NUMERICAL SIMULATION OF NONLINEAR RESONANCE SPECTRA IN APERTURE-LIMITED LIGHT BEAMS

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The frequency dependence of the energy losses of a spatially limited beam under conditions of strong saturation is analyzed. The result presented agree well with experiment and give a detailed picture of the evolution of a beam in a resonantly absorbing medium.

The use of powerful cw or pulsed lasers in atmospheric-optic experiments is stimulating the development of numerical methods for calculating the changes produced in the characteristics of the medium and the radiation itself by self-action.

The changes occurring in the characteristics of laser beams on extended atmospheric paths with a nonlinearity – are described with the help of algorithms employing conservative difference schemes,[1,2] and methods of solution based on the Fourier-Bessel transformation have recently been developed for axisymmetric beams.[3–6]

An improved computational algorithm of the type given in Ref. 6 is proposed in Ref. 7, where with the help of this algorithm the problem of the propagation of an axisymmetric beam in a transparent nonuniform medium with a cubic nonlinearity is solved.

The detailed description given in Ref. 7 of the computational scheme also permits analyzing the behavior of an axisymmetric, spatially limited, light beam in a nonlinear atmospheric channel with absorption and subject to saturation as the beam intensity increases.

The problem of the propagation of an initially Gaussian beam of light in a gas of two-level molecules under conditions of strong saturation of absorption and dispersion near resonance was solved in Refs. 8–10, where the effect of the nonlinearity on the geometric characteristics of the beam was determined. The frequency dependence of the energy losses of a spatially limited beam under conditions of strong saturation was not analyzed. In this paper this problem is solved using the computational scheme that we proposed in Ref. 7.

Using the well-known form of the resonance susceptibility of a two-level system we shall represent Eq. (1) of Ref. 7 for the field E of a light wave in the medium in the form

$$4i\frac{\partial E}{\partial z} + \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right]E = -4\kappa L_D \frac{\delta + i}{1 + \delta^2 + S|E|^2}E, \quad (1)$$

where κ is the linear absorption coefficient; $L_D = 4\pi a^2/\lambda$ is the diffraction length, which is used as the unit of length for the longitudinal dimensionless coordinate z; a is the characteristic radius of the beam, which is also the unit of length for the transverse coordinate r; S is the saturation parameter; $\delta = (\omega - \omega_0) / \tau$ is the detuning of the light frequency ω from the transition frequency ω_0 in units of the homogeneous width γ of the transition line; and, the value of the field at the point of entry into the medium at the center of the beam is used as the unit of measurement of the field E.

The following quantities were calculated as the characteristics of the shape and spectrum of the beam:

1) the light intensity on the beam axis $I_\omega = |E|^2 = I(r, z, \delta)$;

2) the total beam power $W(z, \delta) = \int_0^\infty I(r, z, \delta) 2\pi r dr$; and,

3) the average beam cross section

$$\sigma^2(z, \delta) = \int_0^\infty r^2|E|^2 2\pi r\, dr \Big/ \int_0^\infty |E|^2 2\pi r\, dr.$$

At the point of entry into the medium the beam was assumed to have a Gaussian profile $E(z = 0, r) = \exp(-\pi r^2)$ and a flat wavefront. Strong absorption ($\kappa L_D = 100$) and strong saturation ($S = 100$) were studied. The choice of parameters r_1, ρ_1, α, and N is largely arbitrary and is determined by two requirements: 1) $E(r)$ and $E(\rho)$ must be small at the boundaries of the transverse grid and at the same time 2) the step along z and ρ must be small enough so as to convey correctly the details of the profile and spectral image of the beam. The calculations showed that in this problem the inverse and not the direct Fourier-Bessel transform is most erratic. The beam profile itself, because of the high absorption at the edges, is an ideal function for the discrete Fourier-Bessel transform: it is smooth and decreases rapidly as r increases. As regards the spectral image, the nonlinear distortions of the beam cause the contribution of the high-order harmonics with large values of ρ to increase. This makes it necessary to increase the parameter ρ_1, sacrificing in so doing the accuracy of the approximation owing to the coarsening of the grid in ρ space. The values $r_1 = 0.08$, $\rho_1 = 0.10$, $\alpha = 0.03$, and $N = 128$ were chosen empirically. When the parameters r_1 and ρ_1 were varied from 0.05 to 0.10 and from 0.06 to 0.12, respectively, the values of I differed by not more than 10% and the values of W and σ^2 differed by not more than 1–2%. When the number of points in the transverse direction were doubled and the step size in the r and ρ directions were correspondingly decreased and the step along z was also changed by a factor of 2 5 the agreement of the results was even better.

