OPTIMAL PHASE FIELD DISTRIBUTION ACCORDING TO MAXIMUM FLUX CRITERION IN TRANSMITTING RADIATION TO A RANDOMLY PLACED RECEIVER

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To determine an optimum phase distribution (PD) of a field the integral equation is obtained which generalizes the law of a phase conjugation for the stochastic optical fields. Investigations have been carried out using the method of phase approximation for the Green's functions and the variational methods. The energy criterion of an optical transmitter quality is proposed. In the absence of medium inhomogeneities the optimization of the PD makes it possible to compensate partially for the diffraction influence, random errors in aiming and receiver installation, and to increase the radiation power flux at the receiver by about 10%.

The effects of diffraction scattering, inhomogeneities of a propagation medium, errors in aiming and receiver installation result in decreasing the radiation power (energy) flux at the receiver and considerably restrict capabilities of optical communication systems.[1,2] Modern controllable optical systems[3] make it possible to control the phase distribution of the field at the output of the optical transmitter and, thus, partially compensate for the effects of undesirable factors. If only diffraction and medium inhomogeneities are taken into consideration, the solution of the problem of maximum flux transmission (in the absence of nonlinear, with respect to the field, effects) is determined by the law of phase conjugation.[2–5] In this paper an optimum PD of the output radiation of a transmitter is studied with the account for communication channel properties and typical errors of communication systems in aiming and errors caused by uncertainty in a receiver position.

Let us derive an equation for determining an optimum PD of the field in the plane of output aperture of an optical transmitter of a communication system. Let us first of all derive a relation for the radiation power flux at the receiver. The distribution of a field U over the plane of the receiver aperture can be represented in terms of the integral relation

$$U(\mathbf{r}, z, \mathbf{q}) = \int d\rho\, A(\rho)\, G(\rho, \mathbf{r}, z) \exp i \{\varphi(\rho) + \varphi_q(\rho, \mathbf{q})\}\,, \quad (1)$$

where $A(\rho)$ is the amplitude distribution of the field in the plane of the transmitter output aperture, $A(\rho) \geq 0$; $\mathbf{r}$ and ρ are the radius vectors of the points in the planes of apertures of the receiver and transmitter in a cylindrical coordinate system; z is the distance from the transmitter to the receiver; $G(\rho, \mathbf{r}, z)$ is the Green's function[2] that takes into account the diffraction effects and medium inhomogeneities; $\varphi(\rho) = \varphi_s(\rho) + \varphi_c(\rho)$ is the phase distribution of the field over the plane of the output aperture of the transmitter; $\varphi_s(\rho)$ is the phase distortion of the field inside the optical transmitter; $\varphi_c(\rho)$ is the controllable component of the PD being introduced into the radiation beam with the help of a phase corrector to compensate for the effects of undesirable factors; $\varphi_q(\rho, \mathbf{q})$ are the phase distortions whose resulting effects are equivalent to the errors in aiming[1] and represent the sum of the distortion (it corresponds to the angular error in aiming), field curvature and astigmatism (they correspond to the errors in focusing onto the receiver); the components of the random vector q are the weights of aberration polynomials that compose the random function $\varphi_q(\rho, \mathbf{q})$.

In view of the stochastic nature of the aiming errors and uncertainties in the receiver position the flux of the radiation power at the receiver can be characterized by an averaged statistical value and is calculated by the formula

$$W = \langle | U(\mathbf{r}, z, \mathbf{q}) |^2 \rangle =$$

$$= \int\int d\rho_1 d\rho_2\, A(\rho_1) A(\rho_2)\, F(\rho_1, \rho_2) \exp i \{\varphi(\rho_1) + \varphi(\rho_2)\}\,, \quad (2)$$

