

Modified method of spherical harmonics for determination of the point spread function of a turbid medium layer

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We have developed a method of solution of radiative transfer equation for the case of a point isotropic source, based on the spherical harmonics method. The convergence of the method has been improved by subtracting from the general solution of the small-angle component containing all the peculiar features of the solution. Thus, the solution is obtained for remaining smooth function containing no singularities. To maintain the stability of solution with the increase of the optical depth, scaling is applied. The numerical calculations made clearly show the physical idea of the small-angle approximation, i.e., the neglect of dispersion of the scattered photon trajectories and backscattering.

Introduction

To solve the problems on recovering the images, distorted by a turbid medium such as the atmosphere or water depth, it is often sufficient to determine the point spread function (PSF) or the optical transfer function (OTF) of the system,¹ i.e., to solve the radiative transfer equation (RTE) for the case of a point isotropic source (PI source). The obtained distribution of the medium brightness illuminated with a PI source, by optical reciprocity theorem,² corresponds to the distribution of illumination with a point unidirectional source (PU source) and, hence, can be used in solution of the problems dealing with laser ranging. Generally speaking, at present the boundary-value problem of RTE for a plane layer should be considered solved. An important task is the development of analytical and numerical methods of solution of three-dimensional radiative transfer problems.³ Therefore, the calculation of radiation field from a PI source is considered the simplest case making it possible to analyze the main issues of the three-dimensional problems.

These problems can be divided into two large groups: (1) the radiation fields of wide beams in the medium where one dimension is much larger than two others, and (2) the radiation fields created by spatially finite sources. For problems in the first group there is quite an efficient approximation,⁴ whereas in the second group the estimate can only be made in small-angle approximation⁵⁻⁷ of the field scattered in the forward direction and only for small optical depths. In Ref. 8 an approach was proposed to solving these problems, based on subtraction of direct and singly scattered components and subsequent RTE solution for a Fourier transform of the initial function, which improves the convergence of the method. However, it does not solve all the problems associated with the presence of peculiarities of solution near the source. In such an approach, we determine only OTF of a layer of the turbid medium,

whose knowledge alone is insufficient for reconstruction of the image, obtained, e.g., from satellite, in which case the image is formed by angular scanning of the object.

In the case of a PI source, the brightness of radiation field L at the point \mathbf{r} along the direction $\hat{\mathbf{I}}$ is spherically symmetrical: $L(\mathbf{r}, \hat{\mathbf{I}}) = L(r, \mu)$, $\mu = (\hat{\mathbf{r}}, \hat{\mathbf{I}})$, $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$, and RTE has the form

$$\mu \frac{\partial L}{\partial r} + \frac{1-\mu^2}{r} \frac{\partial L}{\partial \mu} = -\varepsilon L(r, \mu) + \frac{\Lambda \varepsilon}{4\pi} \oint x(\hat{\mathbf{I}}, \hat{\mathbf{I}}') L(r, \mu') d\hat{\mathbf{I}}' \quad (1)$$

with the boundary conditions:

$$\begin{cases} L(r, \mu)|_{r \rightarrow 0, \mu > 0} = \frac{1}{4\pi r^2} \delta(\hat{\mathbf{I}} - \hat{\mathbf{r}}), \\ L(r, \mu)|_{r \rightarrow \infty} = 0, \end{cases} \quad (2)$$

where ε is the extinction coefficient; $x(\hat{\mathbf{I}}, \hat{\mathbf{I}}')$ is the scattering phase function; and Λ is the single scattering albedo. Here and below, all capped symbols denote unit vectors.

1. Method of spherical harmonics

The method of spherical harmonics (SH) for RTE solution essentially consists in representation of all functions entering the equation by series expansion over Legendre polynomials⁹:

$$\begin{aligned} L(r, \mu) &= \sum_{k=0}^{\infty} \frac{2k+1}{4\pi} C_k(r) P_k(\mu), \\ x(\hat{\mathbf{I}}, \hat{\mathbf{I}}') &= \sum_{k=0}^{\infty} (2k+1) x_k P_k(\hat{\mathbf{I}} \cdot \hat{\mathbf{I}}'). \end{aligned} \quad (3)$$

With the use of Legendre polynomials it is possible to separate variables based on the addition theorem:

$$P_k(\hat{\mathbf{I}} \cdot \hat{\mathbf{I}}) = \frac{4\pi}{2k+1} \sum_{m=-k}^{m=k} Y_k^m(\hat{\mathbf{I}}) \overline{Y_k^m(\hat{\mathbf{I}})}, \quad (4)$$

where

$$Y_k^m(\hat{\mathbf{I}}) = \sqrt{\frac{2k+1}{2\pi}} \sqrt{\frac{(k-m)!}{(k+m)!}} P_k^{|m|}(\mu) e^{-im\varphi}$$

are spherical functions; φ is the azimuth angle of the vector $\hat{\mathbf{I}}$ in the coordinate system chosen.