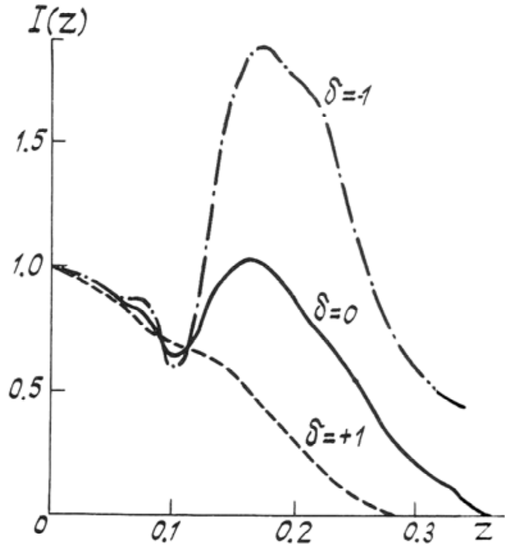

FIG. 1. The longitudinal dependence of the intensity on the axis of the beam for three values of the frequency detuning.

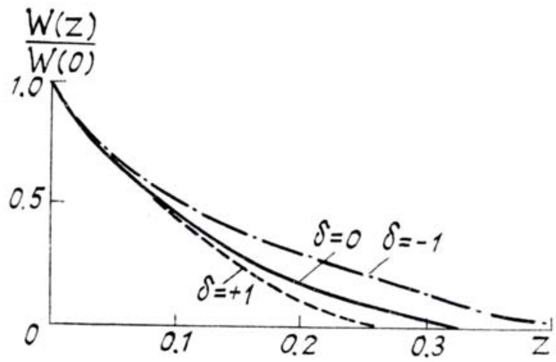

FIG. 2. The total beam power drops along the z axis more rapidly for δ = + 1 (defocusing) and more slowly for δ = –1 (focusing).

Analysis of the calculations shows that the shape and spectrum of the beam evolves along the axis under the influence of several basic factors: 1) diffraction spreading; 2) nonlinear absorption, which forms a channel for bleaching of the medium; 3) diffraction of the Fresnel type by the induce diaphragm formed; and, 4) nonlinear refraction. The latter process gives rise to self-focusing the beam if $\delta < 0$ and self-defocusing if $\delta > 0$. These effects are appreciable in the longitudinal dependence of the axial intensity of the beam (Fig. 1). At the same time diffraction by the bleaching channel (induced distributed diaphragm) occurs and is responsible for the signifies increase in the light energy density on the axis of the beam even in the case $\delta = 0$, when there is no refraction. This effect was observed in Refs, 8 and 9. The total power in the beam (Fig. 2) naturally, diminishes monotonically along the z axis. The sign of the frequency detuning affects the rate of this diminishment because in the presence of focusing the bleaching of the medium increases. The diffraction nature of the enhancement of the intensity on the axis of the beam becomes especially obvious when the beam profile is analyzed (Figs. 4–6). The dips in the curves $I(z)$ correspond to the appearance of a ring structure of the beam with a profile having a minimum at the center (Figs. 4 and 5). Then, with an overall decrease of the area, the beam profile becomes markedly narrowed and sharper, after which significant overall attenuation of the beam occurs, and the further propagation of the beam occurs in the regime closer to the linear regime. Under the conditions of defocusing (Fig. 4) a dip is not formed at the center of the beam profile, and the corresponding curve of the longitudinal change in the axial intensity in Fig. 1 decreases monotonically. In all cases (Figs. 4-6) the intensity at the periphery of the beam approaches zero much more rapidly than the Gaussian exponential; this is caused by the very strong absorption of the field outside the self-bleaching channel ("stripping effect" in the terminology of Refs. 8 and 10). This effect is clearly seen in Fig. 3, where for all values of the frequency detuning the average beam cross section at first decreases, in spite of the fact that the diffraction spreading and, in the case $\delta = +1$, nonlinear refraction deflect the rays away from the axis of the beam toward the periphery. As the beam generally weakens the divergence of the rays starts to compensate the "stripping effect" and the average cross section is stabilized, while the power (Fig. 2) continues to decrease monotonically Finally, at low powers the diffraction spreading starts to dominate, and the average cross section of the beam increases. Since by this time the radius of the beam is much smaller than it is in the point of entry, the diffraction spreading is significantly greater than it would be if the beam were initially Gaussian (a Gaussian beam is characterized by a diffraction length of $1/\pi$, over which in free space the cross section of the Gaussian beam $\exp(-\pi r^2)$ doubles. The expansion of the beam occurs earliest for $\delta = +1$ and somewhat later for $\delta = 0$. For $\delta = -1$ refraction in the region of values studied compensates the increase in the cross section.

