SOME PROBLEMS OF COMPENSATION FOR NONLINEAR DISTORTIONS OF LIGHT BEAMS: ALGORITHMS FOR CONTROL

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The algorithms currently used to control the parameters of light beams and their implementation to numerical experiments aimed at compensating for nonlinear distortions are reviewed. The reasons of the divergence of the algorithm of phase conjugation are discussed. Beam focusing by this algorithm is shown to depend on the chosen integration step along the longitudinal coordinate. The hysteresis dependences of the optical radiation power on the initial power of the light pulse are obtained.

INTRODUCTION

The compensation for amplitude phase distortions (in particular, nonlinear distortions) is of great interest for such practically important problems as transportation of light energy, optical detection and ranging of various objects, information transfer, etc., and it has been the focus of many studies for the last 15 years.[1-5] Many principles of adaptive and programmable control of the light beam parameters were found with the help of mathematical modeling and computer simulation, which provide the basis for the present paper consisting of four parts.

In the first part we consider the problems of organizing control of the light beam wavefront. The main attention is devoted to the problems which have so far remained outside the scope of reviews and have still received only insufficient study. The following problems should be noted among them: the effect of time delay in control channels and in generation of the response signal on the regime of operation of the adaptive system, the existence of invariants of nonlinear interaction of two waves, the existence of bistable dependences of the power received by a prescribed aperture on the initial beam power in the course of dynamic interaction between the waves, and the methods of numerical simulation.

The second part is devoted to the problem of formation of wavefronts by various adaptive systems, in particular, to some problems of location of the actuators of a flexible mirror (there has been an increasing interest in these problems just now[6,7]) and of the necessary shape of the surface by a segmented mirror or by a system of phase conjugation (PC). The problem is also discussed of the number of modes necessary for efficient compensation of random distortions introduced in the wave by a thin turbulent layer.

In the third part we discuss the choice of the optimal beam profile in the class of prescribed functions (hyper–Gaussian, hyper–tubular, and elliptic) under various conditions along the propagation path. In particular, we discuss the problem of jitter of the energy centre of gravity of a profiled beam after it has passed through a layer of a fluctuating medium. The last part of the review covers a comparatively new class of problems in atmospheric optics: distortions of amplitude phase characteristics of light beams due to the change of the composition of the medium caused by chemical reaction within the region occupied by the beam.

It is well known (see, e.g., Refs. 8 and 9) that the rate of absorption of light energy sharply increases near the resonances of the particular groups of molecules in a gaseous mixture. If the pulse duration significantly exceeds the relaxation time of the excited molecule whose energy transforms into heat, we may assume that the translational temperature of the gaseous mixture and the concentrations of its constituents vary upon exposure to the light beam. It is the approximation which is considered in the present paper. Note also that in addition to the problem of transportation of light energy and of information transfer, such a method can be implemented to the problems of atmospheric sensing and of spectroscopy. For this reason we present a brief review of our previous studies and discuss the peculiarities of numerical simulation of the process of interaction of light beams with a gaseous mixture whose composition changes upon exposure to the light beams.

ALGORITHM FOR CONTROL OF THE LIGHT BEAM PARAMETERS

Here we turn our attention to some peculiarities of calculation of the quality criteria and algorithms for control.

One of the methods of control of the light beam parameters is the multidither algorithm which, in general, is described by a system of either differential or finite difference equations of the form[10,11]