Substitution of Eq. (3) in RTE and some manipulations,⁹⁻¹¹ taking into account the properties of the Legendre polynomials, give the following system of equations:

$$\frac{k+1}{r^{k+2}} \frac{d}{dr} [C_{k+1}(r)r^{k+2}] + kr^{k-1} \frac{d}{dr} \left[\frac{C_{k-1}(r)}{r^{k-1}} \right] + (2k+1)\varepsilon(1-\Lambda x_k)C_k(r) = 0. \quad (5)$$

Solution of equation (5) can be written as⁹:

$$C_k(r) = \frac{1}{\sqrt{r}} A_k(v) K_{k+\frac{1}{2}}(vr), \quad (6)$$

where $K_{k+\frac{1}{2}}(z)$ is the modified Bessel function of purely imaginary argument of the second kind (or McDonald function).

Using the properties of McDonald function, we obtain the system of equations:

$$A_{k+1}(v) = \frac{1}{(k+1)} \left[(2k+1) \frac{\varepsilon(1-\Lambda x_k)}{v} A_k(v) - kA_{k-1}(v) \right]. \quad (7)$$

Since the solution of infinite system of equations is impossible, for practical solution it is made finite by assuming that all coefficients ($\forall k > N$): $(C_k(r) \equiv 0)$. The number of terms to be accounted for depends on the properties of the medium, shape of the scattering phase function, and other factors. Thus, we obtain

$$A_{N+1}(v) = 0. \quad (8)$$

Obviously, the Eq. (8) is the equation of $(N+1)$ st power and, correspondingly, it has $(N+1)$ roots. It can be shown¹⁰ that approximation of odd N is better because it leads to $(N+1)/2$ roots identical in absolute value and opposite in sign. In accordance with the boundary condition (2) at infinity, the roots cannot be negative, which leaves just $(N+1)/2$ positive roots. For use of recursion equation (7) it is necessary to know A_0 , which can be set to unity since any value leads to multiplication of the rest A_k coefficients by A_0 . Different roots of Eq. (8) determine linearly independent solutions. Therefore, the general solution can be written in the form

$$L(r, \mu) = \sum_{k=0}^N \frac{2k+1}{4\pi\sqrt{r}} \sum_{q=1}^{(N+1)/2} a_q A_k(v_q) K_{k+\frac{1}{2}}(v_q r) P_k(\mu). \quad (9)$$

The constant coefficients a_q are determined from the boundary conditions. However, the approximate solution cannot satisfy the exact boundary conditions, thus necessitating the use of approximate conditions. The boundary conditions in Marshak form are the best ones:

$$\int_0^1 L(0, \mu) P_{2j-1}(\mu) d\mu = \frac{1}{4\pi r^2} \int_0^1 \delta(\mu-1) P_{2j-1}(\mu) d\mu, \quad (10)$$

$$j \in 1, \frac{N+1}{2}.$$

Substitution of the expression (10) in Eq. (9) yields

$$\forall j \in 1, \frac{N+1}{2}: \sum_{k=0}^N (2k+1)r^2 \sum_{q=1}^{\frac{3(N+1)/2}{q=1}} a_q A_k(v_q) K_{k+\frac{1}{2}}(v_q r) \times \int_0^1 P_{2j-1}(\mu) P_k(\mu) d\mu = 1, \quad (11)$$

that is the system of $(N+1)/2$ linear equations with $(N+1)/2$ unknowns a_q .