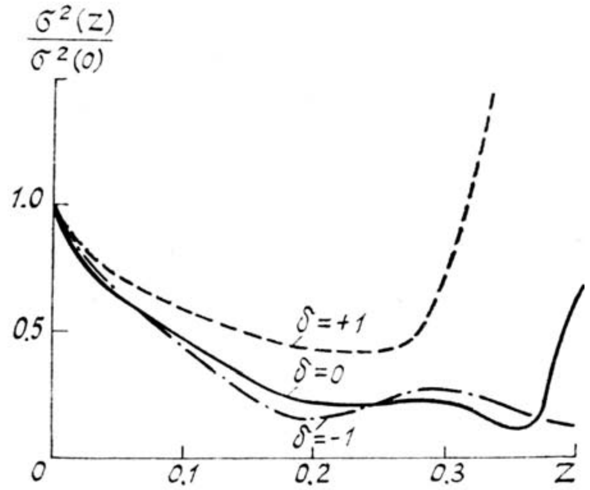

FIG. 3. The "stripping effect", nonlinear refraction, and diffraction spreading which appear in the changes in the average cross section of the beam for three values of the frequency detuning.

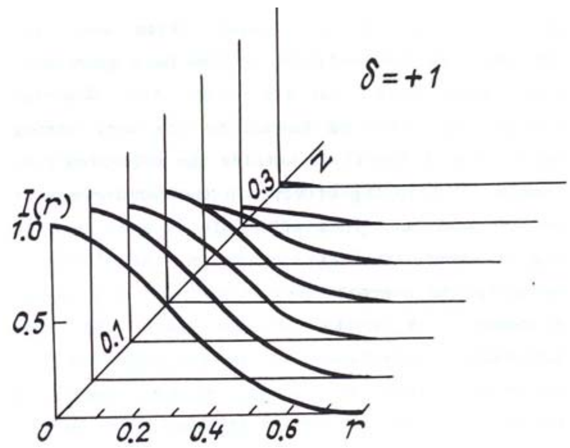

FIG. 4. The evolution of the beam profile as the beam propagates into the medium in the case of defocusing (δ = + 1).

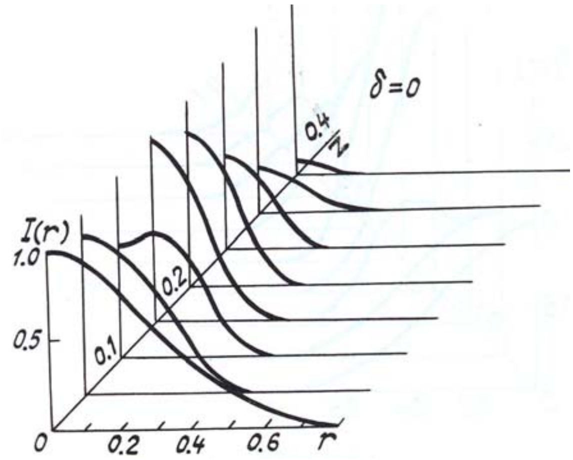

FIG. 5. The evolution of the beam profile as the beam propagates into the medium. At z = 0.1 a dip can be seen at the center. There is no nonlinear refraction (δ = 0) and all changes are associated with diffraction by the induced diaphragm.

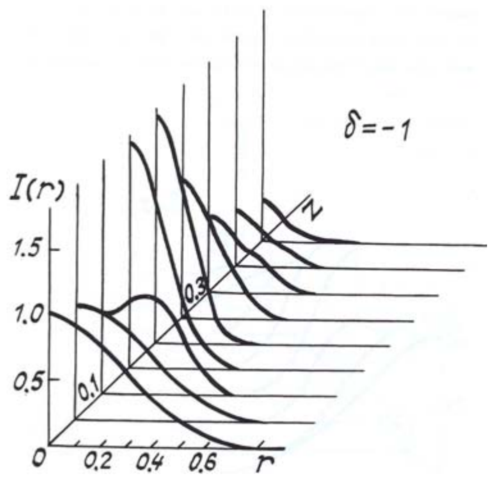

FIG. 6. The longitudinal evolution of the beam profile under combined action of self-diffraction and self-focusing (δ = −1). Self-focusing increases both the depth of the dip at z = 0.1 and the subsequent peaking of the beam profile.