$$L\theta = \hat{\gamma}\mathbf{\Phi}(J(\theta,\ z,\ \alpha))\ , \tag{1}$$

where z is the distance to the receiver normalized to the diffraction length, α is the ratio of the beam power to the characteristic power of self–action, L is the linear operator determined by the adaptive system response function, $\boldsymbol{\theta} = \{\theta_1,\ \dots\ ,\ \theta_{M_0}\}$ is the vector of the light beam parameters being optimized, M_0 is the maximum number of these parameters (i.e., the number of degrees of freedom), $\hat{\gamma}$ is a matrix determining both the current change in the parameter being optimized and the sequence of control θ_j. The vector $\mathbf{\Phi}$ depends on the way of estimating J (the quality of compensation) and on the algorithm implemented to calculate the increment of the functional. In particular, when we employ the gradient methods widely used in the analysis of the systems of cross–aperture sensing,[1–7,10–12] we have $\mathbf{\Phi}_g = \partial J/\partial\boldsymbol{\theta}$. Note that in recent years we have proposed the new methods of optimization of the light beam parameters, which differ from the conventional gradient methods[10,13,14] and are free of some disadvantages typical of the gradient method.

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One more method of organizing control of the wavefronts is based on the algorithm of phase conjugation

$$S_{N+1} = -S^r_N , \qquad N = 0, 1, ..., \tag{2}$$

where N is the iteration number, and S^r is the wavefront of radiation reflected from the object. As was demonstrated in Refs. 15–17, already at moderate nonlinearities the converge of the algorithm substantially degrades. However, since the lack of convergence may be caused by a numerical simulation rather than by the nature of the algorithm (see below), in my opinion it is necessary to study the convergence of the algorithm in ample detail employing a dynamic model of light beam propagation.

During both programmable and adaptive control the method of calculation of the quality criterion is most important. The criterion is used to estimate the results of optimization of the light beam parameters. A number of such methods was proposed in the literature. One of them uses Langrange's identity[18,19] to derive the adjoint system of equations relating the increment to the functional J to the increment to the parameters being optimized. This method is employed for programmable control of the beam parameters.

The second method is based on the calculation of the criterion with the help of the reflected wave,[4,7,16] which propagates in the field of the powerful incident wave. The possible effect of the reflected wave on the medium (i.e., on variation in the refractive index) is neglected. The fundamental property of interaction of counter propagated beams, that is, the symmetry of equations of propagation, is then violated resulting in the absence of invariant[20–22] characterizing the conservation of "phase" of interacting waves. If propagation is nonlinear, this fact undoubtedly affects the process of optimization of the light beam wavefront (i.e., the convergence of the adaptive focusing algorithm). It is also important to emphasize that a stationary model of operation of the adaptive systems (in the sense of propagation of a light beam) was always employed to describe the interaction between the incident and the reflected beams. That circumstance also imposes limitations on the results of modeling in the nonlinear media. This follows from the analogy between the modeling of adaptive focusing of the beam and the PC of the pumping counter propagated beams,[23,24] one of these beams being formed due to the reflection of another beam from a mirror positioned behind the layer of the nonlinear medium: in both cases the iterative process of finding the solution follows the same principles, and the boundary conditions over the cross section of the receiver are interrelated. It is well known[25,26] that in general in a transparent medium one fails to prove the convergence of the iterative process of this kind and may only demonstrate that it remains limited. The numerical simulation of the PC performed by us following the above–indicated scheme demonstrates that these iteration process always converges in the presence of dissipation of optical radiation.

Second, in proceeding from the dynamic model of interaction to the stationary one, the time delay increases because of time needed for the initial wave to pass from the source to the receiver and in the reverse direction (the effect of the additional time delay is discussed below). The role of the time delay in the stationary model is played by the iteration number.

Third, computer analysis of the dynamics of interaction between the counter propagated beams, for example, in the medium with local response, have shown the existence of bistable dependence of the power received by the aperture on the incident power and on the shape of the optical pulse. We shall now look into the problem. (Results on interaction of the counter propagating beams obtained by the author in cooperation with I.G. Zakharova and Yu.N. Karamzin and will be published elsewhere). In the case of significant difference between the lifetimes of lattices of dielectric constants induced by the interacting waves with different spatial periods (which is typical for semiconductors), the interaction of counter propagated pulses in the medium with local response is described by the following system of dimensionless differential variables:

$$\frac{\partial A_+}{\partial t} + \frac{\partial A_+}{\partial z} + i\Delta_\perp A_+ + i\alpha\left(|A_+|^2 + |A_-|^2\right)A_+ = 0 ,$$

$$\frac{\partial A_-}{\partial t} - \frac{\partial A_-}{\partial z} + i\Delta_\perp A_+ + i\alpha\left(|A_+|^2 + |A_-|^2\right)A_- = 0 \tag{3}$$

with the boundary conditions

$$A_+|_{z=0} = \begin{cases} \exp(-((t-t_0)/\tau_p)^2 - x^2(1+i\theta)), \ t \le 2, \ t \ge 7, \\ 1 , \end{cases}$$

$$A_-|_{z=z_0} = A_+|_{z=z_0}(1 - \exp(-(x/R_a)^2)) \exp(I \arctan(x^2/R_m)),$$
$$2 < t < 7 , \tag{4}$$

where $A_\pm$ are the complex amplitudes of the beams, incident upon and reflected from the receiver positioned in the cross section $z_0 = 1$ of the beams and normalized by their maxima, t is time normalized by the time of the pulse propagation to the receiver, $\Delta_\perp = \partial^2/\partial x^2$ is the Laplacian operator with respect to the variable x (for simplicity, we analyze the case of propagation of the slit–shaped beams), R_a is the size of the receiving aperture expressed in units of the initial beam radius, R_m is the curvature of the mirror, τ_p is the parameter characterizing the pulse length and the steepness of the edges of the pulse, and θ characterizes beam focusing. It is apparently that when writing Eq. (4) it was assumed that the power of the beam either is absorbed by the receiver or passes through the aperture. Note that the above–described scheme of wave interaction was used by several authors in the experiments on the transverse optical bistability.

Hysteresis dependences of the output power $P_{out}(t)$ received by a Gaussian aperture of radius R_a on the initial beam power $P_{in}(t)$ were found for a wide ranges of variation of the parameters α, R_m, θ, z_0, R_a, and τ_p. Such a typical dependence is shown in Fig. 1, in which the arrows indicate the time variation of the initial power of optical radiation. The width of the loop and the number of such loops may be controlled by changing the nonlinearity parameter, the beam focusing, the radius of the aperture, and the curvature of the reflecting mirror. Analogous dependences were found for the beam power reflected back into the medium. The reason for such a dependence of the output power on the input one is the presence of a feedback between the incident and the reflected waves. For example, increasing the power of the incident beam intensifies its defocusing. As a result, the output power at the aperture R_a decreases and the fraction of power reflected back into the medium increases which again intensifies defocusing. Note that the central part of the incident beam is not reflected so that the incident optical radiation is focused with the lens induced by the reflected beam due to an intensity minimum on its axis, so that the power of optical radiation entering the aperture R_a increases.

Competition between these two processes results in complicated dependence of P_{out} on P_{in} which is shown in the figure. It can be easily seen that without the dynamic model, when we use the iterative procedure to find the solution, it may not converge, and the power will oscillate, for example, reproducing its own value in one iteration.

It should be also noted that another characteristic time is present in the dynamic model, that is, time τ_d of the pulse propagation from source to receiver and in the reverse direction. For a closed system, in which the information about the reflected beam is used to focus the beam, that time plays the role of the delay time.[27,28] When the ratio of time τ_d to the time constant of adaptation τ_a does not satisfy the condition

$$\tau_d/\tau_a \leq 0.37 , \qquad (5)$$

oscillating (or divergent) regimes of operation of the adaptive system are realized under conditions of the continuous algorithm for control in the system of cross–aperture sensing. Hence the adaptation constant must be limited from below for the stabilization of self–focusing. Note that analogous limitation (however originating from the condition of the maximum speed) appears during dynamic compensation for the beam self–action[29] at $\tau_d = 0$.

