4)\ .$$

Thus, if the lidar ratios along a quasistationary path under investigation are independent or weakly correlated (that corresponds to the majority of atmospheric and hydrospheric paths), then $x_1 x_3 / x_2 x_4 \approx 1$. Taking into account this fact, Eq. (6) takes the form

$$T(r_2, r_3) = [(I_2 I_4)/(I_1 I_3)]^{1/2}\ . \tag{7}$$

The transmission values obtained from Eq. (7) can be used as reference (calibration) values in the known techniques that need for the integral reference values of the transmission of a long portion of the path (a variant of the solution to the problem of integral calibration). The assumptions used in this case are significantly less strict than in the case of the local calibration (5), because they do not impose any limitation on the behavior of the lidar ratio along the path.

The variants of solving the calibration problem are also possible when using the functionals I_1, I_4, and I_5, then at $a_1 \approx a_3$ (Fig. 1*d*) and the constant lidar ratio along the path under investigation

$$T(r_1, r_3) = (I_4/I_1)^{1/2}, \quad (8)$$

$$T(r_2, r_3) = \frac{(I_4 I_5/I_1 + I_4)}{I_4 + I_5}. \quad (9)$$

It should be emphasized that no other and more wide assumptions are needed in this case (in comparison with the assumptions, at which known techniques for determining the profiles of optical parameters) for determining the reference (calibration) values.

2. The aforementioned algorithms for obtaining the reference (calibration) signals from the backscattering signals themselves, informative relative to the profiles of the optical parameters, are constructed at the assumption that $a_1 \approx a_3$. However, the solution to the system of equations from the functionals I_i is also possible when using the assumption[7] that $a_2 \approx a_3$. The arrangement of the functionals I_i corresponding to this variant is shown in Fig. 1*e*. The portions a_2 and a_3 should be placed in this case at the end of a long path and be small, i.e., $(r_2 - r_1) \to 0$, $(r_4 - r_3) \to 0$. Taking into account the assumption that $a_2 \approx a_3$, the solution to the system of equations for the functionals I_1, I_2, I_4, and I_5 (the assumption used is fulfilled practically for all atmospheric and hydrospheric situations, except for the case when the portions are at the boundary of media) relative to $T^2(r_1, r_2)$ and ε on the portion $\Delta r_k = [r_0, r_0 + \Delta r]$ can be written as

$$T^2(r_1, r_2) = \left[\frac{I_2 - I_1}{I_2 - I_1 I_4/I_5}\right]^{1/2}, \quad (10)$$

$$\varepsilon(\Delta r_k) = -\frac{1}{2\,\Delta r_k} \ln\left[1 - I(\Delta r_k)\,\frac{I_5 - I_4}{I_2 I_5 - I_1 I_4}\right] \quad (11)$$

and can be used in the techniques of integral and local calibration, respectively.

Use of the algorithm (10) in the case of inhomogeneous scattering media (with large variance of $g_\pi(r)$) is determined by the presence of portions satisfying the condition[7]

$$\delta\beta_\pi < \exp\{2\varepsilon\Delta r\} - 1 \quad (12)$$

because of the necessity of obtaining correct results (transmission should not be greater than 1 or less than 0), where $\delta\beta_\pi = \Delta\beta_\pi/\beta_\pi$ is the degree of the medium inhomogeneity.

If two adjacent parts, $[r, r + \Delta r]$ and $[r + \Delta r, r + 2\Delta r]$, for which $\overline{\varepsilon}(r, r + \Delta r) \approx \overline{\varepsilon}(r + \Delta r, r + 2\Delta r)$ and $\overline{g}_\pi(r, r + \Delta r) \approx \overline{g}_\pi(r + \Delta r, r + 2\Delta r)$, occur on the path under investigation, one can consider the functionals I_1 and I_2 as the terms of infinitely decreasing progression with the denominator $q = I_2/I_1$. Taking into account that $I_m(r, \infty)$ is calculated as the sum of the geometric progression we have

$$\varepsilon(\Delta r_k) = -\frac{1}{2\,\Delta r_k} \ln\left[1 - \frac{I(\Delta r_k)}{I_1}\,(1 - q)\right]. \quad (13)$$

One can use the expression (13) for obtaining the reference (calibration) value for any of the methods with local calibration. However, the assumption used here requires the presence either of quasihomogeneous portions on the path, that is possible in a limited number of cases, or determination of the functionals I_1 and I_2 on the long portions, what is preferable at sounding of background aerosol.