here the angular brackets designate averaging,

$$F(\rho_1, \rho_2) = F^*(\rho_2, \rho_1) =$$

$$= \langle G(\rho_1, \mathbf{r}, z)\, G^*(\rho_2, \mathbf{r}, z) \exp i \{\varphi_q(\rho_1, \mathbf{q}) - \varphi_q(\rho_2, \mathbf{q})\}\rangle\,, \quad (3)$$

and asterisk denotes the complex conjugate. In formulas (2) and (3) the aiming errors, uncertainties in the position of the receiving aperture and its shape are taken into consideration by the probability distribution function $P(\mathbf{r}, z, \mathbf{q})$ of the aiming error $\mathbf{q}$ and condition that the points $(\mathbf{r}, z)$ belong to the receiving aperture. The function $P(\mathbf{r}, z, \mathbf{q})$ is normalized so that $\int P(\mathbf{r}, z, \mathbf{q})\, d\mathbf{r} d\mathbf{q}/S = 1$ and S is the area of the receiving aperture. If the aiming errors are absent and the position of the receiver in the Fresnel diffraction zone is reliably known,

$$G(\rho_1, \mathbf{r}, z) = (k/2\, i\, \pi z) \exp i \{kz + (k/2\, z)(\rho_1 - \mathbf{r})^2 + \Psi(\rho_1; \mathbf{r}, z)\}\,,$$

and in this case

$$F(\rho_1, \rho_2) = (k/2\, \mathrm{p} z)^2 \exp i \{(k/2\, z)(\rho_1^2 - \rho_2^2)\} \times$$

$$\times \int_S d\mathbf{r} \exp i [-(k/z)(\rho_1 - \rho_2)\mathbf{r} + \Psi(\rho_1, \mathbf{r}, z) - \Psi^*(\rho_2, \mathbf{r}, z)], \quad (4)$$

where $\Psi(\rho, \mathbf{r}, z)$ is an additional shift of the complex phase[2] compared to that in a homogeneous medium for a spherical wave passed from the source at the point ρ to the point

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(**r**, z). The function $\Psi(\rho, \mathbf{r}, z)$ is directly related to the inhomogeneities of a medium and this relation has been considered in a number of papers (see, for example, Ref. 2 and bibliography presented there).

Let us consider the value of the power flux (Eq. (2)) as the functional of the phase distribution φ. Using the method of calculus of variations[6] as applied to relation (2) for determining optimal phase distribution maximizing functional (2) for the cases in which there are no nonlinear, with respect to the field, effects we can obtain the following integral equation:

$$\varphi(\rho_1)=\arctan\frac{\int d\rho_2 A(\rho_2)\sin(\varphi(\rho_2)-\arg F(\rho_1,\rho_2))|F(\rho_1,\rho_2)|}{\int d\rho_2 A(\rho_2)\cos(\varphi(\rho_2)-\arg F(\rho_1,\rho_2))|F(\rho_1,\rho_2)|}; \qquad (5)$$

$$F(\rho_1,\rho_2) = |F(\rho_1,\rho_2)|\exp i \arg F(\rho_1,\rho_2);$$

$$\arg F(\rho_1,\rho_2) = -\arg F(\rho_2,\rho_1).$$

By means of algebraic transformations equation (5) can be reduced to

$$\varphi(\rho_1) = -\arg\int d\rho_2 A(\rho_2)\{\exp - i\varphi(\rho_2)\}\, F(\rho_1,\rho_2). \qquad (6)$$

Equations (5) and (6) generalize the law of the phase conjugation for the stochastic optical fields. In the general case, when determining the power flux and the function $F(\rho_1, \rho_2)$ the effects of all these factors which cannot be taken into account in real time when optimizing the PD are subjected to stochastic and (or) determined averaging. In addition to the aiming errors and uncertainties due to the receiver position, the random errors in forming the compensating phase distribution and, uncontrollable, as a rule, fluctuations of the field amplitude distribution $A(\rho)$ over the plane of the output aperture of the optical transmitter can be mentioned among these factors.