If, on the other hand, the times are related to each other so that the condition

$$\tau_d/\tau_a = \pi/2 + 2\pi n , \qquad n = 0, 1, 2, ..., \qquad (6)$$

is satisfied, the adaptive system realizes a periodic regime of variation of the optimization parameters. For the values of τ_a and τ_d satisfying the inequality

$$2\pi n < \tau_d/\tau_a < \pi/2 + 2\pi n , \qquad n = 1, 2, ..., \qquad (7)$$

the beam is focused according to the principles of damped oscillations, and at times from the interval

$$\pi/2 + 2\pi n < \tau_d/\tau_a < \pi(2n + 1) , \qquad n = 1, ... \qquad (8)$$

the process of focusing diverges. Thus periodic intervals of instability occur in a system with time delay.

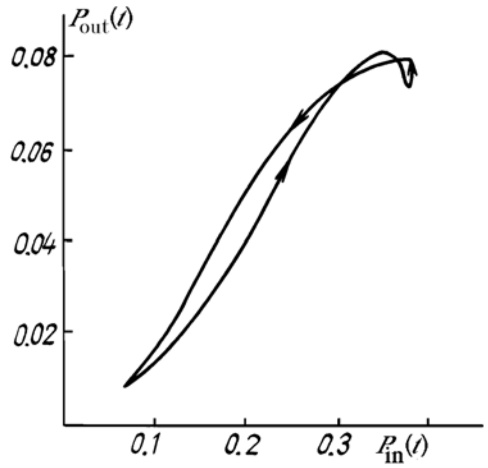

FIG. 1 The dependence of power $P_{out}(t)$ received by the aperture of $R_a = 0.66$ on the initial power $P_{in}(t)$ for a Gaussian pulse. The parameters of the problem are as follows: $\alpha = 20$, $z_0 = 1$, $\tau_p = 2$, $t_0 = 2$, $\theta = 0$, $R_m = 0.33$.

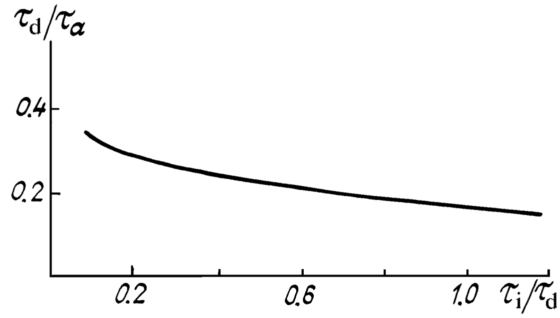

FIG. 2. The dependence of the ratio τ_d/τ_a, at which the maximum speed is attained in the inertial adaptive system, on the inertial time delay τ_i/τ_d.

The presence of inertia in the elements of the adaptive system, characterized by the time τ_i, results in a higher stability of focusing, on the one hand, and on the other – in slower speed of the system at greater times τ_i (see Fig. 2). Note that the fast adaptive system ($\tau_a < \tau_d$) may have periodic variations in the beam parameters with the spectrum that consists of $2n + 1$ harmonics, provided the times satisfy the condition

$$\tau_a \tau_i \tau_d^{-2} = \left(\frac{3\pi}{2} + 2\pi(n + 1) \right)^{-2} . \qquad (9)$$

Hence, with decreasing of τ_a the number of those harmonics grows as $1 + 2n$.

In conclusion of discussion of the problems associated with the algorithms for control we briefly consider some more important questions. It is well known that a speed of the system is very important for both adaptive and programmable control, because it is this characteristic which determines the quality of beam focusing onto the moving receiver during nonstationary self–action. Note that various techniques used to develop a faster adaptive control (and to attain its maximum speed) are minutely described in Ref. 10, in which the algorithms for control of focusing and the wavefront tilts at a maximum possible speed have been summarized. It is important to emphasize that such algorithms admit no further improvement in the class of gradient methods.