3. The algorithm (10) considered in section 2 makes it possible to determine the transmission of the long portions in the beginning of sounding paths. One can obtain similar results for the transmission of the path's portions situated at the end of the paths under investigation, assuming that $a_1 \approx a_2$. The arrangement of functionals shown in Fig. 1,*f* corresponds to this assumption best of all. The parts $[r_1, r_2]$ and $[r_2, r_3]$ corresponding to the functionals I_1 and I_5, respectively, should be small ($[r_1, r_2] \to 0$ and $[r_2, r_3] \to 0$). The system of equations for I_1, I_3, I_4, and I_5 is solved relative to a_1 and a_3. It follows from the first and the last equations of the system, that

$$a_1 = x_1 I_5/x_5 I_1. \quad (14)$$

By substituting a_1 into

$$\frac{I_3}{I_4} = \frac{x_3}{x_4}\,\frac{1}{a_1}\,\frac{(1 - a_1 a_3)}{(1 - a_3)}$$

we obtain

$$a_3 = T^2(r_3, r_4) = \frac{(I_4 x_3 - I_3 I_5 x_4\, x_1/x_5 I_1)}{(I_4 x_3 - I_3 x_4)\,(x_1 I_5/x_5 I_1)}. \quad (15)$$

Within the frameworks of the assumption used in the known techniques, that $g_\pi(r) = \text{const}$ or vary slowly from point to point, Eq. (15) is transformed to

$$T(r_3, r_4) = \left\{\frac{(I_4 - I_3 I_5/I_1)}{(I_4 - I_3)\,(I_5/I_1)}\right\}^{1/2}. \quad (16)$$

Thus, one can also use Eq. (16) for integral calibration in the techniques for reconstructing the optical parameters assuming the lidar ratio to be constant along the path under study.

All the aforementioned algorithms (in the variants 1–3) both for local and integral calibration contain neither instrumentation constants, nor the dependence on the sounding pulse energy. Stability of the algorithms to the variation of sounding pulse energy from pulse to pulse and the absence of absolute calibration follow from this fact, and excludes the errors of determining the instrumentation constants. Moreover, the performance ability of the algorithms to obtain the reference (calibration) values in the variants 2 and 3 is provided even in the presence of a sharp change in the values of optical parameters at the boundary of a media. Indeed, the mean values

$$\bar{g}_\pi(r_0, r) = \frac{1}{N}\sum_{i=1}^{N} g_\pi(\Delta r_i) ,$$

$$\bar{g}_\pi(r_0, r + \Delta r) = \frac{1}{N+1}\sum_{i=1}^{N+1} g_\pi(\Delta r_i)$$

for long portions including the dividing boundaries, differing from each other by only small value of the spatial resolution Δr are practically equal at big N ($N = (r_0 - r)/\Delta r$, $\Delta r \to 0$) and $m \approx 1$, that leads to the stability of calibration algorithms to the dividing boundaries (sharp change of the values of optical parameters) of the media.

At the same time, all the aforementioned algorithms of integral calibration by the variants 1–3, as well as the algorithm of the local calibration by Eq. (4) are stable to the presence of the layer with sharply varying optical parameters (for example, emissions of the stacks of industrial enterprises, at sounding of the atmosphere from onboard an aircraft, etc.) on the portions of determining the transmission (for Eq. (4) at any point of the interval $[r_1, r_4]$). It follows from the fact that these layers are simultaneously included into the functionals I_i for the long portions, different by the value of the spatial resolution usually small ($\Delta r \to 0$), and the mean values $\bar{g}_\pi(r_i, r_j)$, $\bar{g}_\pi(r_i, r_j + \Delta r)$ of these portions are practically equal to each other.

