PROPAGATION OF AN OPTICAL BEAM OF VARIABLE RADIUS UNDER CONDITIONS OF GRAVITATIONAL CONVECTION AND AIR BLOW

A.N. Kucherov, N.K. Makashev, and E.V. Ustinov

N.E. Zhukovskii Central Aerohydrodynamic Institute, Moscow

Received September 22, 1993

A possibility of estimating beam perturbations due to gravitational convection and transverse air blow from the thermal blooming effect is demonstrated in this paper for optical beams of variable radii propagating along a model laboratory path. Perturbations of focused and self–focused beams occurring on the propagation path are proposed to be estimated based on the value of their mean radii, for which we have derived approximate analytical relationships.

An intense optical beam considered in the laboratory experiment was distorted due to thermal blooming. This paper describes the investigation carried out with gravitational convection in a stationary medium as well as with a forced blow with a transverse gas flow, which can lead to a reduction of perturbations. The beam path contains, as a rule, beam folding and focusing (defocusing) mirrors and lenses, telescopes, segments with a widened or narrowed transverse size of the beam and so on.

Dimensionless equations of paraxial optics ($a/L \ll 1$, a is the characteristic radius of the beam, and L is the characteristic length of the path) in the geometric–optics approximation ($F = 2\pi a^2/\lambda L \to \infty$, λ is the radiation wavelength) can be written in the form

$$\left[\frac{\partial}{\partial z} + (\theta,\, \nabla_\perp)\right] \ln I + (\nabla_\perp,\, \theta) = -\, N_\alpha \;; \tag{1}$$

$$\left[\frac{\partial}{\partial z} + (\theta,\, \nabla_\perp)\right] \boldsymbol{\theta} = N\, \nabla_\perp \rho_1(x,\, y,\, z;\, I); \qquad \nabla_\perp = \mathbf{e}_x \frac{\partial}{\partial x} + \mathbf{e}_y \frac{\partial}{\partial y}$$

;(2)

$$I\big|_{z=0} = I_0(x,\, y)\;; \quad \boldsymbol{\theta}\big|_{z=0} = \boldsymbol{\theta}_0(x,\, y)\,. \tag{3}$$

Here, the intensity I is related to the characteristic intensity I_*, the angle $\boldsymbol{\theta}$ of beam deviation from the propagation direction z is related to the value α/L, the coordinate z is related to the characteristic path length L, and the coordinates x, y are related to the characteristic transverse size of the beam a. The absorption parameter $N_\alpha = \alpha L$, where α is the coefficient of radiation absorption with the medium, the self–blooming parameter $N = \varepsilon(L/a)^2(n_0 - 1)/n_0$, where n_0 is the index of nonperturbed medium refraction; ε is the scale of the medium density perturbation; $\rho_1 = \Delta\rho/\varepsilon\rho_0$ is the dimensionless function of density perturbation; ρ_0 is the density of nonperturbed medium; $I_0(x, y)$, $\theta_0(x, y)$ are the preset initial distributions of the intensity and the angle of beam deviation (divergence); $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors along the axes x and y. In the general case the function ρ_1 is determined from the solution of the system of hydrodynamics equations (conservation of mass, momentum, energy, and equation of state). With the transverse air flow moving with the velocity V_0 much lower than the speed of sound the density perturbation is described by the transfer equation which is obtained from the linearized equation of energy conservation:

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y}\right) \rho_1 = -\, I\;;\; \varepsilon = \alpha\, I_*\, a/\rho_0\, h_0\, V_0\;;\; I_* = W_0/\pi a^2. \tag{4}$$

Here W_0 is the total initial power of the beam; h_0 is the enthalpy of the nonperturbed medium. The time t is related to the characteristic time of a liquid (gas) particle travel across the beam a/V_0. The y axis is directed along the blow velocity.