Relations (5) and (6) are the initial ones for studying the optimal PD. In the general case an analytical solution of the obtained equations cannot be found, therefore, an iteration method can be used for solving them numerically. Below we shall consider only some particular cases of Eqs. (5) and (6).

1. Small residual errors of compensation. The measure of compensation of undesirable factors, as it follows from analysis of Eqs. (2) and (5), is the value

$$\sigma_w^2 = \frac{\int\int d\rho_1 d\rho_2 A(\rho_1) A(\rho_2)\, Q\, |F(\rho_1,\rho_2)|}{2\int\int d\rho_1 d\rho_2 A(\rho_1) A(\rho_2) |F(\rho_1,\rho_2)|}. \qquad (7)$$

where $Q = \{\varphi(\rho_1) - \varphi(\rho_2) - \arg F(\rho_1,\rho_2)\}^2$. The value σ_w^2 is related to the power flux by the approximate formula

$$W \simeq \exp(-\sigma_w^2) \int\int d\rho_1 d\rho_2 A(\rho_1) A(\rho_2) |F(\rho_1,\rho_2)|,\ \sigma_w^2 \ll 1. \qquad (8)$$

Formulas (7) and (8) for W and σ_w^2 are obtained by separating out the phase component from the integrand in relation (2), and by expansion of the corresponding exponential function into the Taylor series over the phase value up to the second power terms and by the inverse transformation of the obtained expression into the exponential function. Because of changes in the controllable phase distribution $\varphi(\rho)$ the value σ_w^2 is directly related to W, i.e., the value W increases (decreases) if σ_w^2 decreases (increases). If a source of the field phase distortions is inside the transmitter and $\psi(\rho, \mathbf{r}, z) = 0$ and $\varphi_q(\rho, \mathbf{q}) = 0$, the value calculated by Eq. (7) for a point receiver takes the form of the quality index σ^2 (mean square of residual phase distortions[3]) while for an extended receiver it takes the form of the index g^2 (mean square of the gradient of the residual phase distortions[7]). These conclusions have been drawn based on the fact that the function $F(\rho_1, \rho_2)$ entering into Eq. (7) and determined by relation (4) is comparable with S for a receiver close to a point one but the function $F(\rho_1, \rho_2)$ can be approximated by $\delta(\rho_1 - \rho_2)$ in the case of an extended receiver. Thus, the value σ_w^2 can be considered as the energy index of quality of the optical transmitter of a communication system.

For small values of the index of Eq. (7) Eq. (6) becomes linear with respect to the function of the optimal PD and takes the form of the integral equation

$$\varphi(\rho_1) \simeq \frac{\int d\rho_2 A(\rho_2)\,[\varphi(\rho_2) - \arg F(\rho_1,\rho_2)]\,|F(\rho_1,\rho_2)|}{\int d\rho_2 A(\rho_2) |F(\rho_1,\rho_2)|}. \qquad (9)$$

Equation (9) can also be derived (using the variation method) based on relation (7) and the condition of the minimum of σ_w^2. Relation (9) is reduced to an nonhomogeneous integral equation of the second kind, the methods of its solution are developed in Ref. 8.

Relations (7)–(9) make it possible to use already known techniques[3] of investigations of controllable optical systems in order to find optimal signals controlling the actuators of the phase corrector and to estimate the residual error and the efficiency of compensation taking the phase corrector characteristics into account. Let, for example, a deformable mirror be used to form an optimal PD. In this case[3]

$$\varphi_c(\rho) = \sum_1^N \alpha_i R_i(\rho), \qquad (10)$$

where $R_i(\rho)$ is the response function of the ith actuator, a_i is the amplitude of the control signal applied to the ith actuator, and N is the total number of actuators of the deformable mirror. Substituting Eq. (10) into Eq. (7) by virtue of the relation $\varphi = \varphi_c + \varphi_s$ and minimizing σ_w^2 as a function of a_i for optimal values a_i we obtain that