However, solution for the case of a PI source has peculiarities not only in angle, but also in radius, requiring consideration of infinitely many terms of the series, both in angular and spatial variable, rendering the problem (11) mathematically ill-posed for any finite N . We will consider the case of nonscattering medium, with $\Lambda = 0$ and $\varepsilon = \kappa$. In this case, equation (7) can be written in the following form

$$A_{k+1}(v) = \frac{1}{(k+1)} \left[(2k+1) \frac{\kappa}{v} A_k(v) - kA_{k-1}(v) \right], \quad (12)$$

whence $A_{k+1}(v) \equiv P_k(\kappa/v)$ are the Legendre polynomials. Solution for this case is known and reads:

$$L(r, \mu) = \frac{e^{-\kappa r}}{4\pi r^2} \delta(1-\mu): C_k(r) = \frac{e^{-\kappa r}}{r^2}, \quad (13)$$

making it possible to assert that:

$$\sqrt{\frac{\pi}{2}} \frac{1}{r} \sum_{q=1}^{(N+1)/2} a_q \frac{A_k(v_q)}{\sqrt{v_q}} e^{-v_q r} \sum_{n=0}^k \frac{c_n}{(v_q r)^n} = \sqrt{\frac{\pi}{2}} \frac{1}{r} \sum_{n=0}^k \frac{c_n}{r^n} \sum_{q=1}^{(N+1)/2} a_q \frac{A_k(v_q)}{v_q^{n+1/2}} e^{-v_q r}, \quad (14)$$

where we have used the expansion of McDonald functions of half-integer order into a series¹²:

$$K_{k+\frac{1}{2}}(z) = \sqrt{\frac{\pi}{2z}} \sum_{n=0}^k \frac{c_n}{z^n} e^{-z}.$$

This requires that all sums over q equal zero for all $n \neq 1$ when $N \rightarrow \infty$. This statement is paradoxical because it is incorrect to pass to the limit $N \rightarrow \infty$ for

description of local spatial peculiarity. We can contend that this case requires nontrivial passage to the limit, analogous to the quasi-classical approximation known from the literature.¹²

2. Modification of the spherical harmonics method

Let us introduce the new function

$$C_k(r) = Y_k(r)/r^2. \tag{15}$$

In this case, RTE can be written as follows

$$\frac{1}{2k+1} \frac{d}{dr} [(k+1)Y_{k+1}(r) + kY_{k-1}(r)] + \frac{k(k+1)}{(2k+1)r} [Y_{k+1}(r) - Y_{k-1}(r)] + b_k Y_k(r) = 0. \tag{16}$$

In solution of the last equation in small-angle modification of spherical harmonics method (SHM),⁷ the substitution

$$\frac{k(k+1)}{(2k+1)r} [Y_{k+1}(r) - Y_{k-1}(r)] \approx \frac{x}{r} \frac{\partial}{\partial x} Y(x, r), \quad x = \sqrt{k(k+1)}, \tag{17}$$

is used and solution takes the form⁷:

$$Y_k(r) = \exp \left[-\varepsilon r \left(1 - \frac{\Lambda}{\ln g} \frac{g^{\sqrt{k(k+1)}} - 1}{\sqrt{k(k+1)}} \right) \right], \tag{18}$$

where the scattering phase function is taken in Henyey-Greenstein approximation $x_k = g^k$ or $b_k = \varepsilon(1 - \Lambda g^k)$.

In formula (17) the SHM takes into account that, when peculiarities with respect to angle are present in solution $L(r, \mu)$, its angular spectrum $Y_k(r)$ slowly decreases starting from the index k , which allows⁷ one to introduce continuous function $Y(k, r)$ coinciding at integer-valued points with $Y_k(r)$. The function $Y(k, r)$ is a slow, monotonically decreasing function of k , which precisely permits to admit the approximation (17).

We will use the representation (17) to transform the second term in equation (16) and write

$$\frac{k(k+1)}{(2k+1)r} [Y_{k+1}(r) - Y_{k-1}(r)] \approx \varepsilon \Lambda \left(g^x - \frac{g^x - 1}{x \ln g} \right) Y_k(r). \tag{19}$$

Now, Eq. (16) can be written in the following form

$$\frac{1}{2k+1} \frac{d}{dr} [(k+1)Y_{k+1}(r) + kY_{k-1}(r)] + \varepsilon \left(1 - \frac{\Lambda}{\ln g} \frac{g^{\sqrt{k(k+1)}} - 1}{\sqrt{k(k+1)}} \right) Y_k(r) = 0, \tag{20}$$

which is equivalent to RTE for the case of a PI source under normal incidence upon the layer, but

with much stronger forward-peaked scattering phase function

$$x'_k = \frac{g^{\sqrt{k(k+1)}} - 1}{\sqrt{k(k+1)} \ln g}.$$

Taking into account that, in passing from equation in the spherical geometry to the equation in a plane geometry, the scattering phase function of the medium has been stronger forward-peaked function of the angle (Fig. 1), that is physically equivalent to account for the angular peculiarities of the solution, the number of terms of the series needed for describing the solution increases from hundreds to thousands. The matrix of this system becomes weakly defined, making the equation practically insolvable.