For a self–induced gravitational convection along the horizontal beam path, the density perturbation is determined by the system of hydromechanics equations in the Boussinesq approximation:

$$\begin{cases} \mathrm{div}\mathbf{V} = 0;\;\; \varepsilon_\mathrm{c} = \alpha I^* a/\rho_0 h_0 V_\mathrm{c};\; V_\mathrm{c} = (\alpha I^* \mathrm{g} a^2/\rho_0 h_0)^{1/3}, & (5) \\ \mathrm{dV/dt} + \nabla_\perp \rho_1 = \mathrm{e_y}\rho_1;\; \rho = 1 + (\mathrm{Eu/Fr})(-\, y + \varepsilon\rho_1 + \ldots), & (6) \\ \mathrm{d}\rho_1/\mathrm{dt} = -\, I(x,\, y,\, z,\, t);\; d/d\mathrm{t} = \partial/\partial\mathrm{t} + (\mathbf{V},\, \nabla_\perp). & (7) \end{cases}$$

where g is the acceleration due to gravity, $\mathrm{Eu} = \rho_0\, V_\mathrm{c}^2/P_0$ is the Euler number; P_0 is the pressure of nonperturbed gas; V_c is the characteristic rate of gravitational convection; $t_\mathrm{c} = a/V_\mathrm{c}$ is the characteristic time of its development; $\mathrm{Fr} = V_\mathrm{c}^2/ag$ is the Froude number. Viscosity and thermal conductivity can be neglected in the majority of cases. It should be noted that at air blow velocities $V_0 \gg V_\mathrm{c}$ the scale of density perturbations ε and the parameter of thermal blooming N are much smaller than those under gravitational convection. Hence using forced air blow we can substantially reduce the thermal blooming.

Consider now a model laboratory path divided into three segments. At the start of the first segment the beam has the radius $a_1 = a$, and the segment length is L_1. At the end of this segment a telescope with magnification factor k_1 is placed. At the end of the second segment of the length L_2 a focusing mirror (or lens) is positioned. When there is no initial divergence and perturbations on the path, the beam, on the third segment of the length L_3, is focused into a point.

In vacuum the trajectories of rays (corresponding, e.g., to an exponential radius) are described by the following expressions:

0235-6880/93/12 873-04 $02.00

$a)\ 0 \le z \le z_1;\ z_1 = L_1/L:$

$$\theta_{01} = \theta_0; \tag{8}$$

$$r_{01}(z) = a_1/a + z\,\theta_{01}; \qquad r_{011} = r_{01}(z_1); \tag{9}$$

$b)\ z_1 \le z \le z_2;\ z_2 = (L_1 + L_2)/L:$

$$\theta_{02} = \theta_{01}/k_1; \tag{10}$$

$$r_{02}(z) = r_{011}\,k_1 + (z - z_1)\,\theta_{02}; \qquad r_{022} = r_{02}(z_2); \tag{11}$$

$c)\ z_2 \le z \le z_3;\ z_3 = (L_1 + L_2 + L_3)/L:$

$$\theta_{03} = \theta_{02} - r_{022}/f; \qquad f = L_3/L; \tag{12}$$

$$r_{03}(z) = r_{022} + (z - z_2)\,\theta_{03}. \tag{13}$$

In expressions (10) and (11) it was taken into account that the telescope expands the beam by a factor of k_1 and decreases the beam divergence by a factor of $1/k_1$.

In a nonlinear medium[1,2] the perturbations of the divergence angle and radius of beam by order of magnitude are estimated from the relations

$$B_1(z) = \frac{N}{r_0(z)} \int\limits_0^z \frac{(\exp(-N_\alpha z'))^m}{r_0^n(z')}\,\mathrm{d}z';\quad \Delta\theta/r_0(z) \sim \Delta B_1(z); \tag{14}$$

$$\frac{\Delta r}{r_0(z)} \sim B_2(z) = \int\limits_0^z \frac{B_1(z')\,\mathrm{d}z'}{r_0(z')}. \tag{15}$$