The result presented are in good agreement with experiment and the theoretical concept presented in Refs. 8, 10, and 11, and they make it possible to reconstruct the detailed picture o the evolution of the beam in a resonant) absorbing medium. An important difference between our investigations and those of Refs. 8, 10, an 11 on resonance self-action is that we an interested primarily in the spectroscopic aspect of this problem. The basic problem which « consider below is to determine the difference between the strongly saturated absorption spectrum in a plane wave and in an initially Gaussian beam. As far as we know, such spectral studies have never been performed.

Figures 7 and 8 show the frequency spectra of the intensity on the axis and the total power W at different distances from the point of entry into the medium. Under the conditions of a plane wave the spectrum I would have the well-known Lorentzian shape, broadened by saturation approximately up to 10γ, where γ is the homogeneous width of the unsaturated-transit ion line. Nonlinear refraction and self-diffraction markedly change the line shape. The maximum absorption (minimum Intensity of the transmitted light) is significantly shifted into the region of low frequencies. In the region of higher frequencies, however, there appears a spectral peak of the transmission; the maximum intensity in this peak reaches a value more than two times higher than the maximum intensity at the inlet (curve 4 in Fig. 7). It is interesting that there is a clear correlation between the shape of the spectrum and the shape of the profile of the transverse distribution of the intensity. For example, when a dip is present at the center of the beam profile (Figs. 5 and 6) two maxima and two minima appear in the intensity spectrum (curve 2 in Fig. 7). All these intricate changes in the spectrum are caused by transverse redistribution of the radiation intensity; the character of this redistribution is determined by the detuning δ. Since the total power does not change significantly in the process, the power spectrum should not be affected as strongly by self-action effects. Figure 8 shows that this is indeed so. However these effects give rise to appreciably asymmetry and a significant red shift of the saturation absorption line by an amount of the order of γ and more. The obvious reason for this shift is that the concentration of radiation in the region of the beam near the axis owing to self-focusing for $\delta < 0$ gives rise to bleaching of the medium, and hence increases W.

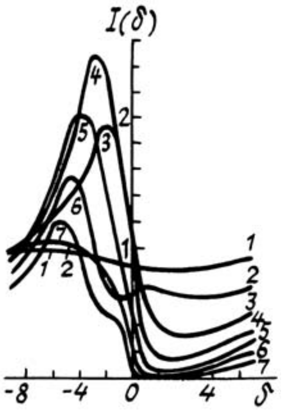

FIG. 7. The spectrum of the intensity on the axis of the beam at different distances from the point of entry into the medium.

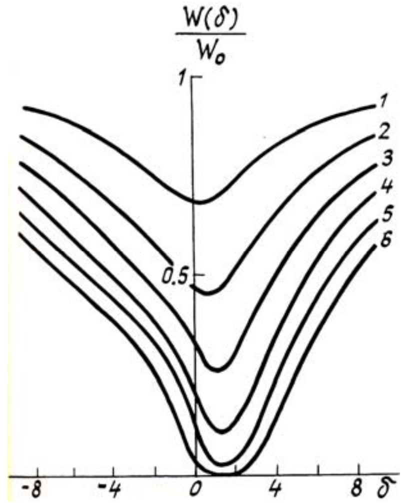

FIG. 8. The spectrum of the total beam power at different distances from the point of entry into the medium.

Thus the self-action-induced asymmetry of the spectral lines can strongly affect the result of spectral measurements in the case of high beam intensities. In particular, the shift in the absorption line increases substantially as the thickness of the absorbing layer increases (Fig. 8). The size of the radiation receiver is important, since the spectrum of the total power is altogether different from the power spectrum for the narrow part of the beam near the axis. The latter region is especially sensitive to self-action effects. The spectrum there can be changed so as to be unrecognizable by simply increasing the thickness of the absorbing layer. All this leads to the conclusion that in the spectroscopy of saturated absorption, in particular, in Doppler-free spectroscopy as well as when probing extended gaseous media with intense light beams, self-action effects must be carefully taken into account.

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