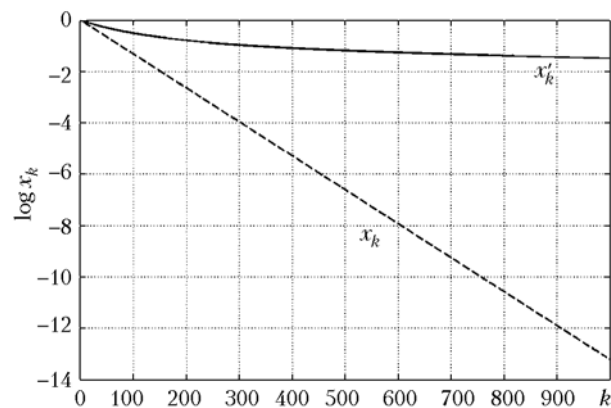


Fig. 1. Dependence of the expansion coefficients of scattering phase function on the number k .

On the other hand, such a transformation stronger distorts the dependence of harmonics on distance at small optical depths, where the angular distribution of the brightness of low scattering orders has the following specific features¹¹: the peculiarity is of the form $\frac{1}{r^2} \delta(\hat{1} - \hat{r})$, in zero scattering order,

$$\frac{1}{r\sqrt{1-\mu^2}} + \ln[r^2(1-\mu^2)]$$

in the first order of scattering, $\ln[r(1-\mu) + \ln r]$ in the second order, and no peculiarities are present in the third and higher orders of scattering. Representation (20) smoothes out the peculiarities of brightness pattern in the first and second orders of scattering.

To eliminate this effect, we will present the solution in the form of the sum:

$$L(r, \mu) = \tilde{L}(r, \mu) + L_{\text{SHM}}(r, \mu), \tag{21}$$

where the solution in small-angle modification of spherical harmonics method (SHM)⁶ has the form

$$L_{\text{SHM}}(r, \mu) = \sum_{k=0}^{\infty} \frac{2k+1}{4\pi r^2} \exp \left[-\varepsilon r \left(1 - \frac{\Lambda}{\ln g} \frac{g^{\sqrt{k(k+1)}} - 1}{\sqrt{k(k+1)}} \right) \right] P_k(\mu) = \sum_{k=0}^{\infty} \frac{2k+1}{4\pi r^2} Z_k(r) P_k(\mu).$$

Since the small angle part of solution in expression (21) contains all peculiarities of the exact solution,^{6,7} then $\tilde{L}(r, \mu)$ is a smooth function.

For further analysis, we will pass to the matrix form of RTE (20), analogous to that presented in Ref. 13:

$$\tilde{A} \frac{d}{d\tau} \mathbf{Y}'(\tau) + \tilde{D} \mathbf{Y}'(\tau) = 0, \quad (22)$$

where $\tau = \varepsilon r$ is optical path, symbol “ $\tilde{\cdot}$ ” above a letter means matrix;

$$(\tilde{A})_{i,i+1} = \frac{i}{2i-1}, (\tilde{A})_{i,i-1} = \frac{i-1}{2i-1}, \mathbf{Y}' = \{\tilde{Y}_{i+1}(r)\},$$

$$\tilde{D} = \text{Diag} \left\{ \varepsilon \left(1 - \frac{\Lambda}{\ln g} \frac{g^p - 1}{p} \right) \right\}, p = \sqrt{(i+1)(i+2)},$$

and i is the running index of matrix elements and columns of the system (22).