Here $r_0(z) = r_{01}(z);\ r_{02}(z);\ r_{03}(z)$ are the variable radii of a beam in vacuum. The exponents m and n in Eq. (14) are: $m = 1$, $n = 1$ with transverse air blow; $m = 2/3$ and $n = 1$ with gravitational convection. The factors $B_1(z)$ and $B_2(z)$ were obtained from the linearized solution of equations (1) and (2) which is strictly valid as $N \to 0$. A comparison with numerical calculations[1] showed that, at least for averaged characteristics, these values give satisfactory results for moderate values $N \sim 1$. At the same time, calculation of integrals (14) and (15) is simpler than the numerical solution of Eqs. (1) and (2). In some situations the approximated analytical relations for the functions B_1 and B_2 are valid. They allow one to rapidly estimate the contribution of any segment of the path to the beam perturbation.

Taking into account the fact that the absorption parameter is, as a rule, small ($N_\alpha \ll 1$), the divergence angle and the mean radius of a beam in a nonlinear medium can be evaluated from the formulas

$a)\ 0 \le z \le z_1:\ \theta_1(z) \simeq \theta_{01} + r_{01}(z)\,B_1(z)\,C;$

$$\begin{cases} B_1(z) = \dfrac{N_1}{1 + \theta_0 z}\,\dfrac{\ln(1 + \theta_0 z)}{\theta_0}\Bigg|_{\theta_0 \to 0} \simeq N_1 z;\quad \theta_{11} = \theta_1(z_1), & (16) \\ r_1(z)\Big|_{\theta_0 = 0} = 1 + CB_2(z);\ r_{11} = r_1(z);\ B_2(z)\Big|_{\theta_0 = 0} = N_1\,\dfrac{z^2}{2}; & (17) \end{cases}$$

$b)\ z_1 \le z \le z_2:\ \theta_2(z) \simeq \theta_{11}/k_1 + r_{02}(z)\,C\,\Delta B_1(z);$

$$\begin{cases} B_1(z) = B_1(z_1) + \\ + \dfrac{N_2}{r_{02}(z)}\,\dfrac{\ln\left[1 + \dfrac{\theta_{01}}{k_1 r_{011}}\,(z - z_1)\right]}{(\theta_{01}/k_1)}\Bigg|_{\theta_0 \to 0} \to B_1(z_1) + \dfrac{N_2(z - z_1)}{r_{022}^2}, & (18) \\ \theta_{22} = \theta_2(z_2); \\ \dfrac{r_2(z)}{r_{02}(z)} = 1 + CB_2(z); \\ B_2(z)\Big|_{\theta_0 = 0} = B_2(z_1) + B_1(z_1)\,\dfrac{z - z_1}{r_{022}} + \dfrac{N_2(z - z_1)^2}{2\,r_{022}^3}, & (19) \\ r_{22} = r_2(z_2); \end{cases}$$

$$c)\ z_2 \le z \le z_3:\ B_1(z) = B_1(z_2) + \frac{N_3}{r_{03}(z)\,\theta_{03}}\ln\left[1 + \frac{\theta_{03}}{r_{022}}(z - z_2)\right];$$

$$\begin{cases} \theta_3(z) = \theta_{22} + N_3\,\dfrac{\ln\left[1 + \dfrac{\theta_{03}}{r_{022}}(z - z_2)\right]}{\theta_{03}}\Bigg|_{\theta_0 \to 0} \to \theta_{22} + \dfrac{N_3(z - z_2)}{r_{022}^2}, \\ \dfrac{r_3(z)}{r_{03}(z)} = 1 + CB_2(z); & (20) \\ B_2(z) = B_2(z_2) + \dfrac{B_1(z_2)}{\theta_{03}}\ln\left[1 + \dfrac{\theta_{03}}{r_{022}}(z - z_2)\right] + & (21) \\ + \dfrac{N_3}{r_{022}\theta_{03}^2}\left\{\dfrac{\theta_{03}(z - z_2)}{r_{03}(z)} + \ln\left[1 + \dfrac{\theta_{03}}{r_{022}}(z - z_2)\right]\right\}. \end{cases}$$

The parameter N is written with indices (segment numbers) with the account that different segments can contain different substances under different conditions (e.g., the absorption coefficients and the air blow velocities are different on different segments of the path).