Another comment is about the way of organizing control described in Ref. 30. The problems of stability of control according several simultaneous criteria of quality for different components of the vector $\{\theta_i\}$ (focusing and tilt) were touched upon elsewhere.[31,32] In contrast to Ref. 30, those studies were based on the gradient method, which was preferable due to the following reasons. First, for the gradient method it is well known the approach which can be used to obtain the maximum speed and the optimal operation regime of the adaptive system. Second, in contrast to Ref. 30, it is (in general) not necessary to invert the matrix of gradients of functionals relative to the control parameters, which would otherwise considerably complicate the operation of the control system, as in the case of nonlinear propagation. Third, it is not necessary to know the extremal criteria, which are well known only for the linear medium. Fourth, the used iterative process[30] is a three–point process, which also complicates the operation of the adaptive system. (This follows, for example, from Ref. 31, which suggests and analyzes three–point algorithms based on the gradient method). It is also important to emphasize that optimal conditions may only be realized in the case in which the extremal value of the functional is known (apparently, it may only be found *a priori* for a linear medium). Otherwise, as demonstrated by the analysis, if the parameter being optimized enters a certain interval around its optimum, the process of adaptation may be ceased. The length of the interval is determined by the error at which the extremum of the functional is prescribed.

TABLE I. The dependence of the power of a Gaussian beam received by the aperture of R_a = 0.25 on the integration step along the longitudinal coordinate in the course of light beam focusing by the algorithm of phase conjugation.

h_z	0.1	0.083	0.05	0.04	0.01	0.005
P	0.164	0.171	0.192	0.2	0.223	0.238

It should be also noted that the analysis of focusing of the Gaussian beam onto the Gaussian receiving aperture with R_a = 0.25, positioned at the beam cross section z = 1 of the linear medium, by the phase conjugation algorithm has shown that, depending on the step of integration along the longitudinal coordinate with the fixed cubic grid along the transverse coordinate, various concentrations of power could be attained at the receiver (see Table I). As for the nonlinear propagation (for example, in the cubic medium) various oscillating regimes were realized, in addition, the average level of these oscillations was determined by the step of integration along the longitudinal coordinate. Thus, to find a final solution of the problem of the nature of such oscillations in the quality criterion of the algorithm of phase conjugation one should model the process of focusing at the nodes of grids adaptable to the solution.

The last comment is about the use of aberration–free description of beam propagation to analyze various algorithms for control. The dependences found from such an analysis were repeatedly confirmed by our numerical experiments. Similar confirmations may now be found in papers of other authors (see Ref. 7, for example), which yield the dependences of optimization efficiency on the path length and on the parameter of nonlinearity. It may be easily seen that they agree quite well with similar dependences reported in Refs. 34 and 35.