The representation (21) leads¹⁴ to appearance of residual terms in the right-hand side of equation (22), namely

$$\Delta = -\frac{1}{2k+1} [(k+1)b'_{k+1}Z_{k+1}(r) + kb'_{k-1}Z_{k-1}(r)] + \frac{k(k+1)}{(2k+1)r} [Z_{k+1}(r) - Z_{k-1}(r)] + b_k Z_k(r) \quad (23)$$

or in the matrix form:

$$\tilde{\Delta} = (\tilde{A} - \tilde{I})\tilde{D}\mathbf{Z}(\tau) + a_{N+1}\mathbf{Z}_{N+1}(r), \quad (24)$$

while the SHM-based system of equations (22) assumes the form

$$\tilde{A} \frac{d}{d\tau} \mathbf{Y}(\tau) + \tilde{D} \mathbf{Y}(\tau) = (\tilde{A} - \tilde{I})\tilde{D}\mathbf{Z} + a_{N+1}\mathbf{Z}_{N+1}(r), \quad (25)$$

where the terms

$$a_{N+1} = \frac{N+1}{2N+1} b'_{N+1}, \mathbf{Z}_{N+1} = \left\{ \underset{N}{0 \dots 0}, Z_{N+1}(r) \right\}$$

appear due to the use of $(N+1)$ st term of small angle approximation.

The boundary conditions also can be written in the matrix form¹⁴:

$$\begin{bmatrix} \tilde{I} & \tilde{G} \end{bmatrix} \tilde{P} \mathbf{C}(0) = \mathbf{0}, \begin{bmatrix} \tilde{I} & -\tilde{G} \end{bmatrix} \tilde{P} \mathbf{C}(\tau_0) = -\mathbf{X}(\tau_0), \quad (26)$$

where $X_j(\tau) = Z_{2j-1}(\tau) - \sum_{i=0}^N G_{ji} Z_{2i-2}(\tau)$; \tilde{P} is the matrix of sorting odd and even elements of the form

$$\tilde{P} \mathbf{C} \equiv \begin{bmatrix} \mathbf{C}_{\text{odd}} \\ \mathbf{C}_{\text{even}} \end{bmatrix}. \quad (27)$$

The matrix of the system consisting of several hundred equations is weakly defined that leads to increase of the calculation error with the growth of the optical depth. This effect is eliminated through introduction of scaling,¹³ which allows the solution

to remain stable with the growth of the optical depth. As a result, the system of equations in the matrix form is as follows¹⁴:

$$-\tilde{S} \tilde{U}^{-1} \mathbf{C}(0) + \tilde{H} \tilde{U}^{-1} \mathbf{C}(\tau_0) = \mathbf{S} \mathbf{J}(\tau_0), \quad (28)$$

where

$$\tilde{S} = \begin{bmatrix} 0 & \tilde{I} \\ e^{\tilde{\Gamma} \tau_0} & 0 \end{bmatrix}, \tilde{H} = \begin{bmatrix} 0 & e^{-\tilde{\Gamma} \tau_0} \\ \tilde{I} & 0 \end{bmatrix}, \tilde{\Gamma} = \begin{bmatrix} \tilde{\Gamma}^- & 0 \\ 0 & \tilde{\Gamma}^+ \end{bmatrix}$$

is the matrix of eigenvalues of the matrix $\tilde{B} = \tilde{A}^{-1} \tilde{D}$, sorted in increasing order; \tilde{U} is the matrix of the corresponding eigenvectors; $\tilde{T} = \tilde{U}^{-1} (\tilde{I} - \tilde{A}^{-1}) \tilde{D}$;

$$\mathbf{J} = \left\{ \sum_{j=1}^{N+1} (T_{ij}) \frac{[1 - \exp[-\Gamma_{ii} \tau_0 - (1 - \Lambda b'_k) \tau_0]]}{\Gamma_{ii} + (1 - \Lambda b'_k)} \right\} + a_{N+1} (\tilde{\Gamma} + b_{N+1} \tilde{I})^{-1} \left\{ \tilde{I} - \exp[-(\tilde{\Gamma} + b_{N+1} \tilde{I})] \right\} \tilde{U}^{-1} A_{N+1}^{-1}.$$

This task was solved using TheMathWorks[®] Matlab software. To assess the accuracy of the obtained solution, comparison against single scattering case was made. In Fig. 2, solid line shows backscattering calculated with the algorithm proposed, while dashed line shows that calculated in single scattering approximation for parameters of the medium being as follows: $g = 0.8$, $\Lambda = 0.8$, and $\tau = 0.1$. For achieving an acceptable accuracy the calculations used 1501 Legendre polynomials.