Using the approximate (16)–(21) and exact, (14) and (15), values of the factors B_1 and B_2, we analyzed the beam perturbations along the paths with one or two telescopes, along the paths including segments with air of different humidity and segments with technical nitrogen containing different concentrations of oxides which effectively absorb radiation in the IR range under study. We considered a mode of self–induced convection and transverse air blow of the beam. For providing control we made numerical calculations by Eqs. (1) and (2) by dividing the beam into an array (of the order of 10000) of elementary ray tubes, to each of which a portion of beam energy and an initial angle were prescribed. The variation of the angle and, hence, the coordinates of an individual tube were calculated from Eq. (2). Energy decrease in the tube due to absorption was calculated by the Bouguer–Beer law rather than by Eq. (1). Diffraction on nonlinear inhomogeneities of the medium was neglected. The Boussinesq equations (5)–(7) were solved using the algorithm from Ref. 3. Depicted in Fig. 1 are the plots of the mean beam radius

$r_m = \int\limits_{-\infty}^{+\infty}\!\!\int [\,x^2 + (y - \Delta y)^2]\,I\,\mathrm{d}x\,\mathrm{d}y/W$ (where

$\Delta y = \int\limits_{-\infty}^{+\infty}\!\!\int y\,I\,\mathrm{d}x\,\mathrm{d}y/W$ is the center of gravity displacement,

$W = \int\limits_{-\infty}^{+\infty}\!\!\int I\,\mathrm{d}x\,\mathrm{d}y$ is the total beam power) vs the factor of thermal broadening $B_2(z = z_3)$ for different situations described. For moderate values of the factor $B_2(z_3) < 6.5$ the mean radius is directly proportional to the value B_2. For the values B_2 approaching 10, linearity of the dependence $r_m(B_2)$ is violated. For the beam focused at the end of the path the factors B_1 and B_2 have singularities at focus since the beam radius in vacuum tends to zero in the absence of initial divergence (within the framework of the wave theory the beam radius tends to diffraction limit ~ $1/F$). The values of the factors B_1 and B_2 take the values larger than 10 in many situations.

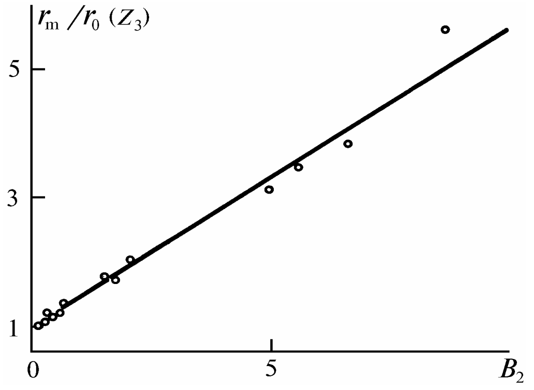

FIG. 1. A plot of the mean beam radius in a cross section under control vs a factor of thermal blooming.

To estimate beam perturbations in such and similar situations, let us introduce a model of a beam with an elliptic cross section with varying size along the coordinates $r_x(z)$ and $r_y(z)$ and the corresponding divergence angles $\theta_x(z) = \mathrm{d}r_x/\mathrm{d}z$ and $\theta_y(z) = \mathrm{d}r_y/\mathrm{d}z$ as well as the center of gravity displacement coordinate of intensity distribution $r_d(z)$ and the corresponding angle $\theta_d(z) = \mathrm{d}r_d/\mathrm{d}z$.