The measured functionals I_i are used in algorithms 1–3 of the integral calibration as a ratio of the adjacent readouts, with different Δr value. Contributions from the multiple scattering C_i are practically the same for the adjacent readouts, taken at $\Delta r \to 0$. Thus, one can write the algorithms of the form (4) and (10) as follows:

$$a_1 = \frac{C_1 I_3}{C_1 I_2} , \quad a_1 = \frac{mC_1 I_2 - C_1 I_1}{mC_1 I_2 - nC_1 C_3 I_1 I_4 / C_3 I_3} ,$$

because $C_3 \approx C_2$ for (4) and $C_1 \approx C_2$, $C_3 \approx C_4$ for (10).

Independence of the calibration algorithms of C_i characterizing the contribution of the multiple scattering, leads to the insignificant influence of the multiple scattering on the results of determining the calibration values T and ε. Insignificant influence of the multiple scattering contribution into the measured signals in the algorithms for calculating the optical parameters using signals in the form of the relative behavior of neighbor readouts differing by Δr was discussed earlier in Refs. 1–6.

The algorithms proposed for determining the integral and local calibration values use information from the backscattering signals themselves, and there is no need (than it is necessary for providing the performance ability of known techniques for determining the profiles of optical parameters along the path) for other simplifying and model ideas about the optical properties of media under study. Indeed, the algorithms of the variants 2–3 require the smoothness of $g_\pi(r)$ and equality of the neighbor values of $\varepsilon(r)$ on the arbitrary (short or long) portions of the medium. These requirements are less strict in comparison with the requirements of homogeneity of the medium or constant g_π and ε in the neighbor layers (then it is necessary to know the initial value ε and to have *a priori* information about the behavior of g_π between the layers), characteristic of the so-called numerical methods of solving the optical location equation (by the classification of Ref. 6). The same applies to the techniques of analytical solution[6] requiring the constant g_π value along the path or knowledge of the relation between g_π and ε. Moreover, the absence of limitations on the length of the portion where the functionals I are determined in the algorithms proposed increases their stability to influence of measurement errors.

The algorithms for determining the calibration values of the optical characteristics in its version 1 require the use of minimum assumptions, namely, the approximate equality of transmissions of two short portions (at $\Delta r \to 0$) of the medium under study. Practically that means the equality of transmissions of the portions corresponding to the strobe (channel) of the recording instrumentation, that is fulfilled even at a significant variance of the optical properties ($\exp\{-2\varepsilon\Delta r\} \approx 1$ at $\Delta r \to 0$ and significant variance of ε). For example, $T(\Delta r) = 0.998$ and 0.9998 for $\Delta r = 0.01$ km and $\varepsilon = 0.1$ and 0.01 km^{-1}, respectively. Though, the presence of the medium boundary at $[r_1, r_4]$ is not desirable for these algorithms, because in this case the $g_\pi(r)$ variations start to affect. As it has been analytically shown when describing variant 1, the effect of the variance of $g_\pi(r)$ caused by natural fluctuations, or due to the turbulence, practically does not exist: $(\bar{g}_\pi(r_1, r_2)\cdot\bar{g}_\pi(r_2, r_4) \approx \bar{g}_\pi(r_1, r_3)\bar{g}_\pi(r_3, r_4)$ for the same portion $[r_1, r_4]$. Thus, these assumptions are the weakest from all used in the known methods.

As is seen from the aforementioned, one can select the algorithm for determining the reference (calibration) value of an optical parameter from the measured backscattering signal and to exclude undesirable independent supplementary measurements of the reference values T and ε for any atmospheric or hydrospheric situation. The backscattering signal is used, which is measured for determining the optical parameters along the path under study.

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