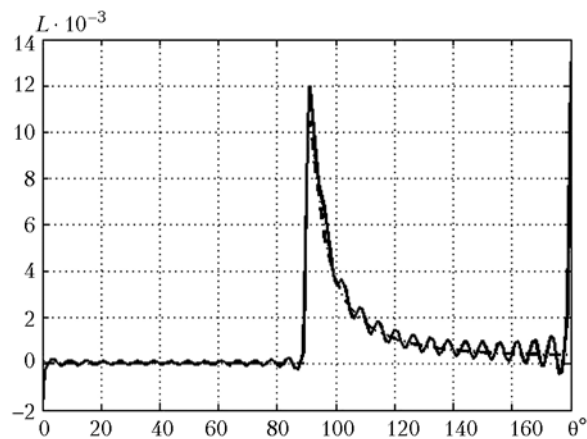


Fig. 2. Comparison of the backscattering brightness calculated using the method proposed and with that calculated using single scattering approximation for small optical depths $\tau = 0.1$.

Shown by solid line in Fig. 3 are results calculated by the algorithm proposed while the dashed line shows that calculated using small-angle modification of the spherical harmonics method for forward scattered field with the parameters of the medium being as follows: $g = 0.8$, $\Lambda = 0.8$, and $\tau = 10$ (here, the logarithms of the corresponding functions are plotted). From Fig. 3 it is seen that the solution for forward scattered radiation converges faster;

therefore, much less harmonics ($N = 301$) can be used in the expansion.

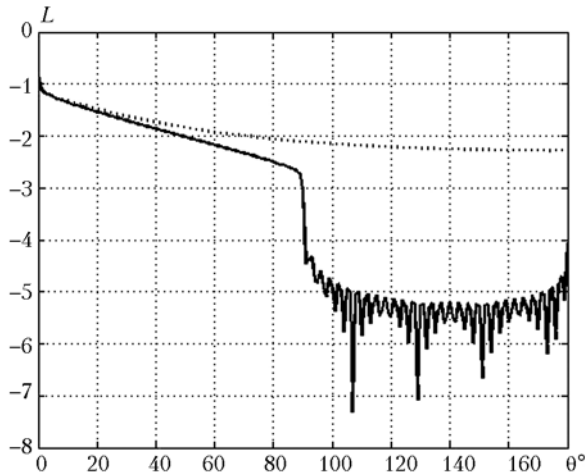


Fig. 3. Comparison of brightness calculations of radiation field in the forward hemisphere of directions against small angle modification of spherical harmonics method for $\tau = 10$.

3. Optical transfer function of a layer of the medium

Optical transfer function of an ideal optical system that views through the depth of a turbid medium is a Fourier transform of the irradiance distribution $E(\rho)$ over the plane containing the image of a luminous point located on the optical axis of the system:

$$T(\mathbf{p}) = \int_{-\infty}^{\infty} E(\rho) e^{-i\mathbf{p}\rho} d^2\rho = \frac{\pi O^2}{4} \int_{-\infty}^{\infty} L(r, \mu) e^{-i\mathbf{p}\rho} d^2\rho, \quad (29)$$

where O is the relative aperture (or F/D ratio) of the system; $\mu = \cos\theta = \sqrt{1 - (\rho/s)^2}$, s is the distance from the exit pupil to the plane of the image analysis; and ρ is the distance from the optical axis to the point of the image in analysis plane.

To introduce the OTF of a vision system, the condition of image invariance (isoplanarity) with respect to shift of the object in the object plane must hold, which, in the general case, is not valid in recording angular brightness distribution at a single point fixed in space.¹⁵ Since to determine the OTF, a luminous point at the optical axis is taken as an object, then $L(r, \mu)$ is the distribution of brightness from a PI source, determined from the boundary-value problem (1). A displacement of the point in the object plane leads to change of r , thereby violating the isoplanarity condition. However, solution of practical problems requires more or less accurate fulfillment of the isoplanarity condition, therefore, it is possible to introduce the conception of isoplanarity zones,¹⁵ within which the subsequent OTF analysis is performed.