The beam intensity varying along the path can be written as $I_{\text{phys}} = W_{\text{phys}}/[\pi\, r_x(z)\, r_y(z)]$, and the density perturbation ρ_1, according to Eqs. (4) or (7), can be estimated by the formulas (at least for a fixed thermal blooming)

$$\rho_1 \sim \exp(-N_\alpha z)/\, r_x(z)\ ;\ \rho_1 \sim [\exp(-N_\alpha z)]^{2/3}/r_x^{2/3}(z)\, r_y^{1/3}(z)$$

for the air blow or gravitational convections, respectively. Based on Eqs. (1) and (2) the following system of equations can be used to estimate the beam perturbations:

$$\frac{\mathrm{d}r_x}{\mathrm{d}z} = \theta_x\ ;\ \frac{\mathrm{d}\theta_x}{\mathrm{d}z} = \frac{b\,[\exp(-N_\alpha z)]^l}{r_x^n\, r_y^m}\ ,\ b = K_x N\ ; \tag{22}$$

$$\frac{\mathrm{d}r_y}{\mathrm{d}z} = \theta_y\ ;\ \frac{\mathrm{d}\theta_y}{\mathrm{d}z} = \frac{b_1\,[\exp(-N_\alpha z)]^l}{r_x^p\, r_y^q}\ ,\ b_1 = K_y N\ ; \tag{23}$$

$$\frac{\mathrm{d}r_c}{\mathrm{d}z} = \theta_c\ ;\ \frac{\mathrm{d}\theta_c}{\mathrm{d}z} = \frac{c\,[\exp(-N_\alpha z)]^l}{r_x^p\, r_y^q}\ ,\ c = K_c N\ ; \tag{24}$$

where, under the air blow, we have $l = 1$, $p = 1$, $q = 1$, $m = 0$, and $n = 2$ and under gravitational convection: $l = 2/3$, $n = 5/3$, $m = 1/3$, $p = 2/3$, and $q = 4/3$. As comparison with the aforementioned numerical calculations showed the constants $K_x = 1$, $K_y = 0.5 \simeq K_c$ under fixed gravitational convection. If we neglect attenuation due to absorption ($N_\alpha = 0$), then under the air blow conditions Eq. (22) is reduced to the equation of free fall (the indices "x" are omitted below):

$$r'' = \pm\, b/r^2\ ;\ r' = \theta\ . \tag{25}$$

Under gravitational convection assuming that $r_x = \text{const}\ r_y$ (for estimating perturbations by the order of magnitude such assumptions is valid) the analysis of solution of system (22)–(24) can be reduced to the analysis of solution of equation (25). The sign "minus" in front of the parameter b in Eq. (25) corresponds to the beam propagation under conditions of self–focusing caused by the shape or the medium properties profile.

Integrating Eq. (25) one time we find

$$r'^2 \pm 2\,b/r = \pm\, A^2\ ;\ \ |\theta| = \sqrt{\mp\,(2\,b/r) \pm A^2}\ ;$$
$$r = 2\,b/(A^2 \mp \theta^2)\ ;\ A^2 = \theta_1^2 \pm (2\,b/r_1)\ , \tag{26}$$

where θ_1 and r_1 are the preset initial values.

Let us integrate Eq. (26) and make a substitution: $\tilde\theta = \theta/A$, $\tilde r = rA/2b$, $\tilde z = (z - z_1)\,A^3/2\,b$. We obtain the following universal solutions for defocusing and self–focusing, respectively:

$$\tilde z = \tilde r\,\tilde\theta + \frac{1}{2}\ln\left|\frac{1+\tilde\theta}{1-\tilde\theta}\right|,\ z_1 = \frac{2b}{A^3}\left(\tilde r_1\,\tilde\theta_1 + \frac{1}{2}\ln\left|\frac{1+\tilde\theta_1}{1-\tilde\theta_1}\right|\right); \tag{27}$$

$$\tilde z = -\,\tilde r\,\tilde\theta - \mathrm{arctg}\,\tilde\theta\ ,\ \ z_1 = \frac{2b}{A^3}\Big(-\,\tilde r_1\,\tilde\theta_1 - \mathrm{arctg}\,\tilde\theta_1\Big)\ , \tag{28}$$

