ON INVARIANTS FOR THERMAL BLOOMING OF OPTICAL RADIATION

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Some invariants and invariant relationships are derived for describing thermal blooming of both light beams propagating through a moving medium and light pulses propagating through a stationary medium. These invariants can be used for checking the results of numerical simulation of thermal blooming of optical radiation.

The invariants of nonlinear self–action of optical radiation are known to be very important for problems of analysis of radiation properties and of data obtained by numerical simulations as well as for the construction of difference schemes. Computer simulation of thermal blooming of light beams has been used for many years for solving the atmospheric optics problems (see, e.g., Refs. 1–5). The invariant of these problems is the initial power (or energy) of optical radiation. In the case of stationary thermal blooming the other values have also been found that remained unchanged in the course of self–action,[6] and the energy conservation laws derived for these values, have been generalized for the case of interaction of two beams propagating in the opposite directions.[7] In contrast to these studies, we present the new integral invariant relations for thermal blooming of light beams propagating through a moving medium and of pulses propagating through a stationary medium.

The process of thermal blooming of optical radiation in a transparent regular medium may be described, within the quasioptical framework of approximation, by a system of dimensionless equations

$$\frac{\partial A}{\partial z} + i\Delta_\perp A + i\alpha TA = 0,\ LT = |A|^2 , \qquad (1)$$

where A is the complex amplitude of the beam normalized by its peak intensity, z is the longitudinal coordinate measured in units of diffraction length ($t_d = 2\kappa a^2$), k is the wave number, a is the initial radius of the beam, $\Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the transverse Laplacian operator, x and y are the transverse coordinates normalized by a, α is the ratio of the initial beam power to characteristic power of thermal blooming, T is the normalized change in temperature of the medium, and L is the linear operator depending on the relation between the parameters of the medium and those of the beam. We have

$$L = \frac{\partial}{\partial x}\ ,\ L = \frac{\partial}{\partial t} - \chi\Delta_\perp\ ,\ L = \frac{\partial}{\partial t} \qquad (2)$$

in the cases of stationary propagation of optical radiation through the moving medium and of the nonstationary self–action with and without an account of thermal diffusion, respectively. The variable t in Eq. (2) is normalized time, χ characterizes the diffusion of heat from the region occupied by the beam. System of equations (2) must be completed by the initial and boundary conditions for A and T which have the form

$$A = A_0(x, y, t),\ A|_{x=0,\,L_x} = A|_{y=0,\,L_y} = 0,\ A|_{t=0} = 0\ ; \qquad (3)$$

for the complex amplitude of the beam, and

$$T_{x=0} = 0,\ T_{x=0,\,L_x} = T|_{y=0,\,L_y} = T|_{t=0} = 0,\ T|_{t=0} = 0 \qquad (4)$$

for temperature, in accordance with operator (2). In the case of stationary self–action, the complex amplitude is independent of time and the last condition of Eq. (3) is dropped. The symbols L_x and L_y in Eqs. (3) and (4) denote the boundaries of the considered region along the coordinates x and y.

For simplicity, we first consider the case of stationary propagation of a slit–shaped beam through the moving medium (the coordinates (x, z)). Multiplying TA^* – the equation of quasioptics and TA – the equation conjugate to it and integrating them over the transverse coordinate, we finally derive

$$\int\limits_0^{L_x} T\frac{\partial|A|^2}{\partial z}\,dx + 2\int\limits_0^{L_x} |A|^2\,\mathrm{Im}\left(A^*\frac{\partial A}{\partial x}\right)dx = 0\ . \qquad (5)$$

Furthermore, integrating the equation describing changes in temperature and differentiating its left and right parts with respect to z, we easily derive the following relation from the equation of quasioptics:

$$\frac{\partial T}{\partial z} = 2\,\mathrm{Im}\left(A^*\frac{\partial A}{\partial x}\right) \qquad (6)$$

or

$$T(z, x) = T(0, x) + 2\int\limits_0^z \mathrm{Im}\left(A^*\frac{\partial A}{\partial x}\right)d\xi\ . \qquad (7)$$