$$\|\alpha_i\| = \|RR\|^{-1}\,\|R\varphi\|, \qquad (11)$$

here $\|\alpha_i\|$ and $\|R\varphi\|$ are the matrix columns, $\|RR\|^{-1}$ is the inverse matrix,

$$\|RR\| = \|\int\int d\rho_1 d\rho_2 A(\rho_1) A(\rho_2)\, R_n(\rho_1) \times$$

$$\times [R_m(\rho_1) - R_m(\rho_2)]\,|F(\rho_1,\rho_2)|\,\|, \qquad (12)$$

$$\|R\varphi\| = \|\int\int d\rho_1 d\rho_2 A(\rho_1) A(\rho_2)\, R_m(\rho_1) \times$$

$$\times [\varphi_s(\rho_1) - \varphi_s(\rho_2) + \arg F(\rho_1,\rho_2)]\,|F(\rho_1,\rho_2)|\,\|, \qquad (13)$$

$m, n = 1, 2, \ldots, N$. If Eq (11) is valid the compensation error is minimum and equal to

$$\min \sigma_w^2 = \sigma_{ws}^2 \Big\{ 1 - \|R\varphi\|^{\mathrm{T}} \|RR\|^{-1} \|R\varphi\| \times$$

$$\times 2 \Big[\int\int \mathrm{d}\rho_1 \mathrm{d}\rho_2 A(\rho_1) A(\rho_2) \times$$

$$\times \left(\varphi_s(\rho_1) - \varphi_s(\rho_2) + \arg F(\rho_1, \rho_2)\right)^2 |F(\rho_1, \rho_2)| \Big]^{-1} \Big\}, \quad (14)$$

Superscript T denotes the transposition procedure, σ_{ws}^2 is the value of the quality index in the absence of compensation ($\|a_i\| = 0$), and the value σ_{ws}^2 is calculated by Eq. (7) taking into account that $\varphi(\rho)$ is replaced by $\varphi_s(\rho)$. Further investigations require that the PD character of the field inside the receiver (either random or regular) and inhomogeneities of the medium, the methods of measuring the field characteristics, and the methods used to control the phase corrector be taken into account.

2. Nearly point receiver of radiation. If there are no uncertainties in the receiver position and errors in aiming relation (4) is reduced to the form

$$F(\rho_1, \rho_2) \simeq (k/2\pi z)^2 S \exp i \Big\{ (k/2z)(\rho_1^2 - \rho_2^2) -$$

$$- (k/z) \int_S \mathrm{d}\mathbf{r} [(\rho_1 - \rho_2)\mathbf{r}]/S + \int_S \mathrm{d}\mathbf{r} [\Psi(\rho_1, \mathbf{r}, z) - \Psi^*(\rho_2, \mathbf{r}, z]/S \Big\}. \quad (15)$$

Formula (15) is derived using a linear expansion of the exponential integrand from Eq. (4) into the Taylor series, taking an integral of this expansion over **r**, and then by performing the inverse transform of the resulting relation into an exponential function. Relation (15) is valid when $kS^{1/2}D/4z \ll 1$ (D is the diameter of the transmitter aperture), while the scale of inhomogeneities of Ψ exceeds $S^{1/2}$ and/or when $|\Psi| \ll 1$. Substituting Eq. (15) into Eq. (5) or Eq. (8) we can find the solution

$$\varphi(\rho) \simeq - (k/2z)\rho^2 + (k/z) \int_S \mathrm{d}\mathbf{r}(\rho\mathbf{r})/S - \int_S \mathrm{d}\mathbf{r}\, \mathrm{Re}\Psi(\rho_1, \mathbf{r}, z)/S. \quad (16)$$

For a point receiver relation (16) results in the known expression

$$\varphi(\rho) = - (k/2z)\rho^2 - \mathrm{Re}\Psi(\rho_1, 0, z).$$

The first term in the Eq. (16) for the optimal phase distribution describes an optimal radiation focusing on a point receiver in a homogeneous medium; the second term is related to asymmetry of the receiving aperture and coordinates the position of a beam energy center with the aperture shape. The third term of Eq. (16) is responsible for the law of forming the field phase distribution at the transmitter output compensating for the medium inhomogeneities in an optimum way according to the criterion of a maximum power flux transmitted to the receiver located in the Fresnel diffraction zone.