Substituting the solution in the form of Eq. (3) into formula (29) for the OTF, we obtain

$$T(\mathbf{p}) = \frac{\pi O^2}{4} \int_{-\infty}^{\infty} \sum_{k=0}^{\infty} \frac{2k+1}{4\pi} C_k(r) P_k(\mu) e^{-i\mathbf{p}\rho} d^2\rho = \frac{\pi O^2}{4} \sum_{k=0}^{\infty} \frac{2k+1}{4\pi} C_k(r) \int_{-\infty}^{\infty} P_k(\mu) e^{-i\mathbf{p}\rho} d^2\rho. \quad (30)$$

The system field of view usually substantially exceeds the object sizes; therefore in the range of small angles the Legendre polynomials can be replaced with Bessel function $P_k(\cos\theta) \approx J_0(k\theta)$, and so, under assumption of axial symmetry of the system, the integral in Eq. (30) can be transformed as follows:

$$\int_{-\infty}^{\infty} P_k(\cos\theta) e^{-i\mathbf{p}\rho} d^2\rho = 2\pi s^2 \int_0^{\infty} P_k(\cos\theta) J_0\left(\rho \frac{ps}{s}\right) \frac{\rho}{s} d\left(\frac{\rho}{s}\right) \approx \int_{-\infty}^{\infty} P_k(\cos\theta) e^{-i\mathbf{p}\rho} d^2\rho = 2\pi s^2 \int_0^{\infty} J_0(k\theta) J_0(ps'\theta) \theta d\theta = s^2 \frac{2\pi}{k} \delta(k - ps) \Big|_{k \gg 1} \approx s^2 \frac{4\pi}{2k+1} \delta(k - ps), \quad (31)$$

where, having in mind the small angle approximation, we used the assumption that $\theta \approx \sin\theta \approx \tan\theta = \rho/s$, which is quite usual for the paraxial optics, and took into account that in the region of small angles the corresponding terms of the series (3) have large numbers.

The last expression leads to the formula for the OTF of a vision system in the following form

$$T(p) = \frac{\pi O^2}{4} C_{k=ps}(r). \quad (32)$$

Figure 4 compares the calculated normalized OTF of a vision system obtained using the solution, described in this paper, with that obtained using small-angle approximation.

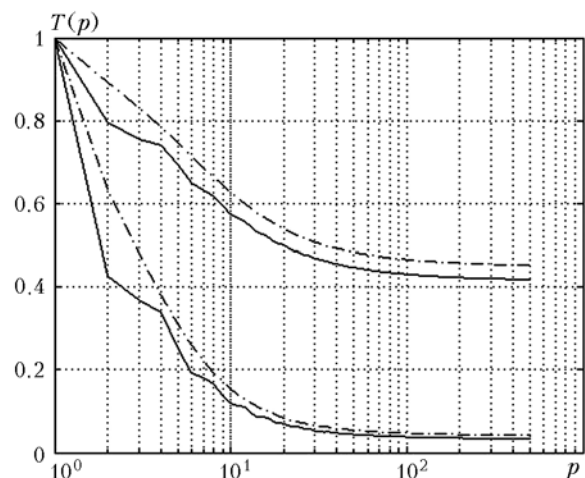


Fig. 4. Normalized OTF of a layer of the turbid medium.

Solid lines in the figure show OTF values obtained by the method proposed, and dashed lines

show the results obtained in small-angle approximation. The calculations assume that $s = 1$. Upper plots correspond to the optical depth $\tau = 1$, and the lower plots to $\tau = 4$.

It is seen that inclusion of the dispersion of scattered photon trajectories reduces OTF values in the region of low spatial frequencies.

Conclusions

1. The method of RTE solution proposed makes it possible to determine the brightness field created by a PI source taking into account multiple reflections within the full solid angle. Separation out of the small-angle component allows one to solve the problem by numerical methods because the remaining function is smooth and does not contain peculiarities.

2. The analysis performed has shown the importance of the account of spatial and angular peculiarities of the solution in the cases of radiative transfer problems with arbitrary three-dimensional geometry.

3. It has been found (see Fig. 3) that the brightness of radiation field calculated in the small-angle approximation overestimates somewhat the field of forward scattered radiation in the region of "large" (in excess of 30°) angles, primarily because small-angle approximation neglects the dispersion of scattered photon trajectories.

4. The influence of dispersion of scattered photon trajectories on the OTF (see Fig. 4) is evident at small frequencies, reducing its values.

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