REFERENCES

1. G.W. Hardy, Proc. IEEE **66**, No. 6, 31–85 (1978).
2. E.A. Vitrichenko, ed., *Adaptive Optics* [Russian translation], (Mir, Moscow, 1980).
3. V.I. Bespalov and G.A. Pasmanik, *Nonlinear Optics and Adaptive Laser Systems* (Nauka, Moscow, 1986), 134 pp.
4. M.A. Voronstsov and V.I. Shmal'gauzen, *Principles of Adaptive Optics* (Nauka, Moscow, 1986), 336 pp.
5. V.P. Lukin, *Atmospheric Adaptive Optics* (Nauka, Moscow, 1986), 284 pp.
6. K.V. Shishakov and V.I. Shmal'gausen, Atm. Opt. **2**, No. 3, 263–265 (1989).
7. F.Yu. Kanev and G.S. Chesnokov, Atm. Opt. **2**, No. 3, 243–244 (1989).
8. V.P. Chebotarev, ed., *Laser Systems* (Nauka, Moscow, 1986), 201 pp.
9. V.M. Shmelev, V.I. Zakharov, and A.I. Nesterenko, Atm. Opt. **2**, No. 6, 489–494 (1989).
10. V.A. Trofimov, Atm. Opt. **3**, No. 1, 57–65 (1990).
11. Yu.N. Karamzin, A.P. Sukhorukov, and V.A. Trofimov, *Mathematical Modeling in Nonlinear Optics* (Moscow Univ. Publ., Moscow, 1989), 154 pp.
12. V.A. Vysloukh et al., Izv. Vyssh. Uchebn. Zaved., Ser. Fizika **28**, No. 11, 30–41 (1985).
13. A.P. Sukhorukov and V.A. Trofimov, Kvant. Elektron. **14**, No. 11, 2261–2263 (1987).
14. A.P. Sukhorukov and V.A. Trofimov, Izv. Akad. Nauk SSSR, Ser. Fizika **52**, No. 2, 377–382 (1988).
15. S.A. Akhmanov et al., Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **23**, No. 1, 3–37 (1980).
16. M.A. Vorontsov and S.S. Chesnokov, Izv. Vyssh. Uchebn. Zaved., Ser. Fizika, No. 10, 15–20 (1980).
17. A.I. Safonov, Avtometriya, No. 2, 13–17 (1981).
18. Yu.N. Karamzin and A.P. Sukhorukov, Izv. Akad. Nauk, Ser. Fizika **42**, No. 12, 2547–2559 (1978).
19. R.P. Fedorenko, *Approximate Solution of Problems of Optimal Control* (Nauka, Moscow, 1978), 408 pp.
20. V.A. Trofimov, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **31**, No. 9, 1146–1147 (1988).
21. S.V. Polyakov and V.A. Trofimov, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **32**, No. 10, 1034–1036 (1989).
22. V.A. Trofimov, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **33** (in print).
23. A.P. Sukhorukov and V.A. Trofimov, Izv. Akad. Nauk SSSR, Ser. Fiz. **51**, No. 2, 340–346 (1987).
24. V.I. Kovalev and V.A. Trofimov, in: *Abstracts of Reports at the Fifth All–Union Conference on Laser Optics* (Leningrad, 1987), p. 357.
25. Yu.N. Karamzin and Yu.B. Konev, "*Numerical modeling of performance of unstable telescopic resonators with an account of diffraction and saturation effect in active media*", Preprint No. 57, M.B. Keldysh Institute of Applied Mathematics of the Academy of Sciences of the USSR, Moscow (1974), 24 pp.
26. I.G. Zakharova and Yu.N. Karamzin, Differential'nye Uravneniya **20**, No. 7, 1213–1221 (1984).
27. V.A. Trofimov, Opt. Spektrosk. **59**, No. 5, 1153–1155 (1985).
28. V.A. Trofimov, Avtometriya No. 2, 29–33 (1987).
29. V.A. Trofimov, Kvant. Elektron. **15**, No. 11, 2379–2381 (1988).
30. V.P. Kandidov et al., Atm. Opt. **2**, No. 11, 1009–1014 (1989).
31. V.A. Trofimov, "*Optimal and adaptive control of the parameters of light beams propagating through media with nonlinear refractive index*", Candidate's Dissertation in Physical and Mathematical Sciences, Moscow (1983), 162 pp.
32. I.N. Kozhevnikova, A.P. Sukhorukov, and V.A. Trofimov, Izv. Vyssh. Uchebn. Zaved., Ser. Fizika **28**, No. 2, 13–19 (1985).
33. V.A. Trofimov, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **30**, No. 4, 505–515 (1987).
34. A.P. Sukhorukov and V.A. Trofimov, Izv. Akad. Nauk SSSR, Ser. Fiz. **46**, No. 10, 1933–1938 (1982).
35. Yu.N. Karamzin, A.P. Sukhorukov, and V.A. Trofimov, Izv. Vyssh. Uchebn. Zaved., Ser. Fizika **48**, No. 7, 1424–1429 (1984).