3. Partial compensation for the diffraction effects. Let the medium of radiation propagation be homogeneous, without errors in aiming and uncertainty of the receiver position. In the case of a linear aperture (a slit–like beam along the Y axis; $\rho = (X, Y)$) and taking into account Eqs. (3) and (4) and the imposed restrictions Eq. (5) takes the form

$$\varphi(x_1) = \arctan \frac{\int \mathrm{d}x_2 A(x_2) \sin(\varphi(x_2))[\sin(\omega(x_1 - x_2))/(x_1 - x_2)]}{\int \mathrm{d}x_2 A(x_2) \cos(\varphi(x_2))[\sin(\omega(x_1 - x_2))/(x_1 - x_2)]}, \quad (17)$$

where $\omega = kD_p D/4z$, D_p is the size of a slit–like receiver; D is the size of the transmitting aperture; and, $x = 2X/D$. From the very beginning the quadratic component $(-(kD^2/8z)x^2)$ of the optimum phase, which determines an optimum focal length of the optical transmitter for focusing radiation onto a point receiver, is excluded from Eq. (17). For $\omega = 0$ there exists a solution of Eq. (17) that can be written as $\varphi(x) = \mathrm{const}$, what well agrees with the results of analysis carried out in Sec. 3. For $\omega \Rightarrow \infty$ the function between brackets in Eq. (17) approaches δ–function, therefore, any function $\varphi(x)$ gives the solution of Eq. (17). For the finite values of ω there exist several solutions of Eq. (17) in addition to the obvious one $\varphi(x) = \mathrm{const}$.

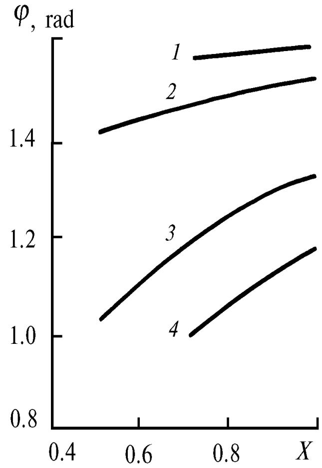

FIG. 1. Optimum phase distribution partially compensating for diffraction effects; $\varphi(x) = -\varphi(-x)$. *1)* $\omega = 2.2$, $\delta W/W = 18\%$; *2)* 2.7, 40%; *3)* 2.2, 22%; *and, 4)* 1.9, 4%.

If the function of the amplitude distribution $A(x)$ is even, the solutions can be grouped into the symmetric solutions $\varphi(x) = \varphi(-x)$ and the asymmetric ones $\varphi(x) = -\varphi(-x)$. To verify these conclusions Eq. (17) has been solved numerically by an iterative method. The functions of optimum phase distribution are shown in Fig. 1 for the case of uniform amplitude distribution of the field over the linear aperture with the symmetric central screening. If the value of ω is within the range from 2 to 4, the optimum PD is nearly linear $\varphi(x) \sim x$, maximum deviation of the phase from a uniform one amounts from 2 to 3 radians or 1/2 ... 1/3 in the units of radiation wavelength. Relative increase δW/W of the radiation power flux reaches 40% in comparison with the case of uniform phase distribution and depends on the receiver size.