Equation (5) then assumes the following form:

$$\int\limits_0^{L_x}\left(T(0, x) + 2\,\mathrm{Im}\int\limits_0^z A^*\frac{\partial A}{\partial x}\,d\eta\right)\frac{\partial|A|^2}{\partial z}\,dx + $$
$$+\int\limits_0^{L_x}|A|^2\frac{\partial}{\partial z}\left(T(0, x) + 2\,\mathrm{Im}\int\limits_0^z A^*\frac{\partial A}{\partial x}\,d\eta\right)dx = 0. \qquad (8)$$

Hence, we derive the integral equation

$$I_1 = \int\limits_0^{L_x}|A|^2\frac{\partial}{\partial z}\left(T(0, x) + 2\,\mathrm{Im}\int\limits_0^z A^*\frac{\partial A}{\partial x}\,d\eta\right)dx = \mathrm{const}, \qquad (9)$$

which is also equivalent to the equality

$$I_2\, T^2(z, L_x) = \mathrm{const}\ . \qquad (10)$$

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Note that using Eq. (5), we may easily write another integral relation

$$I_3 = \int_0^{L_x} \left|\frac{\partial A}{\partial x}\right|^2 \mathrm{d}x + 2\alpha \int_0^{z}\int_0^{L_x} |A|^2 \mathrm{Im}\left(A^* \frac{\partial A}{\partial x} \right) =$$

$$\mathrm{d}x\mathrm{d}\eta = \mathrm{const}\ . \qquad (11)$$

In analysis of the regularities of propagation of a two–dimensional beam along the coordinates (x, y, z), system (5)–(11) acquires a different form. Thus, we obtain instead of Eq. (5)

$$\int_0^{L_x}\int_0^{L_y} \left\{ \mathrm{T}\, \frac{\partial |A|^2}{\partial z} + \right.$$

$$\left. + 2\left(\frac{\partial T}{\partial x} \mathrm{Im}\left(A^* \frac{\partial A}{\partial x}\right) + \frac{\partial T}{\partial y} \mathrm{Im}\left(A^* \frac{\partial A}{\partial y}\right)\right)\right\} \mathrm{d}x\mathrm{dy} = 0\ .$$

(12)

Then I_3 is reduced to the form

$$I_3 = \int_0^{L_x}\int_0^{L_y} \left\{ |\nabla_\perp A|^2 + \quad 2\alpha \int_0^{z} \left(\frac{\partial T}{\partial x} \mathrm{Im}\left(A^* \frac{\partial A}{\partial x} \right) + \right.\right.$$

$$\left.\left. + \frac{\partial T}{\partial y} \mathrm{Im}\left(A^* \frac{\partial A}{\partial y}\right)\right) \mathrm{d}\eta \right\} \mathrm{d}x\mathrm{d}y\ , \quad \nabla_\perp = \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right\}\ , \qquad (13)$$

and the relation

$$I_2 = \int_0^{L_x}\int_0^{L_y} T\ |A|^2\ \mathrm{d}x\mathrm{d}y +$$

$$+ 2\int_0^{z}\int_0^{L_y} \left.\frac{\partial T}{\partial y}\right|_{x=L_x} \int_0^{L_x} \mathrm{Im}\left(A^* \frac{\partial A}{\partial x} \right) \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}\eta \qquad (14)$$

is derived for I_2, which is more complicated then Eq. (10). The relation for I_1 is reduced similarly.

In the case of nonstationary thermal blooming (when $L = \frac{\partial}{\partial t}$ or $L = \frac{\partial}{\partial t} - \chi\, \Delta_\perp$ is chosen from Eq. (2)) Eq. (13) remains valid at any arbitrary time and in general. However, we fail to write any other integral invariants.

In conclusion, it should be noted that numerical experiments were made for a nonlinear symmetric scheme,[8] aimed at studying the conservation of I_3 (see Eq. (13)), for a beam propagating through the moving medium. In particular, setting $|\alpha| = 20$, $z = 0.5$, the number of grid nodes along the transverse coordinates $N_x = N_y = 32$, and along the z axis $N_z = 10$, we found that I_3 changed by no more than 1% of its initial value at $z = 0$ if the relative accuracy of the iterative process was $\varepsilon = 0.01$. The beam profile upon entering the nonlinear defocusing medium was prescribed to be Guaussian

$$A_0(x,\ y) = \exp\left(-2(x - L_x/2)^2 - 2(y - L_y/2)^2 \right)\ , \qquad (15)$$

where $L_x = L_y = 8$. Results of calculations indicated that the invariant relation (13) was valid for the symmetric difference scheme.

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