$$\tilde r = 1/(1 \mp \tilde\theta^2)\ . \tag{29}$$

The origin of the z axis is displaced by the value z_1 by such a way that $\tilde\theta(\tilde z = 0) = 0$, $\tilde r(\tilde z = 0) = 1$. The solutions of Eqs. (27) and (29) corresponding to beam propagation in a defocusing medium (curves *1*) are constructed in Fig. 2. The linear dependence $\tilde r = \tilde z$ is given for comparison too. Depicted here are dependences (28) and (29) related to a self–focused beam (curves *2*).

In a specific problem we realize some portion of universe solutions (27)–(29) which, depending on the value of the known parameter θ_1, r_1, b, and the path length, can contain or not a cross section with a minimum (or maximum) mean radius.

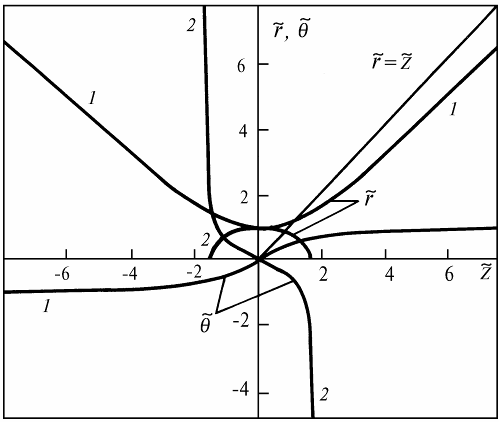

FIG. 2. Variation of the mean beam radius and the divergence angle along the path which was obtained from solution of Eqs. (27)–(29).

The solution (27) and (29) provides asymptotic relationships

$$\tilde{r} = \tilde{z} - 0.5 \ln (4\tilde{z}) + \ldots ; \quad \tilde{\theta} = 1 - 1/2\tilde{z} + \ldots ; \quad \tilde{z} \to \infty , \tag{30}$$

$$\tilde{r} \simeq 1 + \tilde{z}^2/4 + \ldots ; \qquad \tilde{\theta} \mid \tilde{z}/2 + \ldots ; \qquad \tilde{z} \to 0 . \tag{31}$$

Solutions (28) and (29) are defined in the limited region $|z| \le \pi/2$ and provide the asymptotic expressions

$$\tilde{r} \simeq 1 - \tilde{z}^2/4 + \ldots ; \quad \tilde{\theta} \simeq - \tilde{z}/2 + \ldots ; \quad \tilde{z} \to 0 . \tag{32}$$

$$\tilde{r} \approx \left[\frac{3}{2}\left(\mp \frac{\pi}{2} + \tilde{z} \right)\right]^{2/3} + \ldots ; \tilde{\theta} \approx \mp \left[\frac{2}{3(\mp \pi/2 + \tilde{z})}\right]^{1/3} + \ldots ;$$

$$\tilde{z} \to \pm \frac{\pi}{2} . \tag{33}$$

These relations enable one to determine analytical dependences between the known size of the beam at the end of the path (the known allowable level of perturbations) and the required levels of physical parameters: absorbed power, air blow velocity, composition, state of the medium, and so on. In conclusion it should be noted that we proposed an effective algorithm for estimating the beam perturbations due to thermal blooming on complex laboratory paths including many segments with different conditions of propagation. Actually, this approach is also valid for optical beam propagation in the atmosphere.

REFERENCES

1. A.N. Kucherov, N.K. Makashev, and E.V. Ustinov, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **34**, No. 5, 528–535 (1991).
2. A.N. Kucherov, N.K. Makashev, and E.V. Ustinov, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **36**, No. 2, 135–142 (1993).
3. A.N. Kucherov, N.K. Makashev, and E.V. Ustinov, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **35**, No. 2, 145–154 (1992).