The effect of maximization of the radiation power flux for the case of nearly linear PD is shown in Fig. 2, which represents the radiation intensity of the receiver aperture as a function of the normalized coordinate ω. If the receiver aperture is limited by the points I certain displacement of the receiver to the right or to the left, what corresponds to the case of a linear phase distribution over the transmitting aperture, does not result in considerable changes of the radiation power flux because a decrease (increase) of radiation power on the right part of the aperture is compensated for by practically the same increase (decrease) on its left part. If the receiving aperture is limited by points II then a small displacement of the receiver will lead to an increase of the radiation power flux. It can be explained by the fact that when one of the edges of the receiver aperture gets into the region of high power density, while the opposite edge of the receiver aperture reaches the region of low power density and does not leave its boundaries.

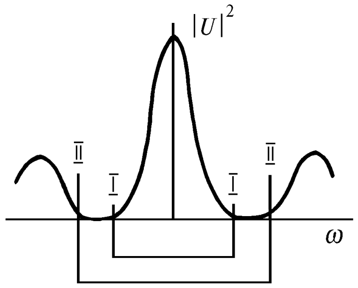

FIG. 2.

It should be noted that the results presented in Sec. 3 are equivalent to the following statement: to maximize the power flux through an extended receiver a nonparabolic forming mirror is needed for transmitting coherent radiation through a homogeneous medium.

4. Partial compensation for random uncertainties in the receiver position or angular errors in aiming. Let the field formed by a linear emitter (with infinite size along the Y axis) be transmitted through a homogeneous medium to a point receiver. The position of the receiver in the direction of the X axis is known correctly to a constant within the interval $[-D_p/2, D_p/2]$ of the probability distribution function $P(X)$, the distance to the receiver aperture is equal to z. In this case the function

$$F(x_1, x_2) \sim \{\exp i(kD^2/8z)(x_1^2 - x_2^2)\} \sin(\omega(x_1 - x_2))/(x_1 - x_2)$$

entering into Eq. (5) and being determined by relations (3) and (4) as well as Eq. (5) for finding the optimum phase distribution in view of uncertainties in the receiver position take the form of integral equation (17) for $\omega = kD_p D/4z$; $x = 2x/D$. The same equation results from the investigation of a compensation for the aiming errors (if there are no uncertainties in the receiver position). In this case the phase distortions of the field equivalent to the angular errors in aiming can be represented in the form[1] $\varphi_q(x) = (k q_x D/2) x$, q_x is the random angular error in aiming (in radians) uniformly distributed over the interval $[-q_{x0}, q_{x0}]$; $|q_{x0}| \ll 1$; $\omega = k |q_{x0}| D/2$. Thus, the results of analysis of Eq. (17) presented in Sec. 3 are applicable to investigations of the partial compensation for uncertainties in the point receiver position or the aiming errors. Hence, to provide a maximum statistical mean power density at the receiver for ω | 2 ... 4 it is reasonable to introduce the PD similar to the additional static angular error in aiming. However, it should be taken into account that the result of PD optimization depends strongly on the probability distribution function $P(\rho, z; q)$, which can be found from the theoretical and (or) experimental investigations[1] of an individual communication system.

Conclusions. The integral equation generalizing the law of phase conjugation for the random optical fields is obtained to determine the phase distribution (PD) of the field that maximizes the power flux through a randomly disposed receiver when transmitting radiation through an inhomogeneous medium with the aiming errors. The investigations were carried out using the Green's functions for a field in an inhomogeneous medium and the variational method. It was shown that the optimum phase distribution can be represented in the form of a sum of three terms. One of them describes the optimum focusing onto a receiver in a homogeneous medium. The second one represents the compensation for medium inhomogeneities. The third term describes the distribution of the power density over the beam cross section in accordance with the position and shape of the receiver aperture. In the absence of medium inhomogeneities the optimization of phase distribution makes it possible to partially compensate for the diffraction effects, random aiming errors, and uncertainties in the receiver position as well as to increase the power flux of radiation through the receiver by 10%.

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